CHAPTER 4. REINFORCED CONCRETE DESIGN

CHAPTER 4

REINFORCED CONCRETE DESIGN

INTRODUCTION

4-1. Purpose

The purpose of this manual is to present methods of design for protective construction used in facilities for development, testing, production, storage, maintenance, modification, inspection, demilitarization, and disposal of explosive materials.

4-2. Objective

The primary objectives are to establish design procedures and construction techniques whereby propagation of explosion (from one structure or part of a structure to another) or mass detonation can be prevented and to provide protection for personnel and valuable equipment.

The secondary objectives are to:

(1) Establish the blast load parameters required for design of protective structures.
(2) Provide methods for calculating the dynamic response of structural elements including reinforced concrete, and structural steel.
(3) Establish construction details and procedures necessary to afford the required strength to resist the applied blast loads.
(4) Establish guidelines for siting explosive facilities to obtain maximum cost effectiveness in both the planning and structural arrangements, providing closures, and preventing damage to interior portions of structures because of structural motion, shock, and fragment perforation.

4-3. Background

For the first 60 years of the 20th century, criteria and methods based upon results of catastrophic events were used for the design of explosive facilities. The criteria and methods did not include a detailed or reliable quantitative basis for assessing the degree of protection afforded by the protective facility. In the late 1960's quantitative procedures were set forth in the first edition of the present manual, "Structures to Resist the Effects of Accidental Explosions". This manual was based on extensive research and development programs which permitted a more reliable approach to current and future design requirements. Since the original publication of this manual, more extensive testing and development programs have taken place. This additional research included work with materials other than reinforced concrete which was the principal construction material referenced in the initial version of the manual.

Modern methods for the manufacture and storage of explosive materials, which include many exotic chemicals, fuels, and propellants, require less space for a given quantity of explosive material than was previously needed. Such concentration of explosives increases the possibility of the propagation of accidental explosions. (One accidental explosion causing the detonation of other explosive materials.) It is evident that a requirement for more accurate design techniques is essential. This manual describes rational design methods to provide the required structural protection.

These design methods account for the close-in effects of a detonation including the high pressures and the nonuniformity of blast loading on protective structures or barriers. These methods also account for intermediate and far-range effects for the design of structures located away from the explosion. The dynamic response of structures, constructed of various materials, or combination of materials, can be calculated, and details are given to provide the strength and ductility required by the design. The design approach is directed primarily toward protective structures subjected to the effects of a high explosive detonation. However, this approach is general, and it is applicable to the design of other explosive environments as well as other explosive materials as mentioned above.

The design techniques set forth in this manual are based upon the results of numerous full- and small-scale structural response and explosive effects tests of various materials conducted in conjunction with the development of this manual and/or related projects.

4-4. Scope

It is not the intent of this manual to establish safety criteria. Applicable documents should be consulted for this purpose. Response predictions for personnel and equipment are included for information.

In this manual an effort is made to cover the more probable design situations. However, sufficient general information on protective design techniques has been included in order that application of the basic theory can be made to situations other than those which were fully considered.

This manual is applicable to the design of protective structures subjected to the effects associated with high explosive detonations. For these design situations, the manual will apply for explosive quantities less than 25,000 pounds for close-in effects. However, this manual is also applicable to other situations such as far- or intermediate-range effects. For these latter cases the design procedures are applicable for explosive quantities in the order of 500,000 pounds which is the maximum quantity of high explosive approved for aboveground storage facilities in the Department of Defense manual, "Ammunition and Explosives Safety Standards", DOD 6055.9-STD. Since tests were primarily directed toward the response of structural steel and reinforced concrete elements to blast overpressures, this manual concentrates on design procedures and techniques for these materials. However, this does not imply that concrete and steel are the only useful materials for protective construction. Tests to establish the response of wood, brick blocks, and plastics, as well as the blast attenuating and mass effects of soil are contemplated. The results of these tests may require, at a later date, the supplementation of these design methods for these and other materials.

Other manuals are available to design protective structures against the effects of high explosive or nuclear detonations. The procedures in these manuals will quite often complement this manual and should be consulted for specific applications.

Computer programs, which are consistent with procedures and techniques contained in the manual, have been approved by the appropriate representative of the US Army, the US Navy, the US Air Force and the Department of Defense Explosives Safety Board (DDESB). These programs are available through the following repositories:

(1) Department of the Army
Commander and Director
U.S. Army Engineer
Waterways Experiment Station
Post Office Box 631
Vicksburg, Mississippi 39180-0631
Attn: WESKA

(2) Department of the Navy
Commanding Officer
Naval Civil Engineering Laboratory
Port Hueneme, California 93043
Attn: Code L51

(3) Department of the Air Force
Aerospace Structures
Information and Analysis Center
Wright Patterson Air Force Base
Ohio 45433
Attn: AFFDL/FBR

If any modifications to these programs are required, they will be submitted for review by DDESB and the above services. Upon concurrence of the revisions, the necessary changes will be made and notification of the changes will be made by the individual repositories.

4-5. Format

This manual is subdivided into six specific chapters dealing with various aspects of design. The titles of these chapters are as follows:

Chapter 1	Introduction
Chapter 2	Blast, Fragment, and Shock Loads
Chapter 3	Principles of Dynamic Analysis
Chapter 4	Reinforced Concrete Design
Chapter 5	Structural Steel Design
Chapter 6	Special Considerations in Explosive Facility Design

When applicable, illustrative examples are included in the Appendices.

Commonly accepted symbols are used as much as possible. However, protective design involves many different scientific and engineering fields, and, therefore, no attempt is made to standardize completely all the symbols used. Each symbol is defined where it is first used, and in the list of symbols at the end of each chapter.

CHAPTER CONTENTS

4-6. General

This chapter is concerned with the design of above ground blast resistant concrete structures. Procedures are presented to obtain the dynamic strength of the various structural components of concrete structures. Except for the particular case of the design of laced reinforced concrete elements, the dynamic analysis of the structural components is presented in Chapter 3.

The dynamic strengths of both the concrete and reinforcement under various stress conditions are given for the applicable design range and the allowable deflection range. Using these strengths, the ultimate dynamic capacity of various concrete elements are given. These capacities include the ultimate moment capacity for various possible cross-section types, ultimate shear capacity as a measure of diagonal tension as well as ultimate direct shear and punching shear, torsion capacity of beams, and the development of the reinforcement through bond with the concrete.

This chapter contains procedures for the design of non-laced (conventional reinforcement) and laced concrete slabs and walls as well as procedures for the design of flat slabs, beams and columns. Procedures are presented for the design of laced and non-laced slabs and beams for close-in effects whereas procedures for the design of non-laced and flat slabs, beams and columns are given for far range effects. It is not economical to use laced slabs for far range effects. Design procedures are given for the flexural response of one- and two-way non-laced slabs, beams and flat slabs which undergo limited deflections. Procedures are also given for large deflections of these elements when they undergo tensile membrane action. Laced reinforced slabs are designed for flexural action for both limited and large deflections. Lastly, the design of columns is presented for elastic or, at best, slight plastic action.

The above design procedures are concerned with the ductile response of structural elements. Procedures are also given for the brittle mode response of concrete elements. The occurrence of both spalling and scabbing of the concrete as well as protection against their effects is treated. In addition, procedures are presented for post-failure fragment design of laced concrete walls and slabs. The resistance of concrete elements to primary fragment impact is considered. For the primary fragments determined in Chapter 2, methods are presented to determine if a fragment is embedded in or perforates a concrete wall. If embedment occurs, the depth of penetration is determined and the occurrence of spalling of the far face can be evaluated. If perforation occurs, the residual velocity of the fragment is determined.

Required construction details and procedures for conventionally reinforced and laced reinforced concrete structures is the last item discussed in the chapter. Conformance to these details will insure a ductile response of the structure to the applied dynamic loads.

BASIS FOR STRUCTURAL DESIGN

4-7. General

Explosive storage and operating facilities are designed to provide a predetermined level of protection against the hazards of accidental explosions. The type of protective structure depends upon both the donor and acceptor systems. The donor system (amount, type and location of the potentially detonating explosives) produces the damaging output while the acceptor system (personnel, equipment, and "acceptor" explosives) requires a level of protection. The protective structure or structural elements are designed to shield against or attenuate the hazardous effects to levels which are tolerable to the acceptor system.

Protective concrete structures are classified as either shelters or barriers. Shelters enclose the receiver system and are generally located far from a potential explosion. Barriers, on the other hand, generally enclose the donor system and, consequently, are located close to the potential explosion. A shelter is a fully enclosed structure which is designed to prevent its contents (acceptor system) from being subjected to the direct effects of blast pressures and fragments. A barrier may be either a fully enclosed structure (containment structure) or an open structure (barricade or cubicle type structure in which one or more surfaces are frangible or open to the atmosphere). Barriers are generally designed to resist close-in detonations. Their purpose is to prevent acceptor explosives, and to a lesser extent, personnel and equipment from being subjected to primary fragment impact and to attenuate blast pressures in accordance with the structural configuration of the barrier.

4-8. Modes of Structural Behavior

The response of a concrete element can be expressed in terms of two modes of structural behavior; ductile and brittle. In the ductile mode of response the element may attain large inelastic deflections without complete collapse. While, in the brittle mode, partial failure or total collapse of the element occurs. The selected behavior of an element for a particular design is governed by: (1) the magnitude and duration of the blast output, (2) the occurrence of primary fragments, and (3) the function of the protective structure, i.e., shelter or barrier depending upon the protection level required.

4-9. Structural Behavior of Reinforced Concrete

4-9.1. General

When a reinforced concrete element is dynamically loaded, the element deflects until such time that: (1) the strain energy of the element is developed sufficiently to balance the kinetic energy produced by the blast load and the element comes to rest, or (2) fragmentation of the concrete occurs resulting in either partial or total collapse of the element. The maximum deflection attainable is a function of the span of the element, the depth of the element, and the type, amount, and details of the reinforcement used in a particular design.

The resistance-deflection curve shown in Figure 4-1 demonstrates the flexural action of a reinforced concrete element. When the element is first loaded, the resistance ideally increases linearly with deflection until yielding of the reinforcement is first initiated. As the element continues to deflect, all the reinforcing steel yields and the resistance is constant with increasing deflection. Within this yield range at a deflection corresponding to 2 degrees support rotation, the compression concrete crushes. For elements without shear reinforcement, this crushing of the concrete results in failure of the element. For elements with shear reinforcement (single leg stirrups shown in Figure 4-2 or lacing shown in Figure 4-3) which properly tie the flexural reinforcement, the crushing of the concrete results in a slight loss of capacity since the compressive force is transferred to the compression reinforcement. As the element is further deflected, the reinforcement enters into its strain hardening region, and the resistance increases with increasing deflection. Single leg stirrups will restrain the compression reinforcement for a short time into its strain hardening region. At four (4) degrees support rotation, the element loses its structural integrity and fails. On the other hand, lacing through its truss action will restrain the reinforcement through its entire strain hardening region until tension failure of the reinforcement occurs at 12 degrees support rotation.

Sufficient shear capacity must be afforded by the concrete alone or in combination with shear reinforcement in order to develop the flexural capacity of an element (Figure 4-1). An abrupt shear failure can occur at any time during the flexural response if the flexural capacity exceeds the shear capacity of the element.

4-9.2. Ductile Mode of Behavior in the Far Design Range

In the far design range, the distribution of the applied blast load is fairly uniform and the deflections required to absorb the loading are comparatively small. Conventionally reinforced (i.e., non-laced) concrete elements with comparatively minor changes to standard reinforcing details are perfectly adequate to resist such loads. While laced reinforcement could be used, it would be extremely uneconomical to do so.

The flexural response of non-laced reinforced concrete elements is demonstrated through the resistance-deflection curve of Figure 4-1. For elements without shear reinforcement, the ultimate deflection is limited to deflections corresponding to 2 degrees support rotation whereas elements with shear reinforcement are capable of attaining 4 degrees support rotation. For ease of construction, single leg stirrups (Figure 4-2) are used as shear reinforcement in slabs and walls. This type of reinforcement is capable of providing shear resistance as well as the necessary restraint of the flexural reinforcement to enable the slab to achieve this increased deflection.

A conventionally reinforced slab may attain substantially larger deflections than those corresponding to 4 degrees support rotations . These increased deflections are possible only if the element has sufficient lateral restraint to develop in-plane forces. The resistance-deflection curve of Figure 4-4 illustrates the structural response of an element having lateral restraint. Initially, the element behaves essentially as a flexural member. If the lateral restraint prevents small motions, in-plane compressive forces are developed. Under flexural action, the capacity is constant with increasing deflection until the compression concrete crushes. As the deflection increases further and the load carried by the slab decreases, membrane action in the slab is developed. The slab carries load by the reinforcement net acting as a plastic tensile membrane. The capacity of the element increases with increasing deflection until the reinforcement fails in tension.

4-9.3. Ductile Mode of Behavior in the Close-in Design Range

Close-in detonations produce nonuniform, high intensity blast load. Extremely high-pressure concentrations are developed which, in turn, can produce local (punching) failure of an element. To maintain the structural integrity of elements subjected to these loads and to permit the large deflections necessary to balance the kinetic energy produced, lacing reinforcement has been developed.

Lacing reinforcement is shown in Figure 4-3 while a typical laced wall is shown in Figure 4-5. A laced element is reinforced symmetrically, i.e., the compression reinforcement is the same as the tension reinforcement. The straight flexural reinforcing bars on each face of the element and the intervening concrete are tied together by the truss action of continuous bent diagonal bars. This system of lacing contributes to the integrity of the protective element in the following ways:

(1) Ductility of the flexural reinforcement, including the strain hardening region, is fully developed.
(2) Integrity of the concrete between the two layers of flexural reinforcement is maintained despite massive cracking.
(3) Compression reinforcement is restrained from buckling.
(4) High shear stresses at the supports are resisted.
(5) Local shear failure produced by the high intensity of the peak blast pressures is prevented.
(6) Quantity and velocity of post-failure fragments produced during the brittle mode of behavior are reduced.

The flexural response of a laced reinforced concrete element is illustrated by the entire resistance-deflection curve shown in Figure 4-1. The lacing permits the element to attain large deflections and fully develop the reinforcement through its strain hardening region. The maximum deflection of a laced element corresponds to 12 degrees support rotation.

Single leg stirrups contribute to the integrity of a protective element in much the same way as lacing, however, the stirrups are less effective at the closer explosive separation distances. The explosive charge must be located further away from an element containing stirrups than a laced element. In addition, the maximum deflection of an element with single leg stirrups is limited to 4 degrees support rotation under flexural action or 8 degrees under tension membrane action. If the charge location permits and reduced support rotations are required, elements with single leg stirrups may prove more economical than laced elements.

4-9.4. Brittle Mode of Behavior

The brittle behavior of reinforced concrete is composed of three types of concrete failure: direct spalling, scabbing and post-failure fragments. Direct spalling consists of the dynamic disengagement of the concrete cover over the flexural reinforcement due to high intensity blast pressures.

Scabbing also consists of the disengagement of the concrete cover over the flexural reinforcement, however, scabbing is due to the element attaining large deflections. Finally post-failure fragments are the result of the collapse of an element and are usually the more serious. Post-failure fragments are generally large in number and/or size with substantial velocities which can result in propagation of explosion. Spalling and scabbing are usually only of concern in those protective structures where personnel, equipment, or sensitive explosives require protection. Controlled post-failure fragments are only permitted where the acceptor system consists of relatively insensitive explosives.

The two types of spalling, direct spalling and scabbing, occur during the ductile mode of behavior. Because direct spalling is dependent upon the transmission of shock pressures, fragments formed from this type of spalling are produced immediately after the blast pressures strike the wall. Scabbing, on the other hand, occurs during the later stages of the flexural (ductile mode) action of the element. Both types of spalls affect the capacity of the element to resist the applied blast load.

Post-failure fragments are the result of a flexural failure of an element. The failure characteristics of laced and unlaced elements differ significantly. The size of failed sections of laced element is fixed by the location of the yield lines. The element fails at the yield lines and the section between yield lines remain intact. Consequently, failure of a laced element consists of a few large sections (Figure 4-6). On the other hand, failure of an unlaced element is a result of a loss of structural integrity and the fragments take the form of concrete rubble (Figure 4-7). The velocity of the post failure fragments from both laced and unlaced elements is a function of the amount of blast overload. However, tests have indicated that the fragment velocities of laced elements are as low as 30 percent of the maximum velocity of the rubble formed from similarly loaded unlaced elements.

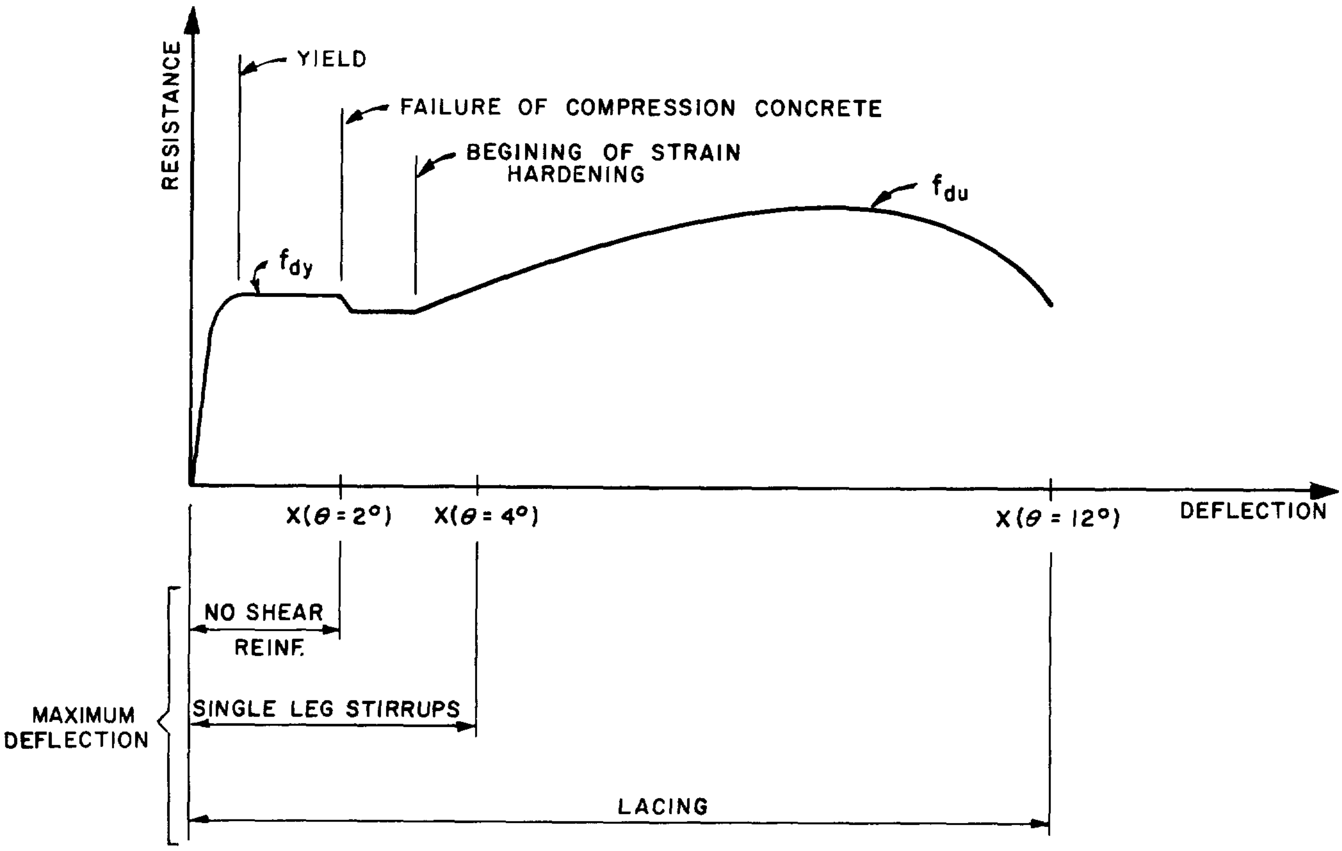


Figure 4-1 Typical resistance-deflection curve for flexural response of concrete elements

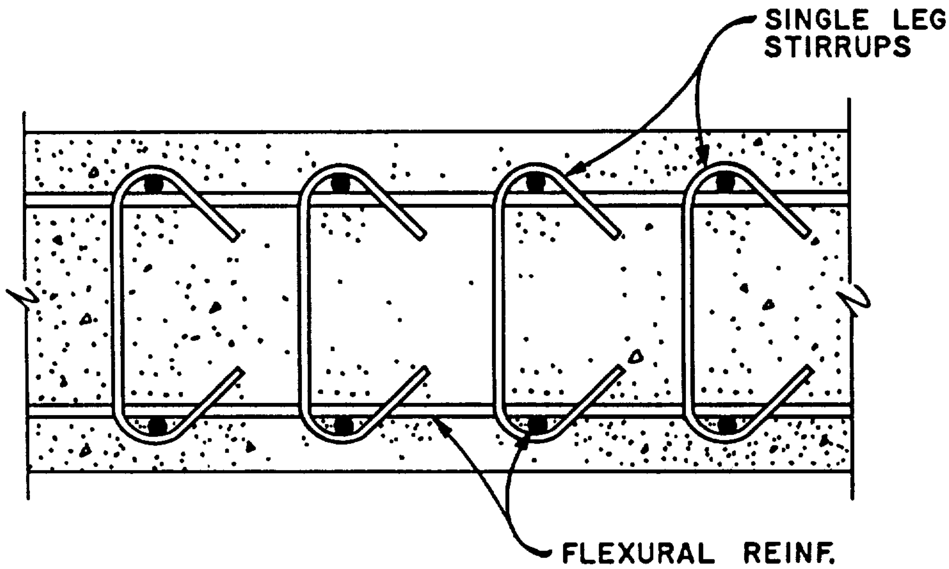


Figure 4-2 Element reinforced with single leg stirrups

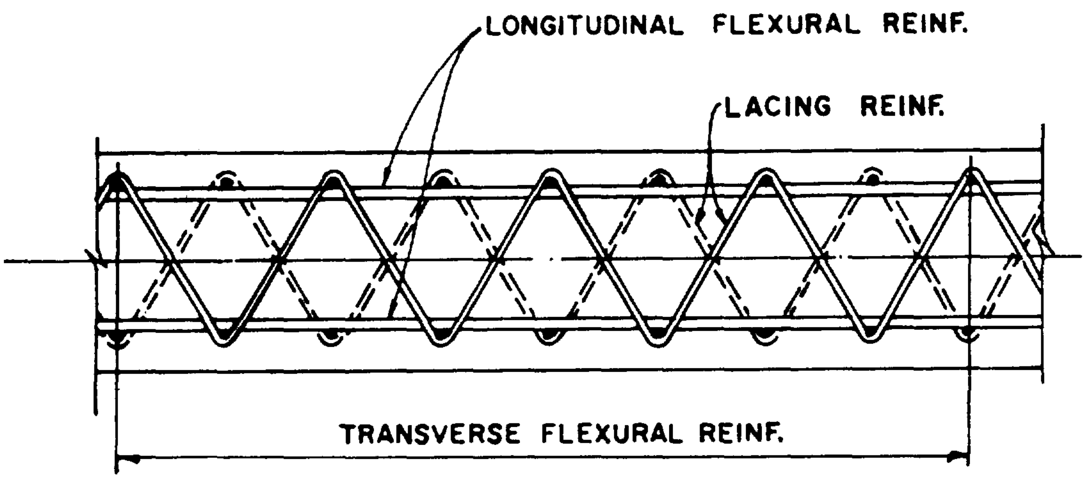


Figure 4-3 Lacing reinforcement

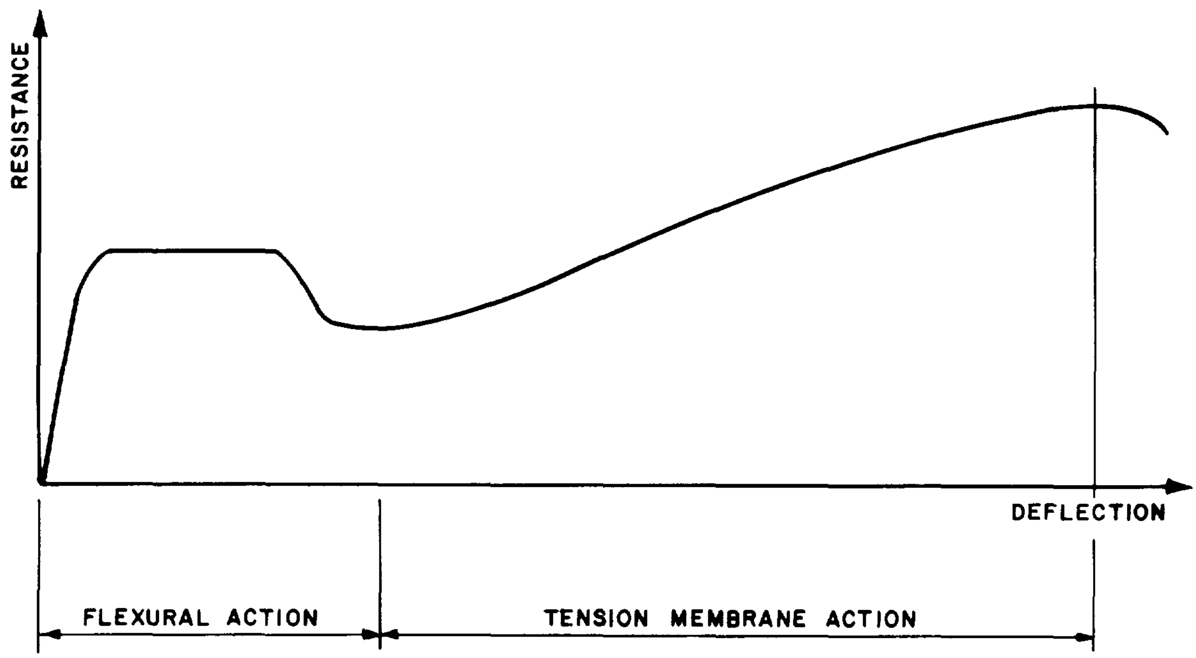


Figure 4-4 Typical resistance-deflection curve for tension membrane response of concrete

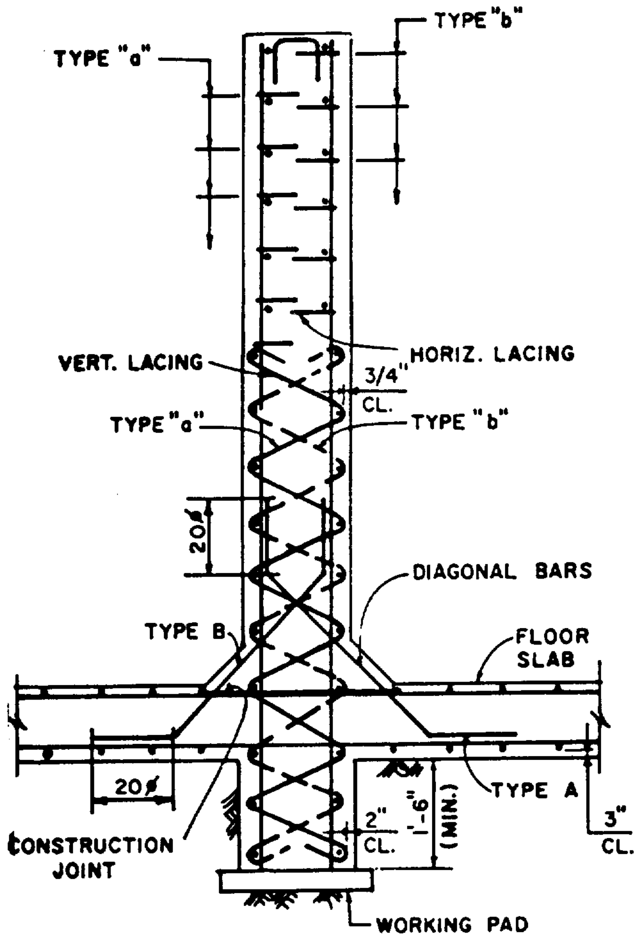


Figure 4-5 Typical laced wall

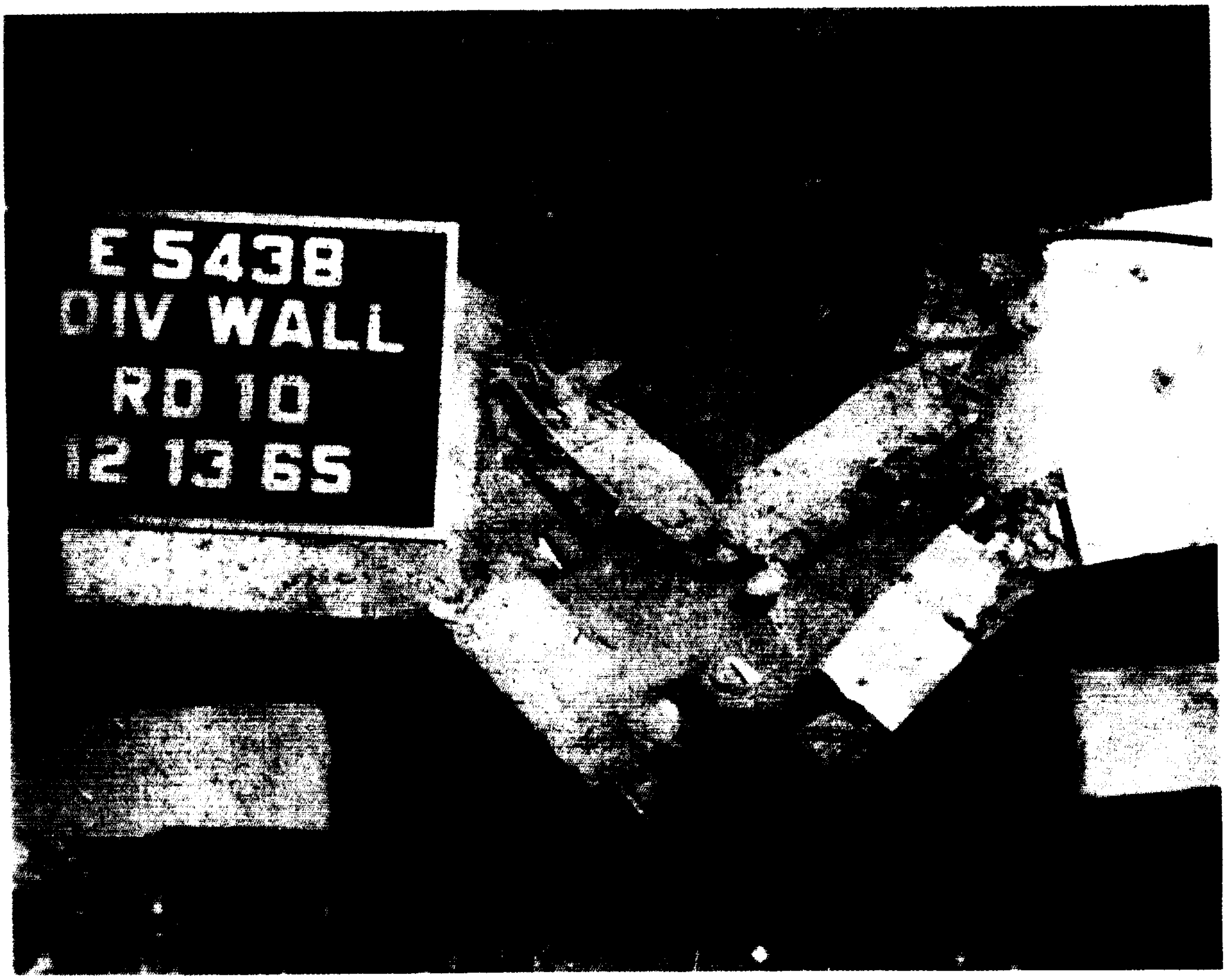


Figure 4-6   Failure of a laced element

Figure 4-7 Failure of an unlaced element

DYNAMIC STRENGTH OF MATERIALS

4-10. Introduction

A structural element subjected to a blast loading exhibits a higher strength than a similar element subjected to a static loading. This increase in strength for both the concrete and reinforcement is attributed to the rapid rates of strain that occur in dynamically loaded members. These increased stresses or dynamic strengths are used to calculate the element's dynamic resistance to the applied blast load. Thus, the dynamic ultimate resistance of an element subjected to a blast load is greater than its static ultimate resistance.

Both the concrete and reinforcing steel exhibit greater strength under rapid strain rates. The higher the strain rate, the higher the compressive strength of concrete and the higher the yield and ultimate strength of the reinforcement.

This phenomenon is accounted for in the design of a blast resistant structure by using dynamic stresses to calculate the dynamic ultimate resistance of the reinforced concrete members.

4-11. Stress-Strain Curve

Typical stress-strain curves for concrete and reinforcing steel are shown in Figure 4-8. The solid curves represent the stress-strain relationship for the materials when tested at the strain and loading rates specified in ASTM Standards. At a higher strain rate, their strength is greater, as illustrated by the dashed curves. Definitions of the symbols used in Figure 4-8 are as follows:

f'_c = static ultimate compressive strength of concrete

f'_{dc} = dynamic ultimate compressive strength of concrete

f_y = static yield stress of reinforcing steel

f_{dy} = dynamic yield stress of reinforcing steel

f_u = static ultimate stress of reinforcing steel

f_{du} = dynamic ultimate stress of reinforcing steel

E_s = modulus of elasticity for reinforcing steel

E_c = secant modulus of elasticity of concrete

ϵ_u = rupture strain

From the standpoint of structural behavior and design, the most important effect of strain rate is the increased yield and ultimate strengths of the reinforcement and the compressive strength of the concrete. For typical strain rates encountered in reinforced concrete elements subjected to blast loads, the increase in the yield strength of the steel and the compressive strength of the concrete is substantial. The ultimate strength of the reinforcement is much less sensitive to the strain rate. The increase in the ultimate strength is slight and the strain at which this stress occurs is slightly reduced. There is essentially no change with strain rate in the modulus of elasticity and rupture strain of the steel. In the case of concrete, as the strain rate increases the scant modulus of elasticity increases slightly, and the strain at maximum stress and rupture remain nearly constant.

4-12. Allowable Material Strengths

4-12.1. General

The behavior of a structural element subjected to a blast loading depends upon the ultimate strength and ductility of the materials from which it is constructed. The required strength of a ductile element is considerably less than that necessary for a brittle element to resist the same applied loading. A ductile element maintains its peak or near-peak strength through large plastic strains whereas a brittle element fails abruptly with little energy absorbed in the plastic range. Reinforced concrete with well tied and anchored ductile reinforcement can be classified as a ductile material.

4-12.2. Reinforcement

Reinforcing steel, designated by the American Society for Testing and Materials (ASTM) as A 615, Grade 60, is considered to have adequate ductility in sizes up to No. 11 bars. The large No. 14 bars also have the desired ductility, but their usage is somewhat restricted due to their special requirements of spacing and anchorage. No. 18 bars are not recommended for use in blast resistant structures. For all reinforcement, ductility is reduced at bends, lapped splices, mechanical splices, etc., and location of these anchorages near points of maximum stress is undesirable and should be avoided.

Reinforcing steel having a minimum yield of 75,000 psi can be produced having chemical properties similar to ASTM A 615, Grade 60. However, production of this steel requires a special order to be placed in which large quantities of individual bar sizes (in the order of 200 tons per bar size) must be ordered. It is recommended that for these high strength bars only straight lengths of bars be utilized, splicing of bars be avoided and application of this reinforcement be limited to members designated to attain an elastic response or a slightly plastic response (X_m/X_E less than or equal to 3).

It is desirable to know the stress-strain relationship for the reinforcement being utilized in order to calculate the ultimate resistance of an element. This information is not usually available; however, minimum values of the yield stress f_y and the ultimate tensile stress f_u are required by ASTM Standards. For ASTM A 615, Grade 60 reinforcement, the minimum yield and ultimate stresses are 60,000 psi and 90,000 psi, respectively. Review of numerous mill test reports for this steel indicate yield stresses at least 10 percent greater than the ASTM minimum, and ultimate stresses at least equal to but not much greater than the ASTM minimum. Therefore it is recommended that for design purposes, the minimum ASTM yield stress be increased by 10 percent while the minimum ASTM ultimate stress be used without any increase. So that, the recommended design values for ASTM A 615, Grade 60 reinforcement, are:

$$f_y = 66,000 \text{ psi}$$

and

$$f_u = 90{,}000 \text{ psi}$$

4-12.3. Concrete

Even though the magnitude of the concrete strength is only significant in the calculation of the ultimate strength of elements with support rotations less than 2 degrees, its effects on the behavior of elements with both small and large support rotations are of equal importance. The shear capacity of an element is dependent upon the magnitude of the concrete strength. For elements with small support rotations (less than 2 degrees), the use of higher strength concrete may eliminate the need for shear reinforcement; while for elements requiring shear reinforcement, the amount of reinforcement is reduced as the concrete strength is increased. For elements with large support rotations (2 to 12 degrees), the cracking and crushing of the concrete associated with the larger rotations is less severe when higher strength concrete is employed. Therefore, the strength of the concrete used in a blast resistant structure may be selected to suit the particular design requirements of the structure. However, under no circumstances should the concrete strength f'_c be less than 3,000 psi. It is recommended that 4,000 psi strength concrete be used in all blast resistant structures regardless of the magnitude of the blast load and deflection criteria.

4-13. Dynamic Design Stresses for Reinforced Concrete

4-13.1. General

Ductility is a significant parameter influencing the dynamic response and behavior of reinforced concrete members subjected to blast loadings. The importance of ductility increases as the duration of the blast load decreases relative to the natural period of the member. In general, to safely withstand a blast load, the required ultimate resistance decreases with increasing ductility of the member. In fact, the ultimate resistance required of ductile members is considerably less than that required for brittle members which fail abruptly with little energy absorbed in the plastic range of behavior.

A ductile member is one that develops plastic hinges in regions of maximum moment by first yielding of the tension reinforcement followed by crushing of the concrete. This behavior is typical of under-reinforced concrete sections. A section can be designed to be very ductile by maintaining an under-reinforced section, adding compression reinforcement, and utilizing lacing bars to prevent buckling of the compression reinforcement. For a laced section, the reinforcement is stressed through its entire strain-hardening region, that is, the steel reaches its ultimate stress f_{du} and fails at its rupture strain ϵ_u. In a flexural member, the straining of the reinforcement, and consequently its stress, is expressed in terms of its angular support rotations.

4-13.2. Dynamic Increase Factor

The dynamic increase factor, DIF, is equal to the ratio of the dynamic stress to the static stress, e.g., f_{dy}/f_y, f_{du}/f_u and f'_{dc}/f'_c. The DIF depends upon the rate of strain of the element, increasing as the strain rate increases. The design curves for the DIF for the unconfined compressive strength of concrete and for the yield stress of ASTM A 615, Grade 60, reinforcing steel, are given in Figure 4-9 and 4-10, respectively. The curves were derived from test data having a maximum strain rate of 10 x 10^{-3} in./in./msec. for concrete and 2.1 x 10^{-3} in./in./msec. for steel. Values taken from these design curves are conservative estimates of DIF and safe for design purposes.

Values of DIF have been established for design of members in the far design range as well as for members in the close-in design range. These design values of DIF are given in Table 4-1. Because of the increased magnitude of the blast loads and subsequent increase in the strain rate, the dynamic increase factors for elements subjected to a close-in detonation are higher than those for elements subjected to an explosion located far from the element.

The design values of DIF presented in Table 4-1 vary not only for the design ranges and type of material but also with the state of stress (bending, diagonal tension, direct shear, bond, and compression) in the material. The values for f_{dy}/f_y and f'_{dc}/f'_c for reinforced concrete members in bending assume the strain rates in the reinforcement and concrete are 0.0001 in./in./ msec. for the far design range and 0.0003 in./in./ msec. in the close-in design range. For members in compression (columns). these strain rates are 0.0002 in./in./msec. and 0.0005 in./in./msec. The lower strain rates in compression (compared to bending) account for the fact that slabs, beams and girders "filter" the dynamic effects of the blast load. Thus, the dynamic load reaching columns is typically a fast "static" load (long rise time of load) which results in lower strain rates in columns. These strain rates and the corresponding values of DIF in Table 4-1 are considered safe values for design purposes.

Available data is not sufficient to permit the construction of a design curve for the DIF for the ultimate stress of ASTM A 615 Grade 60 reinforcing steel. However, it is known that the increase in the ultimate strength of the steel is small and, therefore, not a significant factor in the design of reinforced concrete members. A nominal value of the DIF is given in Table 4-1.

The listed values of DIF for shear (diagonal tension and direct shear) and bond are more conservative than for bending or compression. This conservatism is justified by the need to prevent brittle shear and bond failure and to account for uncertainties in the design process for shear and bond.

A more accurate estimate of the DIF may be obtained utilizing the DIF design curve for concrete and steel given in Figure 4-9 and 4-10, respectively. The increase in capacity of flexural elements is primarily a function of the rate of strain or the reinforcement, in particular, the time to reach yield, t_E, of the reinforcing steel. The average rate of strain for both the concrete and steel may be obtained considering the strain in the materials at yield and the time to reach yield. The member is first designed (procedures given in subsequent sections) using the DIF values given in Table 4-1. The time to reach yield, t_E, is then calculated using the response charts presented in Chapter 2. For the value t_E, the average strain rate in the materials can be obtained. The average strain rate in the concrete (based on f'_{dc} being reached at ϵ_c = 9,992 in./in.) is:

$$\epsilon'_c = 0.002 / t_E \qquad 4\text{-}1$$

while the average strain rate in the reinforcement is:

$$\epsilon'_s = f_{dy} / E_s t_E \qquad 4\text{-}2$$

where

ϵ'_c = average strain rate for concrete

ϵ'_s = average strain rate for reinforcement

t_E = time to yield the reinforcement

For the strain rates obtained from Equations 4-1 and 4-2, the actual DIF is obtained for the concrete and reinforcement from Figure 4-9 and 4-10, respectively. If the difference between the calculated DIF values and the design values of Table 4-1 are small, then the correct values of DIF are those calculated. If the difference is large, the calculated values of DIF are used as new estimates and the process is repeated until the differences between the "estimated" and "calculated" values of DIF are small. The process converges very rapidly and, in most cases, the second iteration of the process converges on the proper values of DIF.

In most cases, the values of DIF obtained from Table 4-1 are satisfactory for design and the determination of the actual DIF values is unwarranted. However, the DIF values can significantly effect the final design of certain members, and the extra calculations required to obtain the actual DIF values are fully warranted. These include deep members, members subjected to impulse-type blast loads and members designed to sustain large deflections. The actual DIF values (usually higher than the design values of Table 4-1) result in a more realistic estimate of the ultimate flexural resistance and, therefore, the maximum shear and bond stresses which must be resisted by the member.

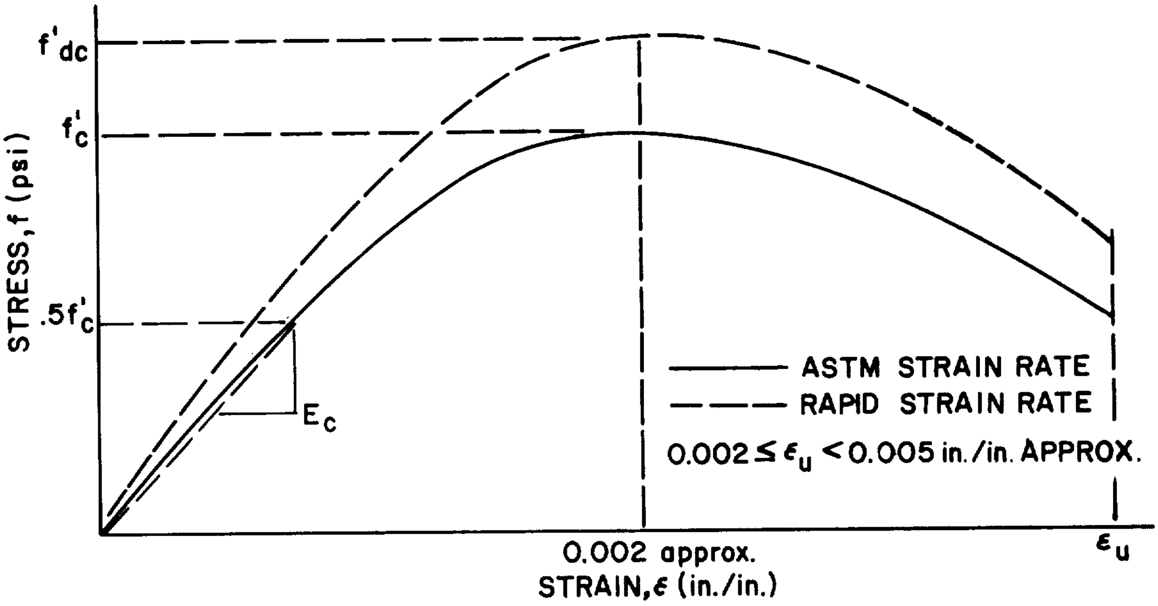
For the elasto-plastic or plastic design of concrete elements, an equivalent elastic curve is considered rather than the actual elasto-plastic resistance-deflection function. The time to reach yield t_E is computed based on this curve using the equivalent elastic deflection X_E and stiffness K_E. Actually, the reinforcement along the supports yield in less time than t_E whereas the reinforcement at mid-span yields at a time greater than t_E. These differences are compensating errors. Therefore, the time to reach yield t_E for the equivalent curve when used in Equations 4-1 and 4-2 produces an accurate average DIF for the concrete and reinforcement at the critical sections throughout a reinforced concrete element.

4-13.3. Dynamic Design Stresses

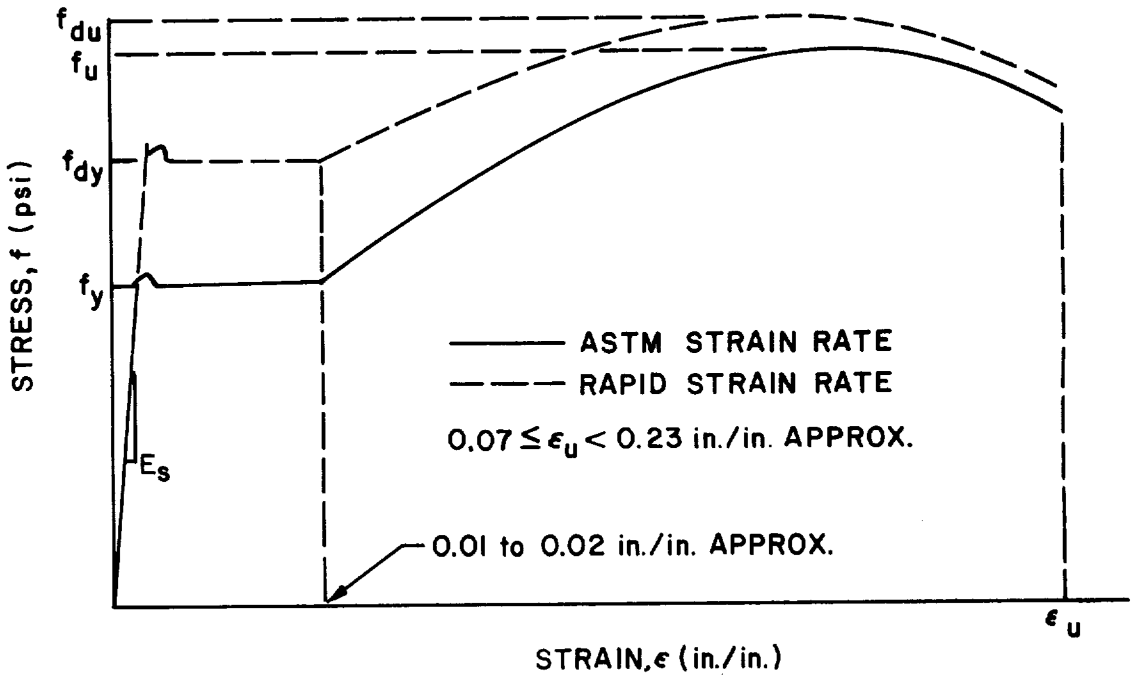
The magnitude of stresses produced in the reinforcement of an element responding in the elastic range can be related directly to the strains. However, in the plastic range the stresses cannot be related directly to the strains. An estimate of the average stress over portions of the plastic range can be made by relating this average stress to the deflection of the element. The deflection is defined in terms of the angular rotation at the supports. The average dynamic stress is expressed as a function of the dynamic yield stress f_{dy} and the dynamic ultimate stress f_{du}.

Criteria for the dynamic stresses to be used in the plastic design of ductile reinforced concrete elements are presented in Table 4-2. The dynamic design stress is expressed in terms of f_{dy}, f_{du}, and f'_{dc}. The value of these terms is determined by multiplying the appropriate static design stress by the appropriate value of the DIF (Table 4-1), so that:

$$f_{(dynamic)} = DIF \times f_{(static)} \qquad 4\text{-}3$$



(a) STRESS-STRAIN CURVE FOR CONCRETE



(b) STRESS-STRAIN CURVE FOR STEEL

Figure 4-8 Typical stress-strain curves for concrete and reinforcing steel

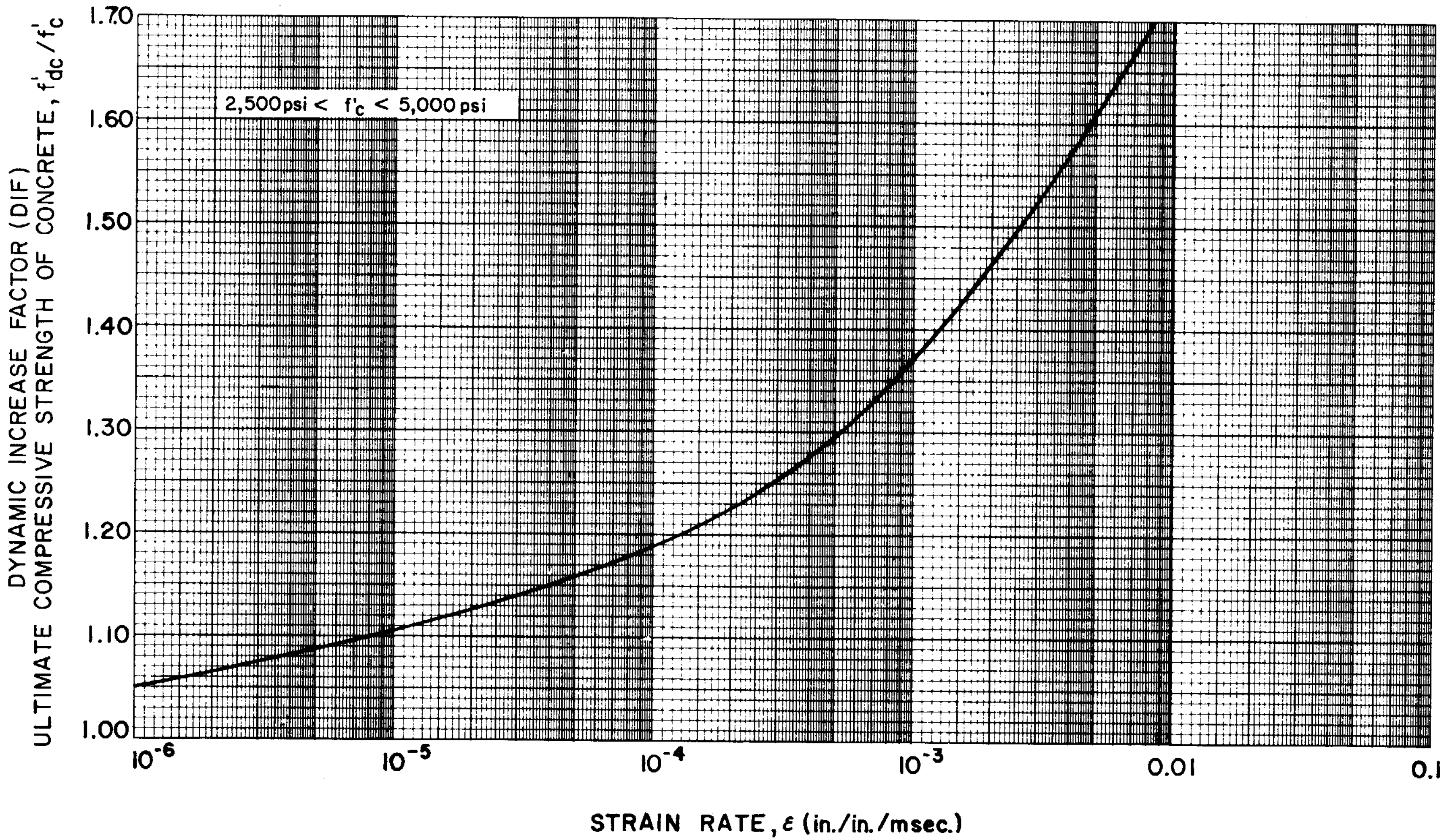


Figure 4-9 Design curve for DIF for ultimate compressive strength of concrete

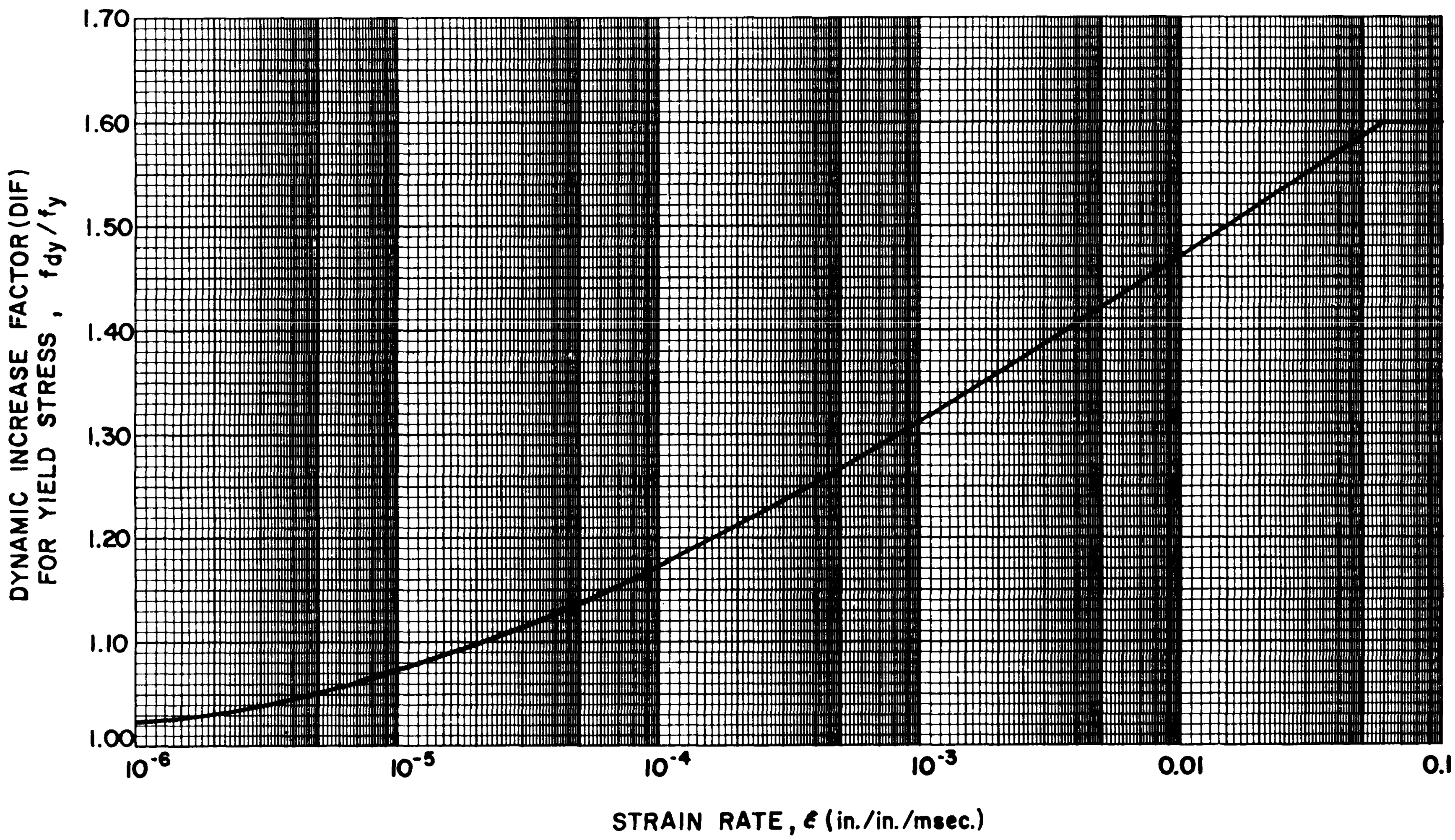


Figure 4-10 Design curve for DIF for yield stress of ASTM A 615 grade 60 reinforcing steel

Table 4-1 Dynamic Increase Factor (DIF) for Design of Reinforced Concrete Elements

TYPE OF STRESS	FAR DESIGN RANGE			CLOSE-IN DESIGN RANGE		
	Reinforcing Bars		Concrete	Reinforcing Bars		Concrete
	f_{dy}/f_y	f_{du}/f_u	f'_{dc}/f'_c	f_{dy}/f_y	f_{du}/f_u	f'_{dc}/f'_c
Bending	1.17	1.05	1.19	1.23	1.05	1.25
Diagonal Tension	1.00	—	1.00	1.10	1.00	1.00
Direct Shear	1.10	1.00	1.10	1.10	1.00	1.10
Bond	1.17	1.05	1.00	1.23	1.05	1.00
Compression	1.10	—	1.12	1.13	—	1.16

Table 4-2 **Dynamic Design Stresses for Design of Reinforced Concrete Elements**

TYPE OF STRESS	TYPE OF REINFORCEMENT	MAXIMUM SUPPORT ROTATION, θ_m (DEGREES)	DYNAMIC DESIGN STRESS	
			REINFORCEMENT, f_{ds}	CONCRETE, f_{dc}
Bending	Tension and Compression	$0 < \theta_m \leq 2$	f_{dy} (1)	f'_{dc}
		$2 < \theta_m \leq 5$	$f_{dy} + (f_{du} - f_{dy})/4$	(2)
		$5 < \theta_m \leq 12$	$(f_{dy} + f_{du})/2$	(2)
Diagonal Tension	Stirrups	$0 < \theta_m \leq 2$	f_{dy}	f'_{dc}
		$2 < \theta_m \leq 5$	f_{dy}	f'_{dc}
		$5 < \theta_m \leq 12$	f_{dy}	f'_{dc}
Diagonal Tension	Lacing	$0 < \theta_m \leq 2$	f_{dy}	f'_{dc}
		$2 < \theta_m \leq 5$	$f_{dy} + (f_{du} - f_{dy})/4$	f'_{dc}
		$5 < \theta_m \leq 12$	$(f_{dy} + f_{du})/2$	f'_{dc}
Direct Shear	Diagonal Bars	$0 < \theta_m \leq 2$	f_{dy}	f'_{dc}
		$2 < \theta_m \leq 5$	$f_{dy} + (f_{du} - f_{dy})/4$	(3)
		$5 < \theta_m \leq 12$	$(f_{dy} + f_{du})/2$	(3)
Compression	Column	(4)	f_{dy}	f'_{dc}

(1) Tension reinforcement only.
(2) Concrete crushed and not effective in resisting moment.
(3) Concrete is considered not effective and shear is resisted by the reinforcement only.
(4) Capacity is not a function of support rotation.

STATIC PROPERTIES

4-14. Modulus of Elasticity

4-14.1. Concrete

The modulus of elasticity of concrete E_c is equal to:

$$E_c = w_c^{1.5}\ 33\ (f'_c)^{1/2} \qquad 4\text{-}4$$

for values of w_c between 90 and 155 lbs/ft^3 where w_c is the unit weight of concrete and normally equal to 150 lbs/ft^3.

4-14.2. Reinforcing Steel

The modulus of elasticity of reinforcing steel E_s is:

$$E_s = 29 \times 10^6 \text{ psi} \qquad 4\text{-}5$$

4-14.3. Modular Ratio

The modular ratio n is:

$$n = E_s/E_c \qquad 4\text{-}6$$

and may be taken as the nearest whole number.

4-15. Moment of Inertia

The determination of the deflection of a reinforced concrete member in the elastic and elasto-plastic ranges is complicated by the fact that the effective moment of inertia of the cross section along the element changes continually as cracking progresses. It is further complicated by the fact that the modulus of elasticity of the concrete changes as the stress increases. It is recommended that the computation of deflections throughout this volume be based upon empirical relations determined from test data.

The average moment of inertia I_a should be used in all deflection calculations and is given by:

$$I_a = \frac{I_g + I_c}{2} \qquad 4\text{-}7$$

For the design of beams, the entire cross-section is considered, so that

$$I_g = \frac{bT_c^3}{12} \qquad 4\text{-}8a$$

and

$$I_c = Fbd^3 \qquad 4\text{-}8b$$

For the design of slabs, a unit width of the cross-section is considered, so that

$$I_g = \frac{T_c^3}{12} \qquad 4\text{-}9a$$

and:

$$I_c = Fd^3 \qquad 4\text{-}9b$$

where:

I_a = average moment of inertia of concrete cross section

I_g = moment of inertia of the gross concrete cross section (neglecting all reinforcing steel)

I_c = moment of inertia of cracked concrete cross section

b = width of beam

T_c = thickness of gross concrete cross section

F = coefficient given in Figures 4-11 and 4-12

d = distance from extreme compression fiber to centroid of tension reinforcement

The moment of inertia of the cracked concrete section considers the compression concrete area and steel areas transformed into equivalent concrete areas and is computed about the centroid of the transformed section. The coefficient F varies as the modular ratio n and the amount of reinforcement in the section. For sections with tension reinforcement only, the coefficient F is given in Figure 4-11 while for sections with equal reinforcement on opposite faces, the coefficient F is given in Figure 4-12.

The variation in the cracked moment of inertia obtained from Figure 4-11 and 4-12 is insignificant for low reinforcement ratios. The variation increases for the larger ratios. Consequently, for the comparatively low reinforcement ratios normally used in slab elements either chart may be used with negligible error. For the higher reinforcement ratios normally used for beams, Figure 4-12 must be used for equally reinforced sections whereas a weighted average from Figure 4-11 and 4-12 may be used for sections where the compression steel is less than the tension steel.

For one-way members the reinforcement ratio p used to obtain the factor F should be an average of the tension steel at the supports and midspan. Also, the effective depth d used to compute the cracked moment of inertia I_c should be an average of the effective depth at the supports and midspan. However, for two-way members, the aspect ratio must be considered in the calculation of the cracked moment of inertia. Average values for the reinforcement ratio p and effective depth d should be used to obtain the cracked moment of inertia in each direction and cracked moment of the member is then obtained from:

$$I_c = \frac{I_{cV}L + I_{cH}H}{L + H} \tag{4-10}$$

where:

I_{cV} = cracked moment of inertia in vertical direction

I_{cH} = cracked moment of inertia in horizontal direction

L = span length

H = span height

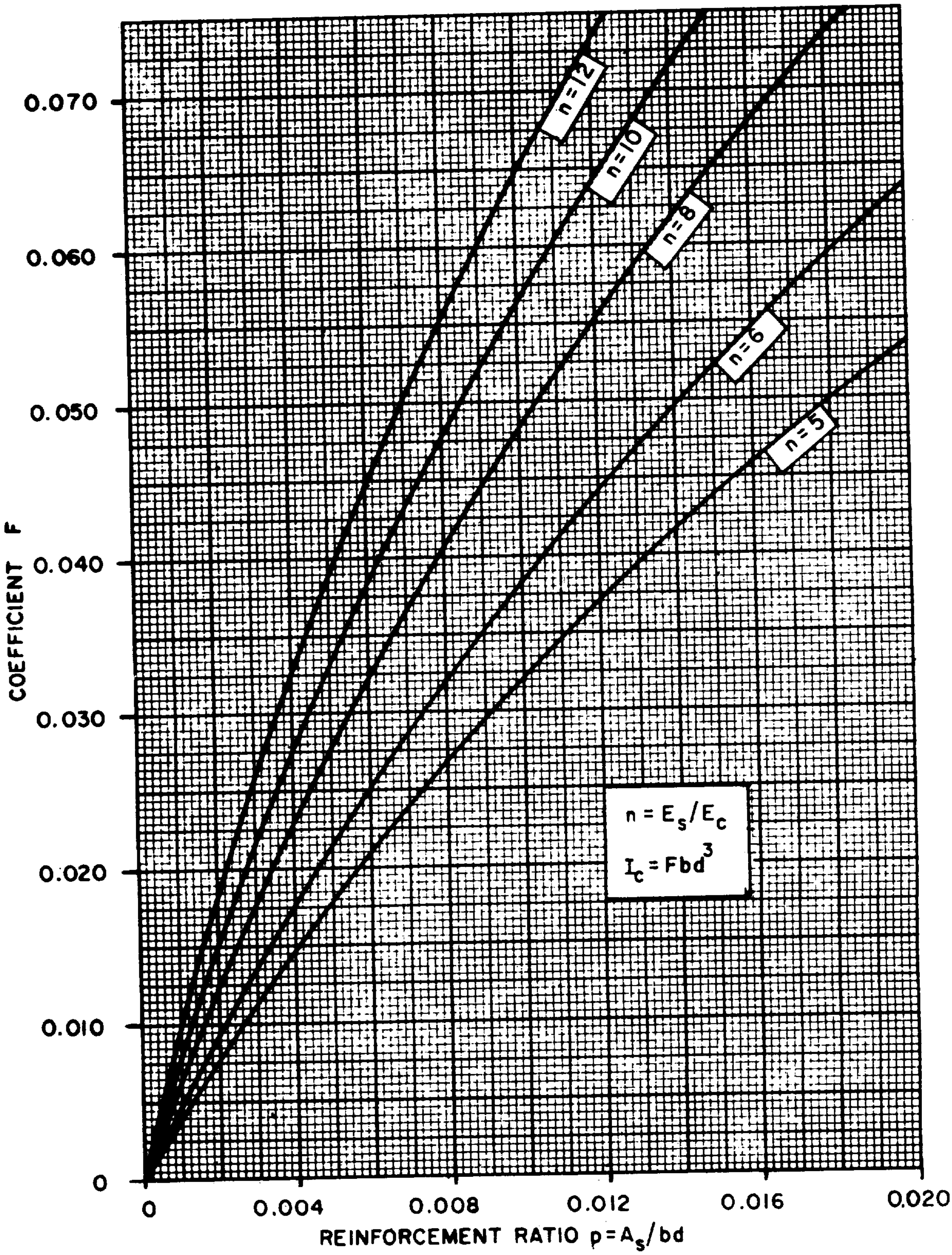


Figure 4-11 Coefficient for moment of inertia of cracked sections with tension reinforcement only

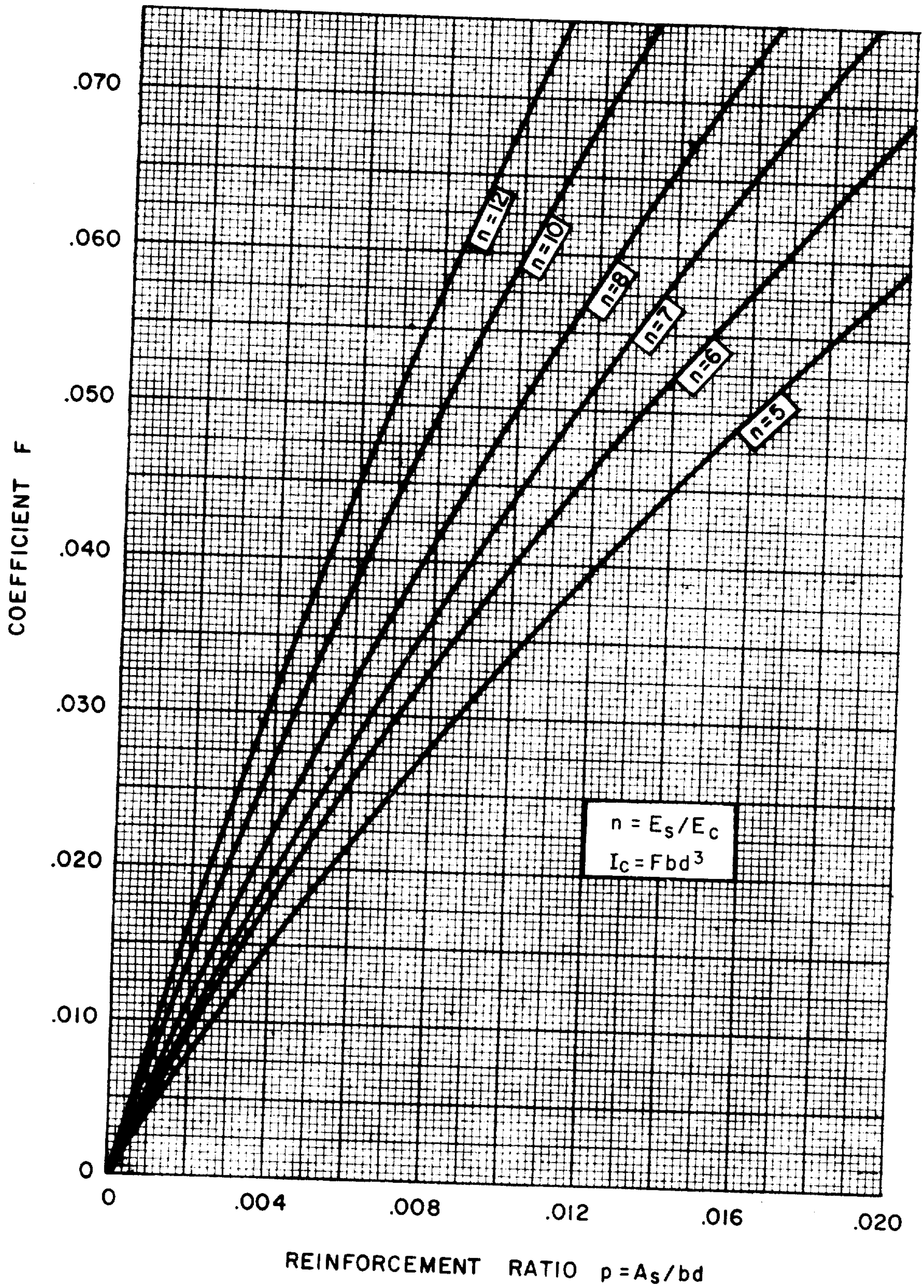


Figure 4-12 Coefficient for moment of inertia of cracked sections with equal reinforcement on opposite faces

ULTIMATE DYNAMIC STRENGTH OF SLABS

4-16. Introduction

Depending upon the magnitudes of the blast output and permissible deformations, one of three types of reinforced concrete cross sections (Figure 4-13) can be utilized in the design or analysis of blast resistant concrete slabs:

a. type I - The concrete is effective in resisting moment. The concrete cover over the reinforcement on both surfaces of the element remains intact.

b. type II - The concrete is crushed and not effective in resisting moment. Compression reinforcement equal to the tension reinforcement is required to resist moment. The concrete cover over the reinforcement on both surfaces of the element remains intact.

c. type III - The concrete cover over the reinforcement on both surfaces of the element is completely disengaged. Equal tension and compression reinforcement which is properly tied together is required to resist moment.

Elements designed using the full cross section (type I) are usually encountered in those structures or portions of structures designed to resist the blast output at the far design range. This type of cross section is utilized in elements with maximum deflections corresponding to support rotations less than 2 degrees. Maximum strength of an element is obtained from a type I cross section. Type I elements may be reinforced on either one or both faces. However, due to rebound forces, reinforcement is required on both faces of an element.

Crushing of the concrete cover over the compression reinforcement is exhibited in elements which undergo support rotations greater than 2 degrees. This failure results in a transfer of the compression stresses from the concrete to the compression reinforcement which, in turn, results in a loss of strength.

Sufficient compression reinforcement must be available to fully develop the tension steel (tension and compression reinforcement must be equal). Elements which sustain crushing of the concrete without any disengagement of the concrete cover are encountered in structures at the far design range when the maximum deflection conforms to support rotations greater than 2 degrees but less than 5 degrees.

Although the ultimate strength of elements with type III cross sections is no less than that of elements with type II cross sections, the overall capacity to resist the blast output is reduced. The spalling of the concrete cover over both layers of reinforcement, caused by either the direction transmission of high pressures through the element at the close-in range or large deflections at the far range, produces a loss of capacity due to the reduction in the concrete mass. A more detailed treatment of the phenomena of crushing and spalling is presented in subsequent sections of this chapter.

The ultimate dynamic strength of reinforced concrete sections may be calculated in accordance with the ultimate strength design methods of the American Concrete Institute Standard Building Code Requirements for Reinforced Concrete (hereafter referred to as the ACI Building Code). The capacity reduction factor 0 which has been established for conventional static load conditions is omitted for the determination of ultimate dynamic strength. Safety or reliability of the protective structure is inherent in the establishment of the magnitude of the blast output for the donor charge, and in the criteria specified for deflection, support rotation, or fragment velocity. Other permissible departures from the criteria for static or gas pressure loadings are described below.

Although certain formulae for elements constructed with conventional weight concrete are given in the following paragraphs of this chapter, more detailed information and design aids are given in the bibliography. Because tests have not yet been conducted to determine the response of lightweight concrete elements designed for close-in and far design ranges, the pertinent formulae for this type of concrete are not included in the manual. However, lightweight concrete may be utilized for structures designed for low pressures (less than 10 psi); but the reduction in mass from conventional weight concrete must be accounted for in the design to maintain the blast resistant capacity of the structure.

4-17. Ultimate Moment Capacity

4-17.1. Cross Section Type I

The ultimate unit resisting moment M_u of a rectangular section of width b with tension reinforcement only is given by:

$$M_u = (A_s f_{ds}/b)[d - (a/2)] \quad \text{4-11}$$

in which:

$$a = A_s f_{ds}/0.85\ b\ f'_{dc} \quad \text{4-12}$$

where:

A_s = area of tension reinforcement within the width b

f_{ds} = dynamic design stress for reinforcement

d = distance from extreme compression fiber to centroid of tension reinforcement

a = depth of equivalent rectangular stress block

b = width of compression face

f'_{dc} = dynamic ultimate compressive strength of concrete

The reinforcement ratio p is defined as:

$$p = A_s/bd \quad \text{4-13}$$

To insure against sudden compression failures, the reinforcement ratio p must not exceed 0.75 of the ratio p_b which produces balanced conditions at ultimate strength and is given by:

$$p_b = (0.85K_1 f'_{dc}/f_{ds})[87{,}000/(87{,}000 + f_{ds})] \qquad 4\text{-}14$$

where:

K_1 = 0.85 for f'_{dc} up to 4,000 psi and is reduced 0.05 for each 1,000 psi in excess of 4,000 psi.

For a rectangular section of width b with compression reinforcement, the ultimate unit resisting moment is:

$$M_u = [(A'_s)f_{ds}/b][d - (a/2)] + (A'_s f_{ds}/b)(d - d') \qquad 4\text{-}15$$

in which:

$$a = (A_s - A'_s)\, f_{ds}/0.85b\, f'_{dc} \qquad 4\text{-}16$$

where:

A'_s = area of compression reinforcement within the width b

d' = distance from extreme compression fiber to centroid of compression reinforcement

a = depth of equivalent rectangular stress block

The reinforcement ratio p' is:

$$p' = A'_s/b \qquad 4\text{-}17$$

Equation 4-15 is valid only when the compression steel reaches the value f_{ds} at ultimate strength, and this condition is satisfied when:

$$p - p' \geq 0.85K_1(f'_{dc}d'/f_{ds}d)[87{,}000/(87{,}000 - f_{ds})] \qquad 4\text{-}18$$

If p - p' is less than the value given by Equation 4-18 or when compression steel is neglected, the calculated ultimate unit resisting moment should not exceed that given by Equation 4-11. The quantity p - p' must not exceed 0.75 of the value of p_b given in Equation 4-14.

4-17.2. Cross Section Types II and III

The ultimate unit resisting moment of type II and type III rectangular sections of width b is:

$$M_u = A_s f_{ds} d_c/b \qquad 4\text{-}19$$

where:

A_s = area of tension or compression reinforcement within the width b

d_c = distance between the centroids of the compression and the tension reinforcement

The reinforcement ratios p and p' are equal to:

$$p_s = p'_c = A /bd \qquad 4\text{-}20$$

The above moment capacity can only be obtained when the areas of the tension and compression reinforcement are equal. In addition, this reinforcement must be properly restrained so as to maintain the integrity of the element when large deflections are encountered.

4-17.3. Minimum Flexural Reinforcement

To insure proper structural behavior under both conventional and blast loadings, a minimum amount of flexural reinforcement is required. This quantity of reinforcement insures that the moment capacity of the reinforced section is greater than that corresponding to the plain concrete section computed from its modulus of rupture. Failure of a plain concrete section is quite sudden. Also, this minimum reinforcement prevents excessive cracking and deformations under conventional loadings.

The minimum reinforcement required for slabs is somewhat less than that required for beams, since an overload would be distributed laterally and sudden failure would be less likely. The minimum reinforcement ratio for dynamic design of slabs is given in Table 4-3. However, this quantity of reinforcement must also satisfy static design requirements. Except for blast loads in the order of magnitude of static loads, the minimum requirements for dynamic loads will control. In cases where minimum requirements for static conditions control, the quantity of reinforcement must be at least 1.33 times the quantity required by static analysis or 0.0018 times the gross concrete area, whichever is less.

Concrete sections with tension reinforcement only are not permitted. For type I sections, compression reinforcement equal to at least one half the required tension reinforcement must be provided. This reinforcement is required to resist the ever present rebound forces. Depending upon the magnitude of these rebound forces, the compression reinforcement required may be greater than one half the tension reinforcement and substantially greater than the minimum quantity given in Table 4-3. For type II and III cross sections, the compression reinforcement is always equal to the tension reinforcement.

4-18. Ultimate Shear (Diagonal Tension) Capacity

4-18.1. Ultimate Shear Stress

The ultimate shear stress v_u, as a measure of diagonal tension, is computed for type I sections from:

$$v_u = V_u/bd \qquad 4\text{-}21$$

and for type II and III sections from:

$$v_u = V_u/bd_c \qquad 4\text{-}22$$

where V_u is the total shear on a width b at either the face of the support, or at the section a distance d (type I) or d_c (types II or III) from the face of the support. For the latter case, the shear at sections between the face of the support and the section d or d_c away need not be considered critical.

For laced elements, the shear stress is always calculated at d_c from the face of the support (or haunch) since the lacing and required diagonal bars provide sufficient corner reinforcement. For unlaced elements, the shear stress is calculated at d from the face of the support for those members that cause compression in their supports (Fig. 4-14a). This provision should not be applied for those members that cause tension in their supports (Fig. 4-14b). For this case, the ultimate shear stress should be calculated at the face of the support. In addition, the shear within the connection should be investigated and special corner reinforcement should be provided.

The ultimate shear stress v_u must not exceed 10 $(f'_{dc})^{1/2}$ in sections using stirrups. The thickness of such sections must be increased and/or the quantity of flexural reinforcement reduced in order to bring the value of v_u within tolerable limits. In sections using lacing, there is no restriction on v_u because of the continuity provided by this type of shear reinforcement. However, for large shear stresses the area of the lacing bars required may become impractical.

4-18.2. Shear Capacity of Unreinforced Concrete

The shear stress permitted on an unreinforced web of a member subjected to flexure only is limited to:

$$v_c = [1.9(f'_{dc})^{1/2} + 2500p] \leq 3.5(f'_{dc})^{1/2} \qquad \text{4-23}$$

where p is the reinforcement ratio of the tension requirement at the support. For the computation of the reinforcement ratio, d is used for type I sections and d_c for type II and III sections. For members subjected to significant axial tension, the shear stress permitted on an unreinforced web is limited to:

$$v_c = 2(1 + N_u/500A_g)\ (f'_{dc})^{1/2} \geq 0 \qquad \text{4-24}$$

while for significant axial compression:

$$v_c = 2(1 + N_u/2000A_g)(f'_{dc})^{1/2} \qquad \text{4-25}$$

where:

N_u = axial load normal to the cross section

A_g = gross area of the cross section

The axial load N_u must occur simultaneously with the total shear V_u on the section in order to apply Equations 4-24 and 4-25.

The value of N_u shall be taken as positive for compression and negative for tension. The simplified dynamic analysis normally performed is not sufficient to accurately determine the time variations between the desired forces and moments from which N_u and V_u are obtained. Unless a time-history analyses is performed, any apparent strength increases due to loading combinations are unreliable. Therefore, it is recommended that the increased shear capacity due to compressive axial loads be neglected and, by the same reasoning, the reduced capacity due to tension forces be included. Both assumptions are conservative.

4-18.3. Design of Shear Reinforcement

Whenever the ultimate shear stress v_u exceeds the shear capacity v_c of the concrete, shear reinforcement must be provided to carry the excess. This shear reinforcement can be either stirrups or lacing, depending upon the magnitudes of the blast loading and support rotation permitted. Stirrups can be used only for elements designed to attain small deflections under flexural behavior. Lacing can also be used for elements designed to attain small deflections; however, lacing must be used for elements designed to attain large deflections. Therefore, stirrups may be used for elements with a type I, II, or III cross section as long as the element is designed to attain small deflections. An exception to this deflection criteria is for the particular case of slabs subjected to tension membrane action. Here the slab can attain large deflections. Lacing may be used for type II and III cross sections designed for either small or large deflections. It would be grossly unwarranted to use lacing for a type I cross section.

The required area of stirrups is calculated from:

$$A_v = [(v_u - v_c)b_s s_s]/\phi f_{ds} \qquad 4\text{-}26$$

while the required area of lacing reinforcement is:

$$A_v = \frac{[(v_u - v_c)\, b_l s_l]}{\phi f_{ds}(\sin\alpha + \cos\alpha)} \qquad 4\text{-}27$$

where:

A_v = total area of stirrups or lacing reinforcement in tension within a width b_s or b_l, and a distance s_s or s_l

$v_u - v_c$ = excess shear stress

b_s = width of concrete strip in which the diagonal tension stresses are resisted by stirrups of area A_v

b_l = width of concrete strip in which the diagonal tension stresses are resisted by lacing of area A_v

s_s = spacing of stirrups in the direction parallel to the longitudinal reinforcement

s_l = spacing of lacing in the direction parallel to the longitudinal reinforcement

ϕ = capacity reduction factor equal to 0.85

α = angle formed by the plane of the lacing and the plane of the longitudinal reinforcement

The angle of inclination α of the lacing bars is given by

$$\cos \alpha = \frac{-2B(1-B) \pm \{[2B(1-B)]^2 - 4[(1-B)^2 + A^2][B^2 - A^2]\}^{1/2}}{2[(1-B)^2 + A^2]} \qquad 4\text{-}28$$

in which

$$A = s_1/d_1 \qquad 4\text{-}29a$$

$$B = \frac{2R_1 + d_b}{d_1} \qquad 4\text{-}29b$$

where

d_1 = distance between centerlines of lacing bends measured normal flexural reinforcement

R_1 = radius of bend in lacing bars (min $R_1 = 4d_b$)

d_b = nominal diameter of reinforcing bar

A typical section of a lacing bar illustrating the terms used in the above equation, is shown in Figure 4-15. To facilitate the design of lacing bars, the angle α can be determined from Figure 4-15.

4-18.4. Minimum Shear Reinforcement

In order to develop the full flexural capacity of a slab, a premature shear failure must be prevented. Shear reinforcement must be provided to resist shear stresses in excess of the capacity of the concrete. However, except for slabs having a type I cross section or subjected to tension-membrane action in the far design range, minimum shear reinforcement must always be provided to insure the full development of the flexural reinforcement and enable the slab to attain large deflections.

Stirrups or lacing must conform to the following limitations to insure a proper distribution of shear reinforcement throughout the element and, in specific cases, to provide a minimum quantity of shear reinforcement.

1. The minimum design stress (excess shear stress $v_u - v_c$) used to calculate the required amount of shear reinforcement, must conform to the limitations of Table 4-4.

2. When stirrups or lacing reinforcement is required, the area A_v should not be less than 0.0015 b_s s_s for stirrups or 0.0015 $b_1 s_1$ for lacing.

3. When stirrups or lacing are provided, the required area A_v is determined at the critical section and this quantity of reinforcement must be uniformly distributed throughout the element.

4. Single leg stirrups should be used for slabs. At least one stirrup must be located at each bar intersection.

5. A lacing bar is bent from a single reinforcing bar. In one direction, the lacing must be continuous across the slab between opposite supports. In all cases, the lacing must be carried past the face of the support and securely anchored within the support.

6. The maximum spacing of stirrups s_s is limited to d/2 for Type I cross sections and $d_c/2$ for Type II and III sections, but not greater than 24 inches.

7. The maximum spacing of lacing s_l is limited to d_c or 24 inches, whichever is smaller.

The spacing of stirrups and lacing is a function of the flexural bar spacing. Consequently, the above limitations for shear reinforcement should be considered in selecting the flexural bar spacing. Once selected, the flexural bar spacing may have to be altered to suit the above limitations.

4-19. Direct Shear Capacity

4-19.1. General

Direct shear failure of a member is characterized by the rapid propagation of a vertical crack through the depth of the member. This crack is usually located at the supports where the maximum shear stresses occur. Failure of this type is possible even in members reinforced for diagonal tension.

Diagonal bars are required at slab supports to prevent direct shear failure: when the design support rotation exceeds 2° (unless the slab is simply supported as given in section 4-19.2); when the design support rotation is ≤ 2° but the direct shear capacity of the concrete is insufficient; or when the section is in tension (as in containment cells). Diagonal reinforcement consists of inclined bars which extend from the support into the slab element.

4-19.2. Direct Shear Capacity of Concrete

If the design support rotation, θ, is greater than 2° ($\theta > 2°$), or if a section (with any support rotation) is in net tension, then the ultimate direct shear capacity of the concrete, V_d, is zero and diagonal bars are required to take all direct shear.

If the design support rotation, θ, is less than or equal to 2° ($\theta \le 2°$), or if the section, with any rotation θ, is simply supported (total moment capacity of adjoining elements at the support must be significantly less than the moment capacity of the section being checked for direct shear), then the ultimate direct shear force, V_d, that can be resisted by the concrete in a slab is:

$$V_d = 0.18\ f'_{dc}\ bd \qquad 4\text{-}30$$

4-19.3. Design of Diagonal Bars

The required area of diagonal bars is determined from:

$$A_d = (V_s b - V_d)/(f_{ds}\sin(\alpha)) \qquad 4\text{-}31$$

where:

$V_d = 0.18\, f'_{dc}\, bd.$ ($\theta \le 2°$ or simple supports),

or $V_d = 0$ ($\theta > 2°$ or section in tension).

and A_d = total area of diagonal bars at the support within a width b

V_s = shear at the support of unit width b

α = angle formed by the plane of the diagonal reinforcement and the longitudinal reinforcement.

4-20. Punching Shear

4-20.1. Ultimate Punching Shear Stress

When a flat slab is supported on a column or a column rests on a two-way slab, failure occurs around and against the concentrated load, punching out a pyramid of concrete from the slab. The ultimate shear stress v_u, as a measure of punching shear, is computed from:

$$v_u = N_u/b_o d_e \qquad 4\text{-}32$$

where:

N_u = the total concentrated axial load or reaction

b_o = failure perimeter located at a distance d/2 from the concentrated load or reaction area

d_e = either d or d_c depending on the type of cross section

4-20.2. Punching Shear Capacity of Concrete

The shear stress permitted for punching shear is limited to:

$$v_c = 4(f'_{dc})^{1/2} \qquad 4\text{-}33$$

Equation 4-33 applies to circular columns and to rectangular columns with aspect ratios no greater than 2. For rectangular areas with aspect ratios greater than 2, the allowable value of v_c should be reduced according to the ACI provisions (not listed in this text).

Shear reinforcement is not permitted to increase the punching shear capacity of a slab. If the ultimate shear stress v_u is greater than the stress permitted for punching shear v_c, the slab thickness must be increased. In flat slab design, the use of a drop panel to increase slab thickness, and/or a column capital to increase failure perimeter, may be employed to prevent punching shear failure. If a drop panel is used, punching shear must be checked at the perimeter of the drop panel, as well as at the top of the column.

4-21. Development of Reinforcement

4-21.1. General

In order to fully develop the flexural and/or axial load capacity of a concrete slab or wall, the full strength of the reinforcement must be realized. At any section along the length of a member, the tensile or compressive force in the reinforcement must be developed on each side of the section by proper embedment length, splices (lapped or mechanical), end anchorage, or for tension only, hooks. At a point of peak stress, this development length or anchorage is necessary on both sides of the point; on one side to transfer stress into and on the other side to transfer stress out of the reinforcing bar.

The types and locations of reinforcement anchorages are severely restricted for blast resistant structures. These restrictions are necessary to insure that the structure acts in a ductile manner. Typical details for both conventionally reinforced and laced reinforced concrete elements are given in latter sections dealing with construction details. Conformance to these details greatly simplifies the calculation of development lengths. The required development lengths to be used in conjunction with the required typical details are given below.

While conformance to the typical details given is mandatory, certain conditions may preclude their use. For these unusual conditions, the required anchorages are calculated according to the procedures given in the ACI Building Code. The basic development length is first calculated and then modified based on the construction details employed to obtain the required end anchorage or splice length. This procedure is outlined in Sections 4-21.4 through 4-21.7. However, it must be repeated that the typical details given must be followed and any deviation from these restrictions and requirements must be carefully considered to insure proper structural behavior.

4-21.2 Provisions for Conventionally Reinforced Concrete Elements

Typical details for conventionally reinforced (non-laced) concrete elements are given in subsequent sections. These details locate splices in reinforcement at points of low stress. This permits the minimum length of lap splices, as well as the development length for end anchorages, to be given by

$$l_d = 40\, d_b \qquad \text{4-34}$$

where

l_d = development length

d_b = diameter of reinforcing bar

The value of l_d shall not be less than 24 inches. Equation 4-34 applies for end anchorage of #14 bars and smaller, and for lap splices of #11 bars and smaller since lap splices of #14 bars are not permitted.

Lap splices of reinforcement must not be located at critical sections. Rather, they must be located in regions of low stress (inflection points) where the area of reinforcement provided is more than twice the area of reinforcement required by analysis. In addition, not more than one-half of the reinforcement may be spliced at one location. The splice of adjacent bars must be staggered at least the required lap length of the bars since overlap of splices of adjacent bars is not desirable. Under these conditions, the required minimum length of lap splices is given in Equation 4-34. If it is impractical to locate splices at the inflection points, then the length of the splice must be calculated according to the provisions of Section 4-21.4.

Typical details for intersecting walls and slab/wall intersections avoid the use of end anchorage of the primary reinforcement. Rather, the reinforcement is anchored by continuing it through the support and bending it into the intersecting wall or slab. This reinforcement is then lap spliced with the reinforcement in the intersecting wall.

4-21.3 Provisions for Laced Reinforced Concrete Elements

Tests of laced elements have indicated that if continuity of the lacing and flexural reinforcement is maintained throughout the element, the required development length for end anchorage as well as the minimum length of lap splices is given by Equation 4-34, but not less than 24 inches. This equation applies for end anchorage of #14 bars and smaller, and for lap lengths of #11 bars and smaller since lap splices of #14 bars are not permitted.

Required construction details and procedures for laced reinforced concrete elements are given in subsequent sections. These details must be followed to insure the full development of both the concrete and reinforcement well into the range of plastic action of the materials. The use of Equation 4-34 to obtain the required development lengths of the reinforcement is predicated on the use of these details.

The typical details for laced reinforced concrete elements require that the reinforcement (flexural as well as lacing bars) must not be spliced at critical sections but rather must be spliced in regions of low stress (inflection points) where the area of reinforcement provided is more than twice the area of reinforcement required by analysis. In addition, not more than one-quarter of the reinforcement may be spliced at one location. The splice of adjacent bars must be staggered at least the required lap length of the bars since overlap of splices of adjacent bars is not desirable.

Specific end anchorage details are required for laced reinforced concrete walls and slabs to enable the reinforcement to attain its ultimate strength. The preferred method of end anchorage is through the use of wall extensions since this method presents the least construction problems. If architectural requirements do not permit the use of wall extensions, the reinforcement is anchored by continuing it through the support and bending it into the intersecting wall or slab. In this latter case, the reinforcement is developed by a combination of anchorage and lap splice. In either case, the lacing extending into the supports provides the necessary confinement which permits the use of Equation 4-34.

4-21.4. **Development Length for Reinforcement in Tension**

The basic development length for #11 bars and smaller which are in tension is given by:

$$l_d = 0.04\ A_b fa_{ds}/(f'_{dc})^{1/2} \qquad \text{4-35a}$$

but not less than:

$$l_d = 0.0004 d_b\ f_{ds} \qquad \text{4-35b}$$

where:

l_d = basic development length

A_b = area of reinforcing bar

d_b = diameter of reinforcing bar

The basic development length for #14 bars in tension is given by:

$$l_d = 0.085\ f_{ds}/(f'_{dc})^{1/2} \qquad \text{4-36}$$

The use of #18 bars is not permitted by this manual.

For top reinforcement, that is, horizontal reinforcement so placed that more than 12 inches of concrete is cast in the slab below the reinforcement, the basic development length must be multiplied by 1.40. This provision applies to horizontal slabs only. Walls with multiple runs of horizontal bars plus vertical bars are not effected by this provision. In addition, the basic development length of all bars may be multiplied by 0.80 if the bars are spaced laterally at least six (6) inches on center. In no case shall the development length be less than 24 inches nor 40 times the diameter of the reinforcing bar.

4-21.5 **Development Length for Reinforcement in Compression**

The basic development length for bars in compression is given by:

$$l_d = 0.02\ d_b\ f_{ds}/(f'_{dc})^{1/2} \qquad \text{4-37a}$$

but not less than:

$$l_d = 0.0003\ d_b\ f_{ds} \qquad \text{4-37b}$$

The development length for compression is not modified for top bars nor lateral bar spacing. In no case shall the development length used be less than twelve (12) inches.

Under dynamic load conditions, members are subject to load reversal or rebound. Reinforcement subject to compressive forces under the primary load may be subject to tensile forces under rebound. Consideration must be given to this stress reversal, since the development length of bars in compression is less than the development length of bars in tension.

4-21.6. Development Length of Hooked Bars

The basic development length for bars in tension which terminate in a standard 90-degree or 180-degree hook is given by:

$$l_{dh} = 0.02\ d_b\ f_{ds}/(f'_{dc})^{1/2} \qquad 4\text{-}38$$

but not less than $8d_b$ or 6 inches, whichever is greater. The development length for a hooked bar l_{dh} is measured from the critical section along the length of the bar to the end of the hook. That is, the length l_{dh} includes the straight length of the bar between the critical section and point of tangency of the hook, the bend radius and one bar diameter. The required hook geometry as specified in the ACI Building Code is given in Figure 4-16.

In the development of hooked bars, no distinction is made between top bars and other bars. However, since hooked bars are especially susceptible to concrete splitting failure if the concrete cover is small, the above equation takes into account the effect of minimum concrete cover. For #11 bar and smaller with the cover not less than 2-½ inches, and for a 90-degree hook, with cover on the bar extension beyond the hook not less than 2 inches, the development length l_{dh} may be multiplied by 0.7.

Hooks are not to be considered effective in developing bars in compression. However, in the design of members subjected to dynamic loads, rebound or load reversal must be considered. That is, under the primary loading, reinforcement is subjected to tensile forces and anchored utilizing a standard hook, but this same hooked reinforcement may be subjected to compressive forces under rebound. Therefore, the straight portion of the hooked bar must be sufficient to develop this compressive force. For those cases where 100 percent rebound is encountered, the straight portion of a hooked bar must be equal to the development length for bars in compression.

4-21.7. Lap Splices of Reinforcement

In blast resistant structures, reinforcing bars may be lap-spliced using only contact lap splices; noncontact lap splices are not permitted. Lap splices shall not be used for reinforcing bars larger than #11 bars. If #14 bars are used, they must be continuous.

Lap splices of adjacent parallel reinforcing bars must be staggered by at least the length of the lap. The minimum length of lap for tension lap splices depends upon the location of the splice. For blast resistant structures, it is strongly recommended that splices be located in regions of low stress, where the area of reinforcement provided is at least twice that required. In such cases, the length of the lap is equal to the basic development length l_d for bars in tension as given by Equation 4-35 or 4-36 and modified, if applicable, for top bars and/or lateral spacing of the bars. In other design situations where the lap splice is not located in regions of low stress, the lap length is equal to 1.3 times the modified development length, l_d.

The minimum length of lap for compression lap splices is equal to the basic development length l_d for bars in compression as given by Equation 4-37. However, due to the occurrence of load reversal, it is recommended that the length of lap splices be based in tension unless it can be shown that the reinforcement will always be in compression.

4-21.8. Mechanical Splices of Reinforcement

Mechanical devices may be used for end anchorage and splices in reinforcement. These devices must be capable of developing the ultimate dynamic tensile strength of the reinforcement without reducing its ductility. Tests showing the adequacy of such devices under dynamic conditions must be performed before these devices are deemed acceptable for use in hardened structures.

4-21.9. Welding of Reinforcement

Welding of reinforcement is to be avoided in blast resistant structures since it results in a reduction of the ultimate strength and ductility of the reinforcing steel. In those cases where welding is absolutely essential, it may be necessary to obtain special reinforcement manufactured with controlled chemical properties. Tests showing the adequacy of the combination of weld and reinforcing steel under dynamic conditions must be performed to demonstrate that this welding does not reduce the ultimate strength and ductility of the reinforcing steel. In lieu of these tests, welding is permitted if the stress in the reinforcement is maintained at a level less than 90 percent of the yield stress.

4-21.10. Bundled Reinforcing Bars

The use of bundled bars may be required for unusual conditions. However, their use is not desirable and should be avoided where possible. A 3 bar bundle should be the maximum bundle employed. The development length and lap splice length of individual bars within a bundle shall be that of the individual bar increased 20 percent for a 2 bar bundle and 33 percent for a 3 bar bundle. In addition, splices of the individual bars within a bundle should be staggered. That is, only one bar of the bundle should be spliced at a given location.

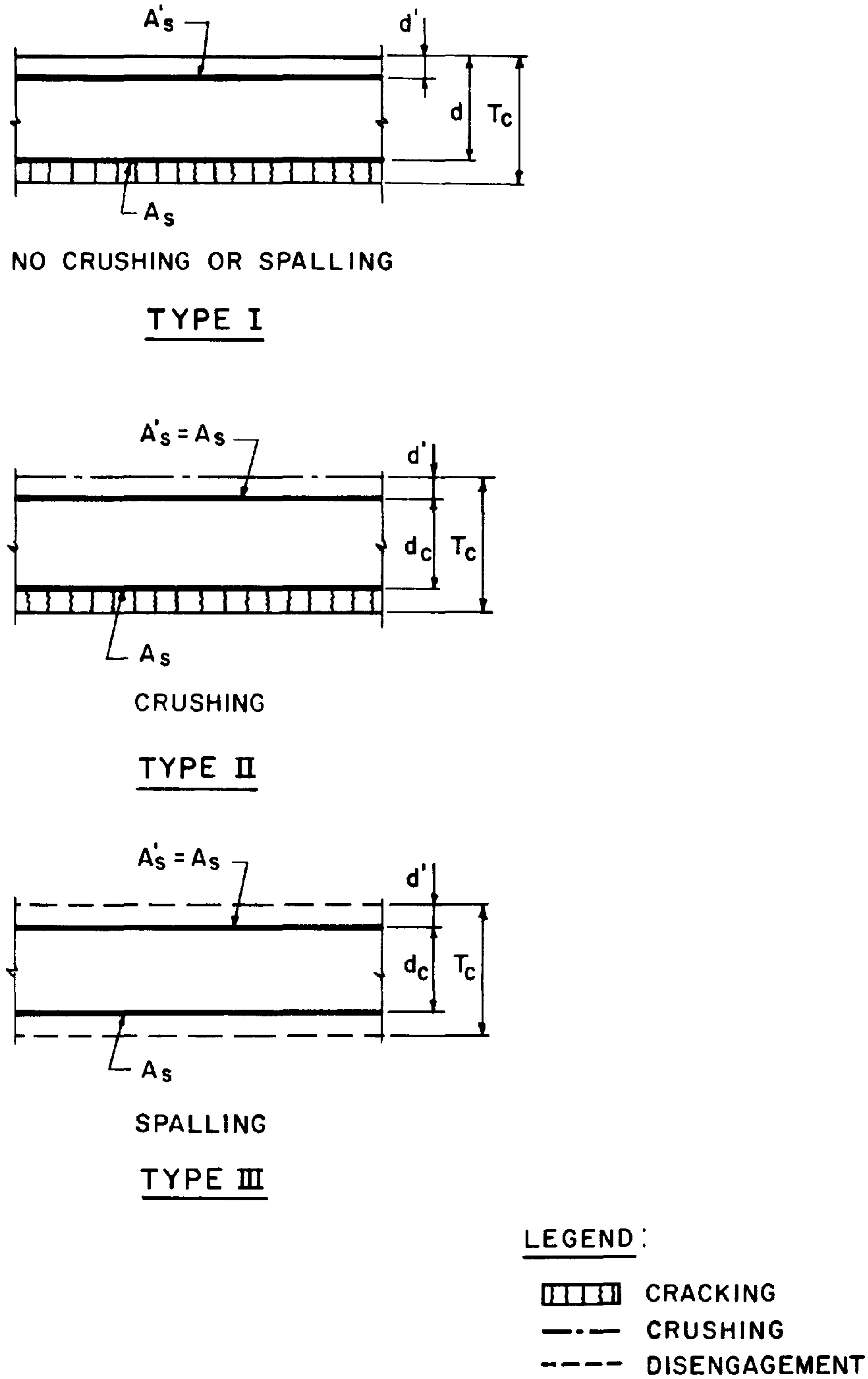
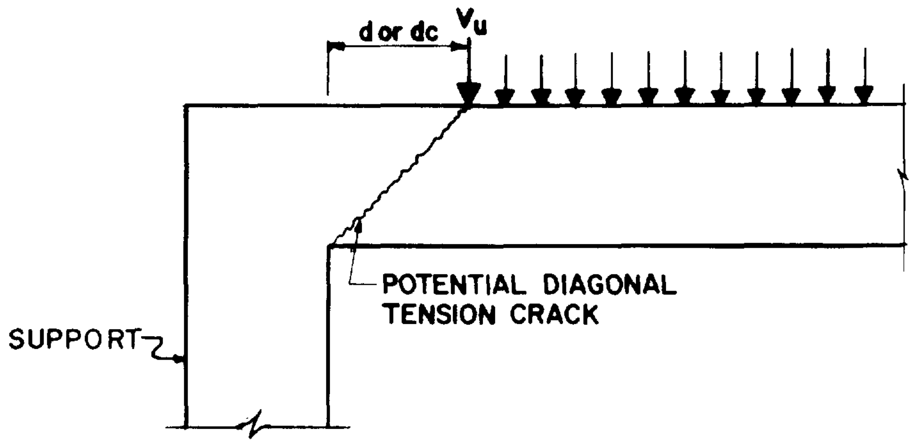
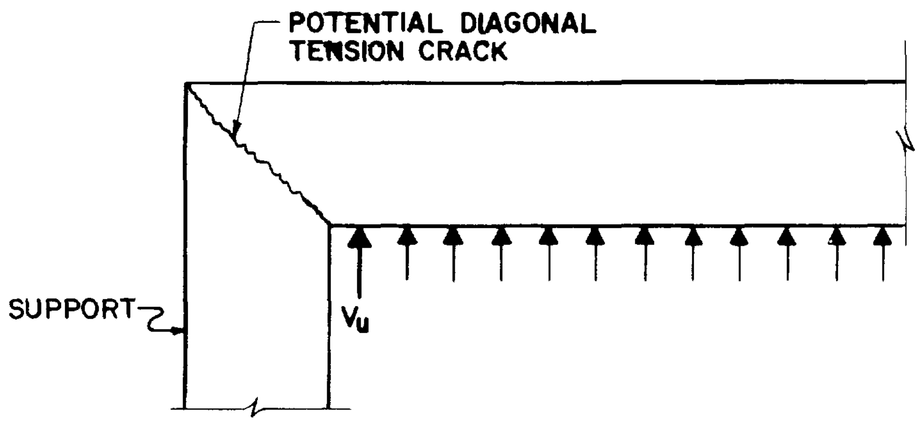


Figure 4-13 Typical reinforced concrete cross sections



a. CRITICAL SECTION AT d or d_c FROM FACE OF SUPPORT.



b. CRITICAL SECTION AT FACE OF SUPPORT

Figure 4-14 Location of critical sections for diagonal tension

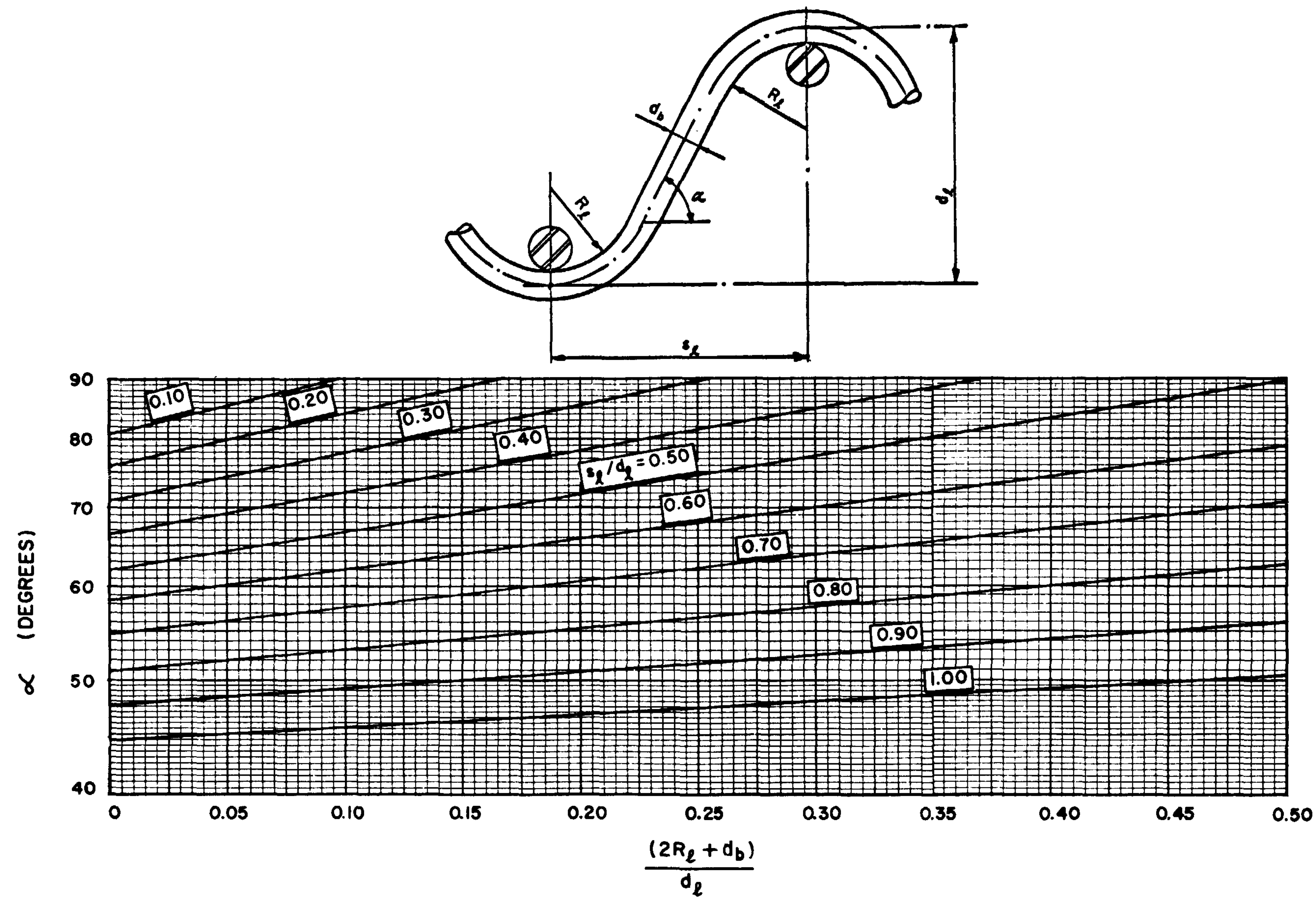


Figure 4-15 Angle of inclination of lacing bars

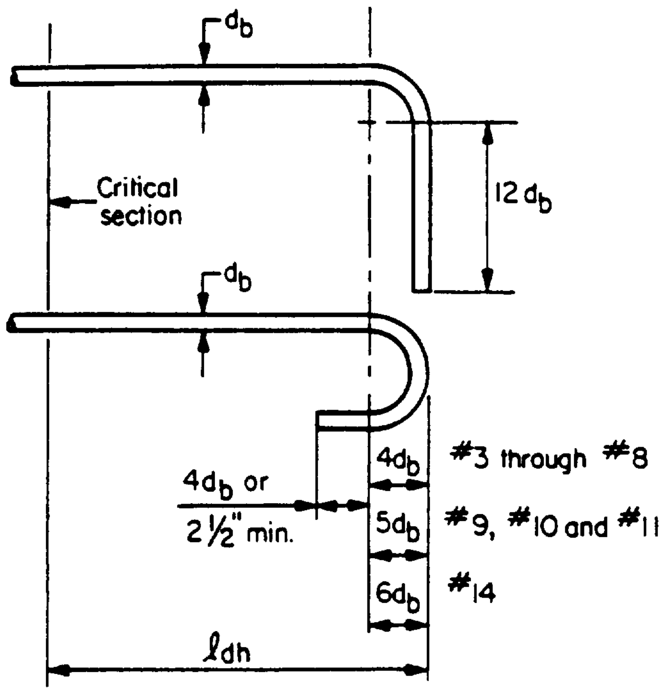


Figure 4-16 Geometry for standard hooked reinforcement

Table 4-3 **Minimum Area of Flexural Reinforcement**

Cross-Section	Reinforcement	One-Way Slabs	Two-Way Slabs
Type I	Main Direction	A_s = 0.0015bd A'_s = 0.0010bd	A_s = 0.0015bd A'_s = 0.0010bd
	Secondary Direction	A_s = 0.0010bd A'_s = 0.0010bd	A_s = 0.0015bd A'_s = 0.0010bd
Type II and Type III	Main Direction	A_s = A'_s = 0.0015bd_c	A_s = A'_s = 0.0015bd_c
	Secondary Direction	A_s = A'_s = 0.0015bd_c*	A_s = A'_s = 0.0015bd_c*

* but not less than $A_s/4$ used in the main direction

Table 4-4 **Minimum Design Shear Stresses for Slabs**

Design Range	Type of Cross-Section	Type of Structural Action	Type of Shear Reinforcement	Excess Shear Stress $v_u - v_c$		
				$v_u \le v_c$	$v_c < v_u \le 1.85v_c$	$v_u > 1.85v_c$
FAR	Type I	Flexure	Stirrups	0	$v_u - v_c$	$v_u - v_c$
	Type II	Flexure	Stirrups	$0.85v_c$	$0.85v_c$	$v_u - v_c$
	Type II & Type III	Tension Membrane	Stirrups	0	$v_u - v_c$	$v_u - v_c$
CLOSE-IN	Type II & Type III	Flexure or Tension Membrane	Stirrups or Lacing	$0.85v_c$	$0.85v_c$	$v_u - v_c$

DESIGN OF NON-LACED REINFORCED SLABS

4-22. Introduction

Conventional reinforced concrete elements are for the purpose of this manual, members without lacing. These non-laced elements make up the bulk of protective concrete construction. They are generally used to withstand the blast and fragment effects associated with the far design range but may also be designed to resist the effects associated with the close-in design range. In the latter case, the distance between the center of the explosive charge and the element must not be less than that corresponding to a scaled distance Z equal to 1.0. Laced elements are required for scaled distances less than 1.0. Non-laced elements may be designed to attain small or large deflections depending upon the protection requirements of the acceptor system.

A non-laced element designed for far range effects may attain deflections corresponding to support rotations up to 2 degrees under flexural action. Single leg stirrups are not required to attain this deflection. However, shear reinforcement is required if the shear capacity of the concrete is not sufficient to develop the ultimate flexural strength. A type I cross-section provides the ultimate moment capacity. The flexural action of a non-laced element may be increased to 4 degrees support rotation if single leg stirrups are provided to restrain the compression reinforcement. In this deflection range, a type II cross-section provides the ultimate moment capacity and mass to resist motion.

Single leg stirrups must be provided when a non-laced element is designed to resist close-in effects. The shear reinforcement must be provided to prevent local punching shear failure. When the explosive charge is located at scaled distances less than 1.0, a laced rather than a non-laced element must be employed. For scaled distances greater than 1.0 but less than 3.0, single leg stirrups must be provided, while for scaled distances greater than 3.0, shear reinforcement should be used only if required by analysis. With single leg stirrups, the member may attain deflections corresponding to support rotations up to 4 degrees under flexural action. A type I and II cross-section provides the ultimate moment capacity and mass to resist motion for elements designed for 2 and 4 degrees support rotation, respectively. If spalling occurs then a type III cross-section would be available. In addition, a non-laced element designed for small deflections in the close-in design range is not reusable and, therefore, cannot sustain multiple incidents.

A non-laced reinforced element may be designed to attain large deflections, that is, deflections corresponding to incipient failure. These increased deflections are possible only if the element has sufficient lateral restraint to develop in-plane forces. The element may be designed for both the close-in and far design range. A type III cross-section provides the ultimate moment capacity and mass to resist motion.

The design of non-laced reinforced elements subjected to a dynamic load involves an iterative (trial and error) design procedure. An element is assumed and its adequacy is verified through a dynamic analysis. The basic data required to determine the ultimate strength of the reinforced concrete section has been presented in previous sections. Procedures to determine the resistance-deflection function used for design, the dynamic properties of the section, and the dynamic analysis required to determine an element's response is presented in Chapter 3. This section contains additional data needed to establish the resistance-deflection curve for design as well as procedures to design the element for shear.

The interrelationship between the various parameters involved in the design of non-laced elements is readily described with the use of the idealized resistance-deflection curve shown in Figure 4-17.

4-23. Distribution of Flexural Reinforcement

4-23.1. General

A prime factor in the design of any facility is construction economy. Construction costs are divided between labor and material costs. Labor cost is further divided into shop and field work, with field labor being generally the more costly. Labor cost can account for as mush as 70% of the cost of blast resistant reinforced concrete. Proper selection of concrete thickness and reinforcement steel will result in a design which optimizes the structural resistance and minimizes required construction materials. Subsequent sections will discuss procedures to optimize the required materials. The designer must then evaluate the issue of constructability. Factors such as standardization of rebar sizes, spacing, congestion, steel erection and concrete placement difficulties. These considerations may require that the initial design, optimized for material quantities, be modified. Such a modification may actually increase the cost of materials while reducing the overall construction cost by reducing labor intensive activities. In addition, improved constructability greatly reduces the risk of quality control problems during construction.

4-23.2. Optimum Reinforcement Distribution

For a given total amount of flexural reinforcement and a given concrete thickness, the dynamic capacity of an element varies with the amount of reinforcement placed in the vertical direction to the amount in the horizontal direction. For a given support condition and aspect ratio L/H, the ideal distribution of the reinforcement will result in the most efficient use of the reinforcement by producing the greatest blast capacity.

The optimum distribution of the reinforcement for a two-way element is that distribution which results in positive yield lines that bisect the 90 degree angle at the corners of the element (45 degree yield lines). For two-way elements there are numerous combinations of support conditions with various moment capacities due either to quantity of reinforcement provided or degree of edge restraint as well as various positive moment capacities again due to variations in the quantity of reinforcement provided. Due to these variations in possible moment capacities, the ratio of the vertical to horizontal reinforcement cannot be expressed as a function of the aspect ratio L/H for different support conditions. Figures 3-4 through 3-20 must be used to determine the moment capacities which will result in a "45 degree yield line". The reinforcement can then be selected from these moment capacities.

In some design cases, it may not be possible to furnish the optimum distribution of reinforcement in a particular element. One such case would be where the optimum distribution violates the maximum or minimum ratio of the vertical to horizontal reinforcement A_{sV}/A_{sH} of 4.0 and 0.25, respectively. A second situation would arise when the optimum distribution requires less than minimum reinforcement in one direction. The most common case would result from the structural configuration of the building in which support moments may not be fully developed (restrained support rather than fully fixed) or from the need of maintaining continuous reinforcement from adjacent elements. In these and other situations where the optimum distribution of the reinforcement cannot be provided, the reinforcement should be furnished to give a distribution as close as the situation allows to the optimum distribution to maintain an economical design.

4-23.3. Optimum Total Percentage of Reinforcement

The relationship between the quantity of reinforcement to the quantity of concrete which results in the minimum cost of an element may be expressed as a total percentage of reinforcement. This total percentage of reinforcement p_T is defined as

$$p_T = p_V + p_H \qquad 4\text{-}39$$

where p_V and p_H are the average percentages of reinforcement on one face of the element in the vertical and horizontal directions, respectively. Based on the average costs of concrete and steel, the optimum percentage for a nonlaced reinforced element using single leg stirrups has been determined to be between 0.6 and 0.8 percent with 0.7 a reasonable design value. For elements which do not contain shear reinforcement, the optimum percentage would be somewhat higher. For large projects, a detailed cost analysis should be performed to obtain a more economical design.

In some design cases, it may be desirable to reduce the concrete thickness below low the optimum thickness. A small increase in cost (10 percent) would be incurred by increasing the value of p_T to one percent. Beyond one percent, the cost increase would be more rapid. However, except for very thin elements, it may be impractical to furnish such large quantities of reinforcement. In fact, in thick walls it may be impractical to even furnish the optimum percentage.

Unless single leg stirrups are required for other than shear capacity such as for close-in effects or to extend flexural action in the far design range from 2 to 4 degrees support rotation, it is more economical to design non-laced elements without shear reinforcement. In this case, the total percentage of reinforcement must be limited so that the ultimate resistance of the element does not produce shear stresses in excess of the concrete capacity.

4-24. Flexural Design for Small Deflections

The design range for small deflections may be divided into two regions; elements with support rotations less than 2 degrees (limited deflections) and support rotations between 2 and 4 degrees. Except for stirrup requirements and the type of cross-section available to resist moment, the design procedure is the same.

In the flexural design of a non-laced reinforced concrete slab, the optimum distribution of the flexural reinforcement must first be determined. A 45 degree yield line pattern is assumed and, based on the support conditions and aspect ratio, the ratio of the vertical to horizontal moment capacities are determined from the yield line location figures of Chapter 3.

Reinforcing bars and a concrete thickness are next chosen such that the distribution of reinforcement is as close as possible to that determined above and such that the total reinforcement ratio p_T is approximately 0.7 for elements utilizing stirrups. For those elements not utilizing shear reinforcement p_T is minimized so that the shear capacity of the concrete is not exceeded. Using the equations of previous sections (eq. 4-11 for type I cross sections, eq. 4-19 for type II or III cross-sections) the moment capacities are computed. The moment capacities are required to calculate the ultimate unit resistance r_u and the equivalent elastic deflection X_E . These parameters along with the natural period of vibration T_N define the equivalent single-degree-of-freedom system of the slab, and are discussed in detail in Chapter 3.

A dynamic analysis (see Section 4-26) is then performed to check that the slab meets the response criteria. Lastly, the shear capacity is checked (Section 4-27). If the slab does not meet the response criteria or fails in shear (or is greatly overdesigned) a new concrete section is assumed and the entire design procedure is repeated.

4-25. Design for Large Deflections

4-25.1. Introduction

Design of non-laced reinforced concrete elements without shear reinforcement (single leg stirrups) for support rotations greater than 2 degrees or elements with single leg stirrups for support rotations greater than 4 degrees depends on their capacity to act as a tensile membrane. Lateral restraint of the element must be provided to achieve this action. Thus, if lateral restraint does not exist, tensile membrane action is not developed and the element reaches incipient failure at 2 degrees (4 degrees if adequate single leg stirrups are provided) support rotation. However, if lateral restraint exists, deflection of the element induces membrane action and in-plane forces. These in-plane forces provide the means for the element to continue to develop substantial resistance up to maximum support rotations of approximately 12 degrees.

4-25.2. Lateral Restraint

Adequate lateral restraint of the reinforcement is mandatory in order for the element to develop and the designer to utilize the benefits of tensile membrane behavior. Sufficient lateral restraint is provided if the reinforcement is adequately anchored into adjacent supporting members capable of resisting the lateral force induced by tensile membrane action.

Tensile membrane behavior should not be considered in the design process unless full external lateral restraint is provided in the span directions shown in Table 4-5. Full external lateral restraint means that adjacent members can effectively resist a total lateral force equivalent to the ultimate strength of all continuous reinforcement in the element passing the boundary identified by the arrows in Table 4-5. External lateral restraint is not required for elements supported on four edges provided the aspect ratio L/H is not less than one-half nor greater than 2. Within this range of L/H, the inherent lateral restraint provided by the element's own compression ring around its boundary is sufficient lateral restraint to develop tensile membrane behavior.

4-25.3. Resistance-Deflection Curve

A typical resistance-deflection curve for laterally restrained elements is shown in Figure 4-18. The initial portion of the curve is due primarily to flexural action. If the lateral restraint prevents small motions, in-plane compressive forces are developed. The increased capacity due to these forces is neglected and is not shown in Figure 4-18. The ultimate flexural resistance is maintained until 2 degrees support rotation is produced. At this support rotation, the concrete begins to crush and the element loses flexural capacity. If adequate single leg stirrups were provided, the flexural action would be extended to 4 degrees. However, due to the presence of continuous reinforcement and adequate lateral restraint, tensile membrane action is developed. The resistance due to this action increases with increasing deflection up to incipient failure at approximately 12 degrees support rotation. The tensile membrane resistance is shown as the dashed line in Figure 4-18.

In order to simplify the design calculations, the resistance is assumed to be due to flexural action throughout the entire range of behavior. To approximate the energy absorbed under the actual resistance-deflection curve, the deflection of the idealized curve is limited to 8 degrees support rotation. Design for this maximum deflection would produce incipient failure conditions. Using this equivalent design curve, deflections between 2 degrees (or 4 degrees if single leg stirrups are provided) and incipient failure cannot be accurately predicted.

For the design of a non-laced laterally restrained element for 8 degrees support rotation, a type III cross-section is used to compute the ultimate moment capacity of the section as well as to provide the mass to resist motion. The stress in the reinforcement f_{ds} would be equal to that corresponding to support rotations $5 \geq \theta_m \geq 12$ given in Table 4-2. At every section throughout the element, the tension and compression reinforcement must be continuous in the restrained direction(s) in order to develop the tensile membrane action which is discussed in detail below.

4-25.4. Ultimate Tensile Membrane Capacity

As can be seen in Figure 4-18, the tensile membrane resistance of an element is a function of the element's deflection. It is also a function of the span length and the amount of continuous reinforcement. The tensile membrane resistance, r_T of a laterally restrained element at a deflection, X, is expressed as:

For one-way elements

$$r_T = X \left[\frac{8T_y}{L_y^2}\right] \qquad 4\text{-}40$$

For two-way elements

$$r_T = \frac{1.5\, X\pi^3 T_y / L_y^2}{4 \sum_{n=1,3,5} \left[\frac{1}{n^3}(-1)^{(n-1)/2} \left[1 - \frac{1}{\cosh \left[\frac{n\pi L_x}{2L_y} \left[\frac{T_y}{T_x} \right]^{1/2} \right]} \right] \right]} \qquad 4\text{-}41$$

in which

$$T_y = (A_s)_y f_{ds} \qquad 4\text{-}42$$

and

$$T_x = (A_s)_x f_{ds} \qquad 4\text{-}43$$

where

r_T = tensile membrane resistance

X = deflection of element

L_y = clear span in short direction

L_x = clear span in long direction

T_y = force in the continuous reinforcement in the short direction

T_x = force in the continuous reinforcement in the long direction

$(A_s)_y$ = continuous reinforcement in the short direction

$(A_s)_x$ = continuous reinforcement in the long direction

Even though the capacity of a laterally restrained element is based on flexural action, adequate tensile membrane capacity must be provided. That is, sufficient continuous reinforcement must be provided so that the tensile membrane resistance r_T corresponding to 8 degrees support rotation must be greater than the flexural resistance r_u. The deflection is computed as a function of the yield line locations (shortest sector length). The force in the continuous reinforcement is calculated using the dynamic design stress f_{ds} corresponding to 8 degrees support rotation (Table 4-2).

4-25.5. Flexural Design

Since the actual tensile membrane resistance deflection curve is replaced with an equivalent flexural curve, the design of a non-laced element for large deflections is greatly simplified. The design is performed in a similar manner as for small deflections. However, sufficient continuous reinforcement must be provided to develop the required tensile membrane resistance. Where external restrain is required, the support must withstand the lateral forces T_y and T_x as given in Equations 4-42, and 4-43, respectively.

4-26. Dynamic Analysis

4-26.1. Design for Shock Load

The dynamic analysis of a slab is accomplished by first representing it as a single-degree-of-freedom system and then finding the response of that system when subject to a blast load. The equivalent single-degree-of-freedom system is defined in terms of its ultimate resistance r_u, equivalent elastic deflection X_E and natural period of vibration T_N. The ultimate unit resistance is calculated from the equations of Chapter 3 for the moment capacities determined according to the previous sections. The procedures and parameters necessary to obtain the equivalent elastic deflection and natural period of vibration can also be found in Chapter 3.

For elements subjected to dead loads in the same direction as the blast loads (for example a roof or retaining wall exposed to an exterior explosion) the resistance available to withstand the blast load is reduced. An approximation of the resistance available is

$$r_{avail} = r_u - r_{DL} \left[\frac{f_{ds}}{f_{dy}} \right] \qquad 4\text{-}44$$

where

r_{avail} = dynamic resistance available

r_{DL} = uniform dead load

Chapter 2 describes procedures for determining the dynamic load which is defined by its peak value P and duration T. For the ratios P/r_u and T/T_N, the ductility ratio X_m/X_E and t_m/T can be obtained from the response charts of Chapter 3. These values, X_m which is the maximum deflection, and t_m, the time to reach the maximum deflection, define the dynamic response of the element.

The effective mass and the stiffness used in computing the natural period of vibration T_N depends on the type of cross section and loadmass factor used, both of which depend on the range of the maximum deflection. When the deflections are small (less than 4 degrees) a type I or type II cross section is used. The mass is calculated using the entire thickness of the concrete element T_c. The spalling that occurs when an element acts under tensile membrane behavior or which may occur due to close-in effects requires the use of a type III cross section to resist moment. Since the concrete cover over the flexural reinforcement is completely disengaged, the mass is calculated based on the distance between the centroids of the compression and tension reinforcement.

When designing for completely elastic behavior, the elastic stiffness is used while, in other cases, the equivalent elasto-plastic stiffness K_E is used. The elastic value of the load-mass factor K_{LM} is used for the elastic range while, in the elasto-plastic range, the load-mass factor is the average of the elastic and elasto-plastic values. For small plastic deformations, the value of K_{LM} is equal to the average of the equivalent elastic value and the plastic value. The plastic value of K_{LM} is used for slabs designed for large plastic deformations.

Due to the large number of variables involved in the design of non-laced reinforced elements, design equations have not been developed. However, design equations have been developed for laced elements subjected to impulse loads and are presented in subsequent sections of this chapter. Use of these procedures for the design of non-laced elements subjected to impulse load will result in a variety of errors depending upon support conditions, thickness of the concrete section, quantity and distribution of the flexural reinforcement, etc. However, these procedures may be used to obtain a trial section which then may be analyzed as described above.

4-26.2. Design for Rebound

Elements must be designed to resist rebound, that is, the damped elastic or elasto-plastic harmonic motion which occurs after the maximum positive displacement X_m has been attained. When an element reaches X_m, the resistance is at a maximum, the velocity is zero, and its deceleration is a maximum. The element will vibrate about the blast load curve (positive and/or negative phase) and/or the zero line (dead load for roofs) depending on the time to reach maximum deflection t_m and the duration of the blast load T.

Usually only those elements with a type I cross-section will require additional reinforcement to resist rebound. Additional reinforcement is not required for type II and III cross-sections since these sections have equal reinforcement on opposite faces and the maximum possible rebound resistance is equal to the ultimate (positive) resistance. However, the supports for all types of cross sections , including the anchorage of the reinforcement (compression reinforcement under positive phase loading is subjected to tension forces under rebound conditions) must be investigated for rebound (negative) reactions. Also, it should be noted that the support conditions for rebound are not always the same as for the positive load.

The negative resistance r^-, attained by an element when subjected to a triangular pressure-time load, is obtained from figure 3-268 of Chapter 3. Entering the figure with the ratios of X_m/X_E and T/T_N, previously determined for the positive phase of design, the ratio of the required rebound resistance to the ultimate resistance r^-/r_u is obtained. The element must be reinforced to withstand this rebound resistance r^- to insure that the slab will remain elastic during rebound. However, in some cases, negative plastic deformations are permissible.

The tension reinforcement provided to withstand rebound forces is added to what is the compression zone during the initial loading phase. To obtain this reinforcement, the element is essentially designed for a negative load equal to the calculated value of r^-. However, in no case shall the rebound reinforcement be less than one-half of the positive phase reinforcement. The moment capacities and the rebound resistance capacity are calculated using the same equations previously presented. Note that while dead load reduces the available resistance for the dynamic loading, this load increases the available resistance for rebound.

4-27. Design for Shear

4-27.1. General

The ultimate shear V_u at any section of a flexural element is a function of its geometry, yield line location and unit resistance r. For one-way or two-way elements the ultimate shear is developed when the resistance reaches the ultimate unit value r_u. In the design of a concrete element, there are two critical locations where shear must be considered. The ultimate shear stress v_u is calculated at a distance d or d_c from the supports to check the diagonal tension stress and to provide shear reinforcement (stirrups) is necessary. The direct shear force or the ultimate support shear V_s is calculated at the face of the support to determine the required quantity of diagonal bars.

4-27.2. Ultimate Shear Stress at d_e from the Support

4-27.2.1. One-Way Elements

The ultimate shear stresses v_u at a distance d_e from the support are given in Table 4-6 for one-way elements. Depending upon the cross section type being considered, d_e can represent either d or d_c. For those cases where an element does not reach its ultimate resistance r_u is replaced by the actual resistance r attained by the element. For those members whose loading causes tension in their supports, the ultimate shear stress is calculated at the face of the support. For those cases, the ultimate support shear V is calculated as explained in the next section. This shear is then divided by the effective cross-sectional area (bd or bd_c) of the element to obtain the ultimate shear stress.

4-27.2.2. Two-Way Elements

For two-way elements the ultimate shear stress must be calculated at each support. The shears acting at each section are calculated using the yield line procedure outlined in Chapter 3 for the determination of the ultimate resistance r_u. Because of the higher stiffness at the corners, the shear along any section parallel to the support varies. The full shear stress V acts along the supports except in the corners where only 2/3 of the shear stress is used (Fig. 4-19). Since the shear is zero along the yield lines, the total shear at any section of the sector is equal to the resistance r_u times the area between the section being considered and the positive yield lines.

To illustrate this procedure, consider a two-way element, fixed on three sides and free on the fourth, with the yield line pattern as shown in Figure 4-19. For the triangular sector I, the shear V_{dV} and shear stress v_{uV} at distance d_e from the support is

$$\frac{2}{3} V_{dv} \left(\frac{L}{4} - \frac{d_e L}{2y} + \frac{L}{4} - \frac{d_e L}{2y}\right) + V_{dv}\left(\frac{L}{2}\right) = r_u \left(\frac{y-d_e}{2}\right)\left(L - \frac{d_e L}{y}\right) \qquad \text{4-45a}$$

$$V_{dv} = \frac{3r_u y\ (1 - d_e/y)^2}{(5 - 4\ d_e/y)} \qquad 4\text{-}45b$$

and since the shear stress v_u is equal to V/bd_e and b equals one inch

$$v_{uv} = \frac{3r_u\ (1 - d_e/y)^2}{\frac{d_e}{y}\ (5 - 4\ d_e/y)} \qquad 4\text{-}46$$

For the trapezoidal sector II

$$\frac{2}{3} V_{dH} \left(\frac{y}{2} - \frac{2d_e y}{L}\right) + V_{dH} \left(H - \frac{y}{2}\right)$$

$$= r_u \left(\frac{L}{2} - d_e\right) \frac{1}{2} \left(H - y + H - \frac{2d_e y}{L}\right) \qquad 4\text{-}47a$$

$$V_{dh} = \frac{3r_u(L - 2\ d_e) \left[2H - y - \frac{2d_e y}{L}\right]}{2 \left[6H - y - \frac{8d_e y}{L}\right]} \qquad 4\text{-}47b$$

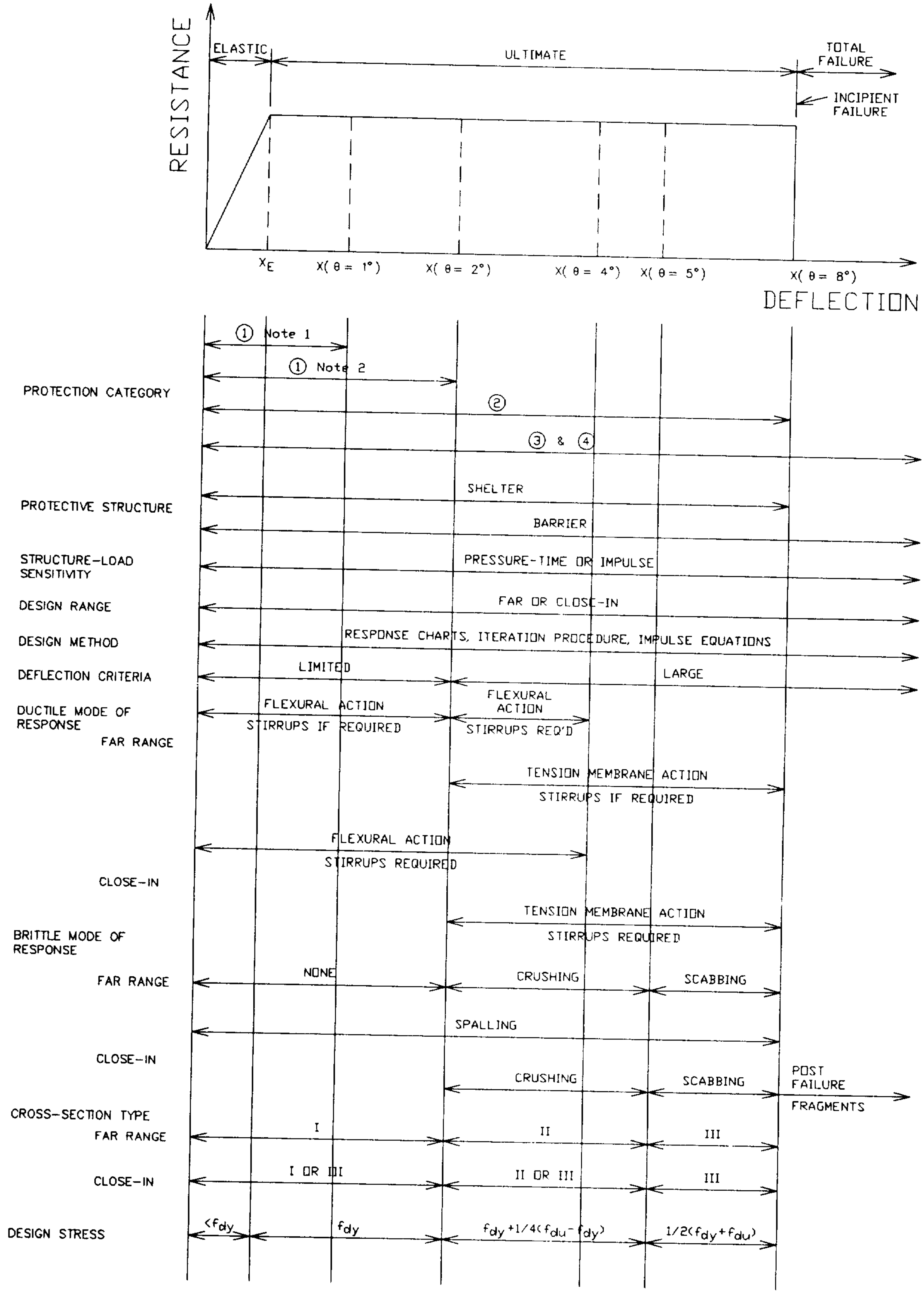
$$v_{uH} = \frac{3r_u \left(1 - \frac{2d_e}{L}\right) \left(2 - \frac{y}{H} - \frac{2d_e y}{LH}\right)}{2 \frac{d_e}{L} \left(6 - \frac{y}{H} - \frac{8d_e y}{LH}\right)} \qquad 4\text{-}48$$

Values of the ultimate shear stresses v_{uH} and v_{uV} at a distance d_e from the support for several two-way elements are given in Table 4-7. As stated above, d_e represents either d or d_c, depending upon the type of cross section being considered. The ultimate shear stress is calculated at the face of the support for those members whose loading condition causes tension in their supports. For these cases, the ultimate support shear V is calculated as explained in the next section. This shear is then divided by the effective cross-sectional area (bd or bd_c) of the element to obtain the ultimate shear stress v.

For the situations where the ultimate resistance of an element is not attained, the maximum shear stress is less than the ultimate value. However, the distribution of the shear stresses is assumed to be the same and, therefore, the shear stresses can be calculated from the equation of Table 4-7 by replacing r_u with the actual resistance attained (r_e, r_{ep}, etc.).

4-27.3. Ultimate Support Shear

See Chapter 3, for procedures used to calculate the ultimate shears of both one-way and two-way elements.



Note 1: Neither shear reinforcement nor tensile membrane action
Note 2: Either shear reinforcement or tensile membrane action

Figure 4-17 Relationship between design parameters for unlaced elements

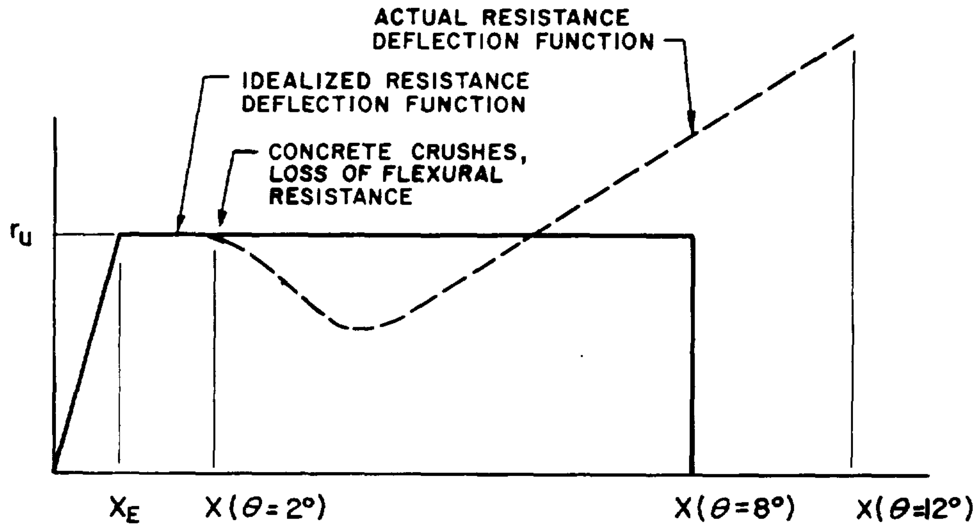


Figure 4-18 Idealized resistance-deflection curve for large deflections

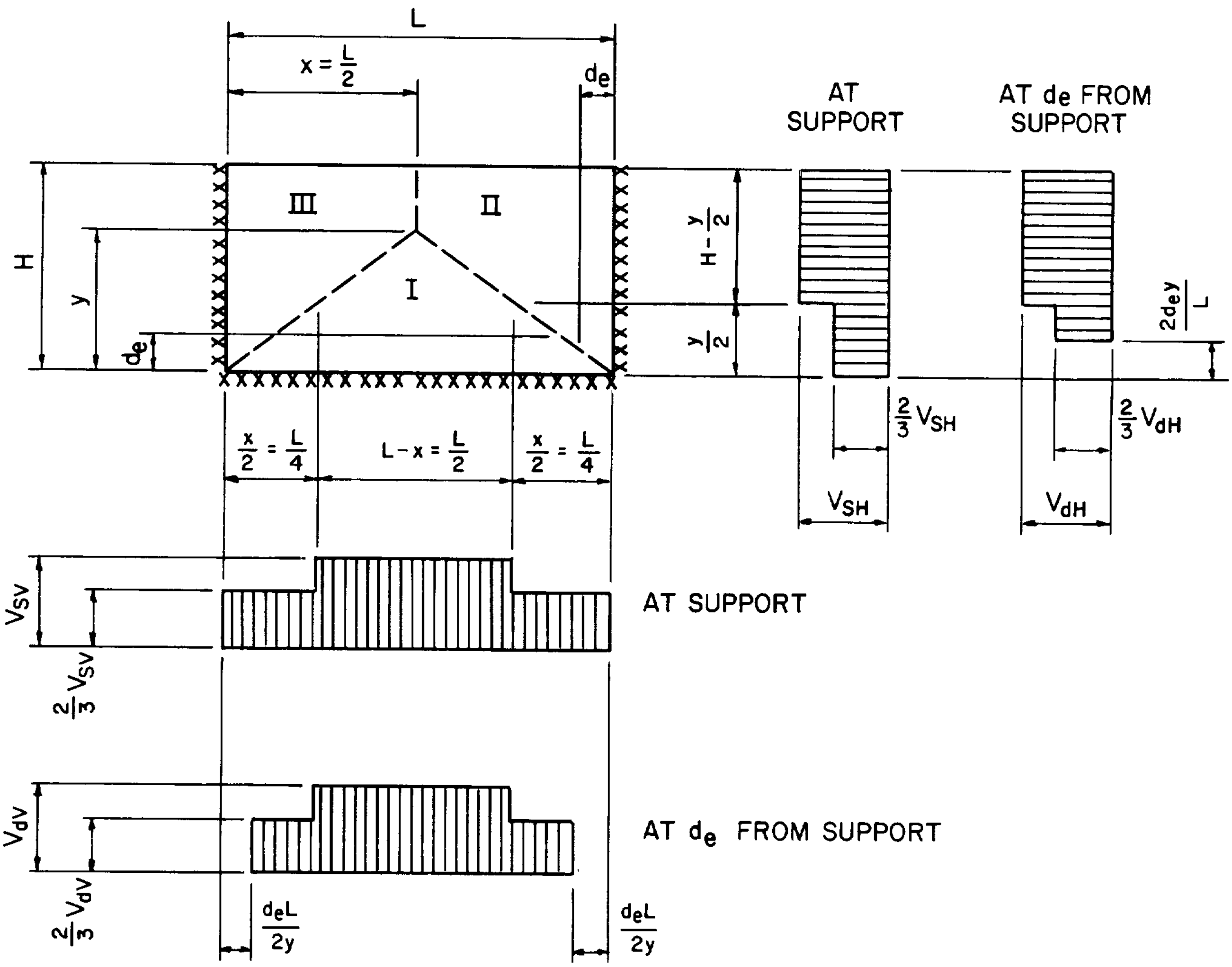
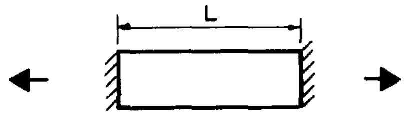
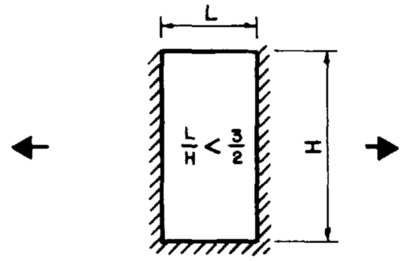
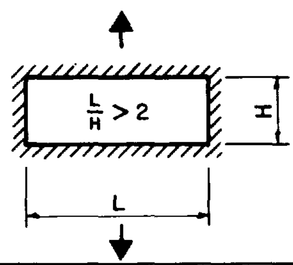
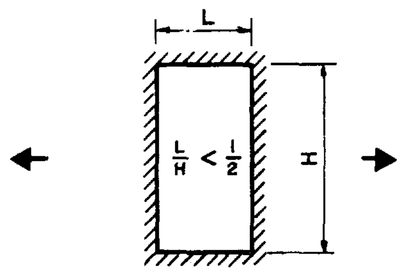
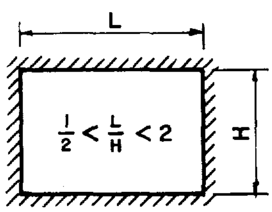


Figure 4-19 Determination of ultimate shears

Table 4-5 Restraint and Aspect Ratio Requirements for Tension Membrane Behavior

Edge Support Conditions	External Lateral Restraint Requirements
	Opposite edges
	Opposite edges
	Opposite edges short direction
	Opposite edges short direction
	None required

Indicates direction of external lateral restraint which must be resisted by adjacent member

Table 4-6 Ultimate Shear Stress at Distance d_e from Face of Support for One-Way Elements

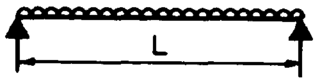
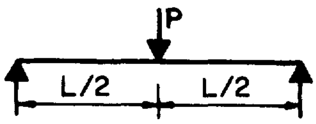
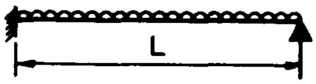
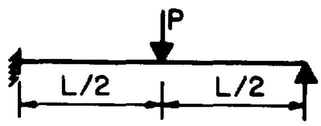
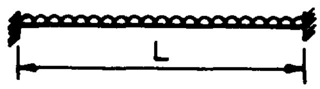
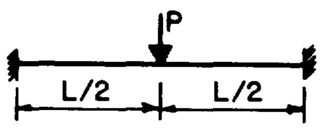
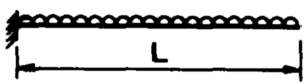
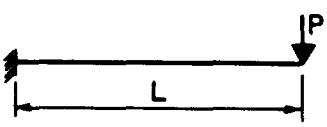
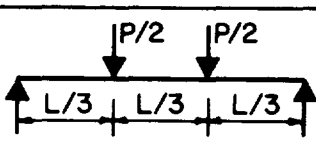
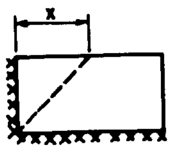
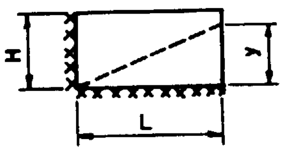
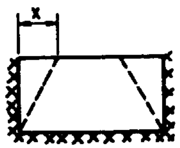
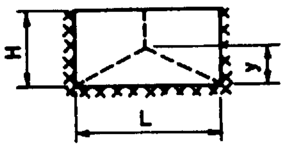
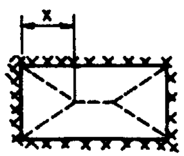
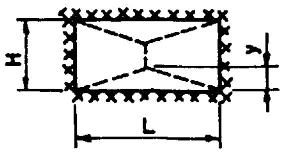
Edge Conditions and Loading Diagrams	ULTIMATE SHEAR STRESS V_u
	$\frac{r_u(\frac{L}{2} - d_e)}{d_e}$
	$\frac{R_u}{2d_e}$
	LEFT SUPPORT $r_u(\frac{5L}{8} - d_e)/d_e$ RIGHT SUPPORT $r_u(\frac{3L}{8} - d_e)/d_e$
	LEFT SUPPORT $\frac{11R_u}{16d_e}$ RIGHT SUPPORT $\frac{5R_u}{16d_e}$
	$\frac{r_u(\frac{L}{2} - d_e)}{d_e}$
	$\frac{R_u}{2d_e}$
	$\frac{r_u(L - d_e)}{d_e}$
	$\frac{R_u}{d_e}$
	$\frac{R_u}{2d_e}$

Table 4-7 Ultimate Shear Stress at Distance d_e from Face of Support for Two-Way Elements

Edge conditions	Yield line location	Limits	Ultimate horizontal shear stress v_{uH}	Limits	Ultimate vertical shear stress v_{uV}
Two adjacent edges fixed and two edges free		$0 \leq d_e/x \leq \frac{1}{2}$	$\frac{3r_u(1-d_e/x)^2}{d_e/x(5-4d_e/x)}$	$0 \leq d_e/H \leq \frac{1}{2}$	$\frac{3r_u(1-d_e/H)(2-x/L-d_ex/HL)}{d_e/H(6-x/L-4d_ex/HL)}$
		$\frac{1}{2} \leq d_e/x \leq 1$	$\frac{r_u(1-d_e/x)}{2(d_e/x)}$	$\frac{1}{2} \leq d_e/H \leq 1$	$\frac{r_u(1-d_e/H)(2-x/L-d_ex/HL)}{2d_e/H(1-d_ex/HL)}$
		$0 \leq d_e/L \leq \frac{1}{2}$	$\frac{3r_u(1-d_e/L)(2-y/H-d_ey/LH)}{d_e/L(6-y/H-4d_ey/LH)}$	$0 \leq d_e/y \leq \frac{1}{2}$	$\frac{3r_u(1-d_e/y)^2}{d_e/y(5-4d_e/y)}$
		$\frac{1}{2} \leq d_e/L \leq 1$	$\frac{r_u(1-d_e/L)(2-y/H-d_ey/LH)}{2d_e/L(1-d_ey/LH)}$	$\frac{1}{2} \leq d_e/y \leq 1$	$\frac{r_u(1-d_e/y)}{2d_e/y}$
Three edges fixed and one edge free		$0 \leq d_e/x \leq \frac{1}{2}$	$\frac{3r_u(1-d_e/x)^2}{d_e/x(5-4d_e/x)}$	$0 \leq d_e/H \leq \frac{1}{2}$	$\frac{3r_u(1-d_e/H)(1-x/L-d_ex/HL)}{d_e/H(3-x/L-4d_ex/HL)}$
		$\frac{1}{2} \leq d_e/x \leq 1$	$\frac{r_u(1-d_e/x)}{2(d_e/x)}$	$\frac{1}{2} \leq d_e/H \leq 1$	$\frac{r_u(1-d_e/H)(1-x/L-d_ex/HL)}{d_e/H(1-2d_ex/HL)}$
		$0 \leq d_e/L \leq \frac{1}{4}$	$\frac{3r_u(1-2d_e/L)(2-y/H-2d_ey/LH)}{2d_e/L(6-y/H-8d_ey/LH)}$	$0 \leq d_e/y \leq \frac{1}{2}$	$\frac{3r_u(1-d_e/y)^2}{d_e/y(5-4d_e/y)}$
		$\frac{1}{4} \leq d_e/L \leq \frac{1}{2}$	$\frac{r_u(1-2d_e/L)(2-y/H-2d_ey/LH)}{4d_e/L(1-2d_ey/LH)}$	$\frac{1}{2} \leq d_e/y \leq 1$	$\frac{r_u(1-d_e/y)}{2d_e/y}$
Four edges fixed		$0 \leq d_e/x \leq \frac{1}{2}$	$\frac{3r_u(1-d_e/x)^2}{d_e/x(5-4d_e/x)}$	$0 \leq d_e/H \leq \frac{1}{4}$	$\frac{3r_u(\frac{1}{2}-d_e/H)(1-x/L-2d_ex/HL)}{d_e/H(3-x/L-8d_ex/HL)}$
		$\frac{1}{2} \leq d_e/x \leq 1$	$\frac{r_u(1-d_e/x)}{2d_e/x}$	$\frac{1}{4} \leq d_e/H \leq \frac{1}{2}$	$\frac{r_u(\frac{1}{2}-d_e/H)(1-x/L-2d_ex/HL)}{d_e/H(1-4d_ex/HL)}$
		$0 \leq d_e/L \leq \frac{1}{4}$	$\frac{3r_u(\frac{1}{2}-d_e/L)(1-y/H-2d_ey/LH)}{d_e/L(3-y/H-8d_ey/LH)}$	$0 \leq d_e/y \leq \frac{1}{2}$	$\frac{3r_u(1-d_e/y)^2}{d_e/y(5-4d_e/y)}$
		$\frac{1}{4} \leq d_e/L \leq \frac{1}{2}$	$\frac{r_u(\frac{1}{2}-d_e/L)(1-y/H-2d_ey/LH)}{d_e/L(1-4d_ey/LH)}$	$\frac{1}{2} \leq d_e/y \leq 1$	$\frac{r_u(1-d_e/y)}{2d_e/y}$

DESIGN OF FLAT SLABS

4-28. Introduction

The typical unhardened flat slab structure consists of a two-way slab supported by columns. Except for edge beams which may be used at the exterior edge of the slab, beams and girders are not used to transfer the loads into the columns. In this case, the columns tend to punch upward through the slab. There are several methods that can be used to prevent this; the upper end of the column can be enlarged creating a column capital, a drop panel can be added by thickening the slab in the vicinity of the column, or both a column capital and a drop panel may be used.

Hardened flat slab structures may be designed to withstand the effects associated with a far range explosion. The flat slab of a hardened structure is similar to an unhardened slab but for a hardened flat slab structure, the exterior supports must be shear walls which are monolithic with the roof. The shear walls transmit the lateral loads to the foundations. Due to the stiffness of the walls, there is negligible sidesway in the columns and hence no induced moments due to lateral loads. Shear walls may also replace a row of interior columns if additional stiffness is required. Earth cover may or may not be used for hardened flat slab structures.

A portion of a typical hardened flat slab structure is shown in Figure 4-20. As depicted, there are generally four different panels to be considered, interior, corner and two exterior, each of which has a different stiffness. The exterior panels are designated as short and long span panels which refers to the length of the span between the columns and the exterior wall. In the typical flat slab, the reinforcement would be distributed according to elastic theory. The elastic distribution of the flexural stresses is approximated by the methods presented in the ACI Building Code, The static design must meet all of the criteria of the code as well as of all applicable local codes.

However, for blast resistant structures, certain design criteria are more restrictive than those given in the ACI Building Code. To ensure two-way action in the slab, the aspect ratio L/H of each panel must be greater than 1 but less than 2. While the ACI code permits unequal span lengths and offset columns, it is strongly recommended that offset columns not be used and the variation in span lengths be limited to 10 percent. Columns and column capitals may have either a round or square cross section, but round columns and capitals are preferred to avoid shear stress concentrations. It is also recommended that haunches be provided at the shear walls.

Flat slabs may be designed to attain limited or large deflections depending upon the magnitude and duration of the applied blast load and the level of protection required by the acceptor system. Under flexural action alone, the slab may attain deflections corresponding to 2 degrees support rotation. The flexural action may be extended to 4 degrees rotation if single leg stirrups are added to restrain the flexural reinforcement. If sufficient continuous flexural reinforcement is provided, the slab may attain 8 degrees support rotation through tension membrane action. Unless required for shear, single leg stirrups are not required for the slab to achieve support rotations less than 2 degrees nor tension membrane action. The stress in the reinforcement as well as the type of cross section used to determine the ultimate moment capacity of the reinforced concrete is a function of the maximum deflection.

The basic data required for determining the ultimate strength of the reinforced concrete, including the ultimate moment capacity and the ultimate shear capacity, have been presented in previous sections. Procedures for performing the dynamic analysis are presented in Chapter 3. Only modifications and additions relating to flat slabs are presented in this section. The interrelationship between the various parameters involved in the design of flat slabs is readily described with the use of the idealized resistance-deflection curve shown in Figure 4-21.

4-29. Distribution of Flexural Reinforcement

4-29.1. General

For a two-way slab continuously supported on its edges, the flexural stresses are distributed uniformly across the entire slab (except for the reduced stresses at the corners). The flexural stresses in a flat slab supported by walls and columns are distributed from one panel to the next depending on the relative stiffness of the supports and the spans of the panels. Flat slabs also distribute the flexural stresses transversely, concentrating the stresses in the vicinity of the column. A uniform distribution of reinforcement would result in a failure due to local "fan" yield lines around the columns at a relatively low resistance. By concentrating the reinforcement over the columns, a higher ultimate resistance is obtained.

An elastic distribution of reinforcement is required in the design procedure presented in this Manual. This distribution will insure the formation of a predictable collapse mechanism. Local failures around the columns, and one-or two-way folding (local one-way action) will be prevented. With an elastic distribution of reinforcement, the yield lines form simultaneously across the entire slab. In addition, the design will be more economical and cracking under service loads will be minimized.

4-29.2 Elastic Distribution of Moments According to the ACI Building Code

Procedures outlined in the ACI Building Code are employed to determine the elastic distribution of the reinforcement (and hence of the moments). The Code presents two design methods, namely the Direct Design Method and the Equivalent Frame Method. The Equivalent Frame Method may be used for all flat slab configurations whereas the Direct Design Method can only be used for three or more spans. Since the Direct Design Method requires fewer calculations, it is the preferred method and is discussed in detail in this section.

For the typical flat slabs with continuous exterior walls and $L/H > 1$, the column strips are $H/2$ in width in each direction. A wall strip is parallel and adjacent to an exterior wall and its width is $H/4$. The remaining portions of the slab are called middle strips.

Using the Direct Design Method as given in Chapter 13 of ACI 318-77 the moments are distributed taking into account the relative flexural and torsional stiffnesses of the wall, slab and beams. Assuming there are no beams or interior shear walls, the ratio of the flexural stiffness of the beam section to the slab section α , is zero. The torsional resistance of a concrete wall monolithic with the slab is very large and, therefore, the torsional stiffness ratio of the wall to the slab β_t may be assumed to be greater than 2.5.

The ratio of the flexural stiffness of the exterior wall and the flexural stiffness of the slab is defined as:

$$\alpha_{ec} = \frac{(4E_c I_w)/\ H_w}{(4E_c I_s)/\ l_s} \qquad 4\text{-}49$$

where

α_{ec} = ratio of the flexural stiffness of the exterior wall to slab

I_w = gross moment of inertia of wall

I_s = gross moment of inertia of slab

H_w = height of wall

l_s = span of flat slab panel

In direction H, Equation 4-49 becomes

$$\alpha_{ecH} = \frac{T_w^{\ 3}\ H}{T_s^{\ 3}\ H_w} \qquad 4\text{-}50$$

and in direction L

$$\alpha_{ecL} = \frac{T_w^{\ 3}\ L}{T_s^{\ 3}\ H_w} \qquad 4\text{-}51$$

where

T_w = thickness of wall

H = short span of flat slab panel

T_s = thickness of flat slab

L = long span of flat slab panel

The unit column and midstrip moments are proportioned from the total span moments. The distributions percentages for a flat slab with equal spans in each direction is as follows (see Fig. 4-22):

For Direction H:

$$m_1^{\ -} = 0.65\ \alpha'_{ecH}\ M_{OH}\ /L \qquad 4\text{-}52$$

$$m_2^{\ +} = 0.40\ (0.63 - 0.28\ \alpha'_{ecH})\ M_{OH}\ /(L - H/2) \qquad 4\text{-}53$$

$$m_3^{\ -} = 0.25\ (0.75 - 0.10\ \alpha'_{ecH})\ M_{OH}\ /(L\text{-}H/2) \qquad 4\text{-}54$$

$$m_4^{\ -} = 0.25\ (0.65)\ M_{OH}\ /(L\text{-}H/2) \qquad 4\text{-}55$$

$$m_5^+ = 0.40\ (0.35)\ M_{OH}\ /(L-H/2) \qquad 4\text{-}56$$

$$m_6^+ = 0.60\ (0.63 - 0.28\ \alpha'_{ecH})\ M_{OH}\ /(H/2) \qquad 4\text{-}57$$

$$m_7^- = 0.75\ (0.75 - 0.10\ \alpha'_{ecH})\ M_{OH}\ /(H/2) \qquad 4\text{-}58$$

$$m_8^- = 0.75\ (0.65)\ M_{OH}\ /(H/2) \qquad 4\text{-}59$$

$$m_9^+ = 0.60\ (0.35)\ M_{OH}\ /(H/2) \qquad 4\text{-}60$$

in which

$$M_{OH} = \frac{wL(H-c)^2}{8} \qquad 4\text{-}61$$

and

$$\alpha'_{ecH} = \frac{1}{1 + (1/\alpha_{ecH})} \qquad 4\text{-}62$$

M_{OH} = total panel moment for direction H

w = applied uniform load

c = width of column capital

For Direction L:

$$m_{10}^- = 0.65\ \alpha'_{ecL} M_{OL}\ /H \qquad 4\text{-}63$$

$$m_{11}^+ = 0.40\ (0.63 - 0.28\ \alpha'_{ecL})\ M_{OL}\ /(H/2) \qquad 4\text{-}64$$

$$m_{12}^- = 0.25\ (0.75 - 0.10\ \alpha'_{ecL})\ M_{OL}\ /(H/2) \qquad 4\text{-}65$$

$$m_{13}^- = 0.25\ (0.65)\ M_{OL}\ /(H/2) \qquad 4\text{-}66$$

$$m_{14}^+ = 0.40\ (0.35)\ M_{OL}\ /(H/2) \qquad 4\text{-}67$$

$$m_{15}^+ = 0.60\ (0.63 - 0.28\ \alpha'_{ecL})\ M_{OL}\ /(H/2) \qquad 4\text{-}68$$

$$m_{16}^- = 0.75\ (0.75 - 0.10\ \alpha'_{ecL})\ M_{OL}\ /(H/2) \qquad 4\text{-}69$$

$$m_{17}^- = 0.75\ (0.65)\ M_{OL}\ /(H/2) \qquad 4\text{-}70$$

$$m_{18}^+ = 0.60\ (0.35)\ M_{OL}\ /(H/2) \qquad 4\text{-}71$$

in which

$$M_{OL} = \frac{wH\ (L-c)^2}{8} \qquad 4\text{-}72$$

and

$$\alpha'_{ecL} = \frac{1}{1 + (1/\alpha_{ecL})} \qquad 4\text{-}73$$

where M_{OL} is the total panel moment for direction L.

For the wall strips in both directions the same reinforcement as in the adjacent middle strips is used. At the column, the larger of the two negative moments is chosen and the positive moment can then be adjusted up to 10 percent so that the total panel moments remain unchanged.

4-29.3 Design for Small Deflections

The resistance-deflection function for flat slabs with small deflections is shown in Figure 4-23a. With an elastic distribution of reinforcement all the yield lines form simultaneously and the slab remains elastic until it reaches its ultimate resistance. Since for small deflections the concrete remains effective in resisting stress, a type I cross section is used to compute the ultimate moment capacities. The slab may undergo a maximum support rotation of 2 degrees at which point the concrete crushes. Shear reinforcement is generally not required for flat slabs, but must be provided if required by analysis. If properly designed, single leg stirrups are provided, the flexural action of the slab may be extended to 4 degrees support rotation. While stirrups may be furnished to resist shear or to extend flexural action, it is usually more cost effective to design flat slabs without shear reinforcement.

4-29.4. Design for Large Deflections

Due to the geometric limitations (aspect ratio L/H of each panel must be greater than 1 but less than 2) imposed on flat slabs designed for blast loads, sufficient lateral restraint is available to develop in-plane forces and induce tension membrane action. This tension membrane action provides the means for the slab to attain deflections corresponding to a maximum support rotation in excess of 12 degrees. Continuous reinforcement must be provided to resist these in-plane tension forces.

A typical resistance-deflection curve for the flat slabs up to incipient failure is shown in Figure 4-23b. The initial portion of the curve is due primarily to flexural action. At 2 degrees support rotation, the concrete begins to crush and the slab loses flexural capacity. However, due to the presence of continuous reinforcement, tension membrane action is mobilized. The resistance due to this action increases with increasing deflection up to incipient failure at approximately 12 degrees support rotation. The tension membrane action is shown as the dashed line in Figure 4-23b.

In order to simplify the design calculations, the resistance is assumed to be equal to the flexural action throughout the entire range of behavior. To approximate the energy absorbed under the actual resistance-deflection curve, the deflection of the equivalent curve is limited to 8 degrees support rotation. This deflection would produce incipient failure conditions. Using this equivalent design curve, deflections between 2 degrees and incipient failure cannot be accurately predicted.

A type II or III cross section is used to compute the ultimate moment capacity of a flat slab designed for large deflections. At every section throughout the slab, tension and compression reinforcement must be continuous in order to develop the tension membrane action (tension membrane capacity is discussed in detail below). It should be noted that in addition to the above requirements for the reinforcement, an elastic distribution of reinforcement must still be maintained.

Shear reinforcement is only provided when required by analysis. If the concrete can resist the shear stresses, shear reinforcement is not required for flexural action (deflections less than 2 degrees) nor for tension membrane action (deflections between 2 and 8 degrees). However, shear reinforcement, in the form of single leg stirrups, does allow the slab to rotate up to 4 degrees under flexural action. There are two design situations where single leg stirrups are desirable when designing for rotations between 2 and 4 degrees: (1) the slab is incapable of developing adequate tension membrane action and (2) the maximum deflection must be accurately predicted (which cannot be done utilizing tension membrane action). In all other design situations, it is usually more economical to eliminate single leg stirrups by increasing the slab thickness (to increase shear capacity) and/or by increasing the amount of continuous reinforcement (to develop adequate tension membrane capacity).

4-29.5. Minimum Reinforcement

To ensure proper structural behavior under dynamic loads and also to minimize excessive deformations under conventional loads, the minimum area of reinforcement must be at least equal to that specified in Table 4-3. With an elastic distribution of reinforcement in a flat slab, the minimum reinforcement generally will occur only in the center of the midstrip and/or in the wall strip. It is important to also check the static requirements for minimum reinforcement. Where static conditions control, the area of reinforcement must be at least equal to 0.0018 times the gross area of concrete or 1.33 times the area required by static loading conditions, whichever is less. Unless the blast loads are in the same order of magnitude as the static loads, this criteria does not control.

Although the spacing of the flexural reinforcement must not exceed two times the slab thickness nor 18 inches, the preferred spacing is 12 inches or less.

There is no minimum shear reinforcement requirement for flat slabs. Shear reinforcement is only provided when required by analysis. However, when a slab is designed to undergo flexural response with support rotations between 2 and 4 degrees (i.e., where tension membrane action is not considered). stirrups are required. The minimum area of the stirrups is given in Table 4-4.

4-30. Dynamic Analysis

4-30.1. General

The dynamic analysis of a structural element is accomplished by first representing the structural element as a single-degree-of-freedom system and then finding the response of that system when subject to a blast load. Chapter 3 presents procedures, figures and response charts for determining the dynamically equivalent system and its response. However, certain parameters of a flat slab, such as the ultimate resistance and the elastic deflections, cannot be calculated using the methods of Chapter 3. Methods for calculating those parameters are presented below.

4-30.2. Ultimate Flexural Resistance

4-30.2.1. General

The ultimate resistance r_u of a flat slab is a function of the strength, amount and distribution of the reinforcement, the thickness and strength of the concrete and the aspect ratios of the panels. The ultimate resistance is obtained using a yield line analysis. Since in-plane compression forces and tension membrane forces are not considered, the ultimate resistance determined from a yield line analysis will generally be lower than the actual resistance.

The first step in finding the ultimate resistance is to assume a yield line pattern consistent with the support conditions and the distribution of the reinforcement. The pattern will contain one or more unknown dimensions which locate the yield lines. The correct solution is the one which gives the lowest value of the ultimate resistance. Figure 4-24 shows the yield line pattern that will form in a multi-panel flat slab with an elastic distribution of reinforcement. The roof-slab interactions must be designed to insure that the perimeter yield lines form in the roof slab and not in the wall. The yield lines at the columns are assumed to form at the face of the column capitals.

The ultimate resistance can be found from the yield line pattern using either the equilibrium method or the virtual work method, both of which have been discussed in Chapter 3. The equilibrium method is one that has been employed in previous examples but, in the case of flat slabs, requires the introduction of nodal forces which are not always readily determined. The virtual work method though more difficult to solve algebraically, does not require the calculations of the nodal forces. Consequently, the virtual work method is the easier method to apply to flat slabs and is the method detailed below.

The virtual work method does not predict the correct yield line pattern but rather gives the minimum resistance of an assumed yield line pattern. If the distribution of reinforcement is not elastic and/or the span lengths are not approximately equal, the minimum resistance found by the virtual work method may not be the ultimate resistance. In these cases, local failures are possible. It is strongly recommended that these design situations be avoided. In the rare instances where they cannot be avoided, the nodal forces must be calculated and the equilibrium method used to predict the correct yield line pattern.

4-30.2.2. Virtual Work Method

In the virtual work method, equations for the external and internal work are written in terms of the unit resistance r_u, the moment capacities and the geometry. The expression for the external work is set equal to that for the internal work, and the resulting equation is solved for the minimum value of r_u and the associated failure mechanism.

A point within the slab boundaries is given a small displacement in the direction of the load. The resulting deflections and rotations of all of the slab segments are determined in terms of the displacement and the slab segment dimensions. Work will be done by the external loads and by the internal reactions along the yield lines.

The external work done by r_u is:

$$W = \Sigma\, r_u A \Delta \qquad 4\text{-}74$$

where:

W = external work
A = area of the sector
Δ = deflection of the sector's centroid

The internal work done by the reactions at the yield lines is due only to the bending moments since the support reactions do not undergo any displacement and the work done by the shear forces (nodal forces) is zero when summed over the entire slab.

The internal work is:

$$E = \Sigma\, m\Theta l \qquad 4\text{-}75$$

where

E = internal work
m = ultimate unit moment
Θ = relative rotation about yield line
l = length of the yield line

In terms of the moments and rotations in the principal reinforcement directions x and y:

$$E = \Sigma\, m_x \theta_x l_y + \Sigma\, m_y \theta_y l_x \qquad 4\text{-}76$$

Equating the external and internal work, W = E

$$\Sigma\, r_u A \Delta = \Sigma\, m_x \theta_x l_y + \Sigma\, m_y \theta_y l_x \qquad 4\text{-}77$$

Particular attention must be paid to the negative moment capacities of the yield lines radiating from the column capitals when determining E. Top bar cut-offs, if present, will reduce the moment capacity on the part of the yield line furthest from the column. In addition, corner effects must be considered where the two walls intersect. That is, as a result of the increased stiffness at the corners, the ultimate moment of the reinforcement is reduced to 2/3 of its capacity over a length equal to ½ the length of the positive yield line.

To illustrate the application of Equation 4-77, consider the flat slab shown in Figure 4-25. This flat slab is the roof of a square structure with one central column, and is symmetrical about the x and y axes.

Note that Sectors I and III, and Sectors II and IV are identical because of symmetry. To simplify calculations, each sector has been resolved into a rectangle and a triangle. The external work for each sector is:

$$W_I = W_{III} = r_u x(L - x)\,(\Delta/2) + r_u x(x/2)\,(\Delta/3) \qquad 4\text{-}78$$

$$W_{II} = W_{IV} = r_u c(L - x - c)(\Delta/2)$$

$$+ r_u (L - x - c)[(L - x - c)/2](2\Delta/3) \qquad 4\text{-}79$$

Substituting L = 41 and summing

$$\Sigma W = 2 (r_u\Delta/6)(32\ 1^2 - 41x + cx - 4cl - c^2) \qquad 4\text{-}80$$

where

c = width of column capital
L = length of panel
x = horizontal location of the yield line
l = width of ½ of the column strip
Δ = maximum deflection of slab

The internal work for each sector is:

$$E_I = E_{III} = [\frac{2}{3} m \frac{x}{2} + m (L - \frac{x}{2})] \Theta_A$$

$$+ [\frac{2}{3} (2m) \frac{x}{2} + 2m(31 - \frac{x}{2}) + 3ml]\Theta_A \qquad 4\text{-}81$$

$$E_{II} = E_{IV} = [3ml + 2m(L - x - 1)]\Theta_B$$

$$+ [4.5ml + 1.5ml (L - x - 1)]\Theta_B \qquad 4\text{-}82$$

Substituting L = 41, $\Theta_A = \Delta/x$ and $\Theta_B = \Delta/(L - x - c)$

$$\Sigma E = 2\Delta m [\frac{13\ 1}{x} - \frac{1}{2} + \frac{18\ 1 - 3.5x}{41 - x - c}] \qquad 4\text{-}83$$

Equating W = E and solving for r_u

$$r_u = \frac{6m [\frac{131}{x} - \frac{1}{2} + \frac{181 - 3.5x}{41 - x - c}]}{[321^2 - 41x + cx - 4cl - c^2]} \qquad 4\text{-}84$$

with x as the only unknown. The minimum value of r_u is readily determined by trial and error.

A complete design example is presented in Appendix 4A.

In general, the virtual work equation will contain more than one unknown, and it will be correspondingly more difficult to obtain the minimum ultimate resistance. However, a trial and error process rapidly converges on the correct solution.

A trial and error procedure to solve for the minimum value of the resistance function (r_u/M_0 for a preliminary design and r_u for a final design) with two unknown yield line locations, x and y, can be accomplished as follows:

1. Start with both yield lines located close to the centerline of the respective middle strips.

2. Vary x, holding y constant, in the direction which minimizes the resistance function until it begins to increase.

3. Hold x constant and vary y in the minimum direction until the resistance function begins to increase.

4. Once this minimum point is achieved, shift each yield line to either side of the minimum location to check that a further refinement of the yield line is not necessary to minimize the resistance function.

It should be noted that if the yield line should shift out of the middle strip, a new resistance function equation must be written and the procedure then repeated since the magnitude of the unit moments acting on the yield lines would change.

4-30.2.3. Effect of Column Capitals and Drop Panels

Although column capitals and drop panels are primarily used to prevent shear failures, they have a significant effect on the ultimate resistance. The addition of a column capital or revision of the size of the capital changes the clear span of a flat slab and requires the re-evaluation of a slab's ultimate resistance.

Drop panels increase the ultimate resistance by increasing the depth of the section and thus the moment capacity in the vicinity of the column. This effect can be countered by decreasing the amount of reinforcement to maintain the same moment capacity. If the drop panel is used to increase the negative moment capacity, it must extend at least 1/6 of the center-to-center span length in each direction. The width of the drop panel may be up to 20 percent larger than the column strip. When the drop panel is larger than the column strip, the percentage of reinforcement calculated for the column strip shall be provided throughout the drop panel. Additional reinforcement must be provided in the bottom of the drop panel to prevent it from scabbing and becoming hazardous debris. For a type II cross section, the reinforcement in the drop panel is the same as the negative reinforcement over the column. Only ½ the amount of the negative reinforcement is required in a drop panel for a type I cross section.

4-30.3. Ultimate Tension Membrane Capacity

When the support rotation of a flat slab reaches 2 degrees, the concrete begins to crush and flexural action is no longer possible. However, the slab is capable of sustaining large rotations due to tension membrane action. As previously explained, the actual resistance-deflection curve describing the tension membrane action has been replaced with an equivalent curve which considers flexural action only (Fig. 4-23b). Using this idealized curve, incipient failure is taken to occur at 8 degrees which corresponds to an actual support rotation of approximately 12 degrees.

It can be seen from Figure 4-23b, that the tension membrane resistance is a function of the deflection. It is also a function of the span length and the amount of the continuous reinforcement. Data is not presently available to obtain the tension membrane capacity of a flat slab. However, an approximation may be made using the equation developed for two-way slabs. Therefore, the tension membrane capacity, r_T, of a flat slab is given by:

$$r_T = \frac{1.5X\pi^3 \, T_H / L_H^2}{4 \sum_{n=1,3,5} \left[\frac{1}{n^3} (-1)^{(n-1)/2} \left[1 - \frac{1}{\cosh\left[\frac{n\pi L_L}{2L_H} \left[\frac{T_H}{T_L} \right]^{1/2} \right]} \right] \right]} \qquad 4\text{-}85$$

where

r_T = tension membrane resistance

X = deflection of slab

L_H = clear span in short direction

L_L = clear span in long direction

T_H = force in the continuous reinforcement in short span direction

T_L = force in the continuous reinforcement in long span direction

Although the capacity of a flat slab is based on flexural action, adequate tension membrane capacity must be provided. That is, r_T corresponding to 8 degrees support rotation must be greater than the flexural resistance r_u when designing for large deflections. The deflection is computed as a function of the yield line locations (shortest sector length). The force in the continuous reinforcement is calculated using the dynamic design stress corresponding to 8 degrees (Table 4-2). The clear span L_H and L_L are calculated as the clear distance between the faces of the supports (face of the column if no column capital is used, face of the column capital, face of the wall if no haunch is used or the face of haunch).

4-30.4. Elastic Deflections

The elastic deflection of various points on an interior panel of a flat slab are given by the general equation

$$X_e = \frac{C r_u L^4 (1 - \nu^2)}{E_c I_a} \qquad 4\text{-}86$$

where

X_e - elastic deflection

C - deflection coefficient from Table 4-8

L - long span of panel

ν - poisson's ration - 0.167

I_a - average of the cracked and gross moment of inertia of the concrete slab

The deflection coefficient varies with the panel aspect ratio L/H, the ratio of the support size to the span C/L and the location within the panel. The values of the deflection coefficient given in Table 4-8 are based on a finite difference method and are given for the center of the panel C_C and the midpoints of the long and short sides, C_L and C_S, respectively.

The deflection for the interior panel is determined by using C_C in the above expression. For the long and short span panels and the corner panel (Fig. 4-20). No simplified solution for the center deflections are currently available. Generally, the deflections for these panels will be smaller than the deflection of the interior panel because of the restraining effects of the exterior walls. These deflections can be approximated by using the following expressions:

Long Span Panel $C = C_C - C_S/2$ 4-87

Short Span Panel $C = C_C - C_L/2$ 4-88

Corner Panel $C = C_C - C_S/2 - C_L/2$ 4-89

where the values of C_C, C_S and C_L are those for the interior panel from Table 4-8.

The dynamic response of a flat slab is more sensitive to the elastic stiffness when the maximum allowable deflection is small. The possible error diminishes with increasing allowable maximum deflection.

4-30.5. Load-Mass Factors

4-30.5.1. Elastic Range

No data is currently available to determine the loadmass factor, K_{LM}, of a flat slab in the elastic range of behavior. It is, therefore, recommended that the values listed in the table of the load-mass factors for two-way elements be used (Chapter 3). The slab should be considered as fixed on all four edges with the appropriate L/H ratio. Since an average value of the elastic and plastic load-mass factor is used in determining the natural period of vibration, the possible error incurred will diminish with increasing allowable maximum deflection.

4-30.5.2. Plastic Range

The load-mass factor on the plastic range is determined using the procedure outlined for two-way elements in Chapter 3. The supports for the individual sectors are at the face of the exterior walls (or haunches, if present) or at the face of the column capitals. Flat slabs without drop panels have a uniform thickness and the equation for determining the load-mass factor may be expressed in terms of the area moment of inertia and the area of the individual sectors. For flat slabs with drop panels, the equations must be expressed in terms of the mass moment of inertia and the non-uniform mass of the individual sectors to account for the non-uniform slab thickness.

4-30.6. Dynamic Response

The equivalent single-degree-of-freedom system of the flat slab is defined in terms of its ultimate resistance r_u, elastic deflection X_E and its natural period of vibration T_N. The procedure for determining the value of T_N has been presented in Chapter 3 while the calculation of r_u and X_E has been presented above. The resistance deflection curve used in the dynamic analysis is shown in Figure 4-26. The resistance available to withstand the blast loads must be reduced by the dead loads. An approximation of the resistance available is

$$r_{avail} = r_u - r_{DL} \left(\frac{f_{ds}}{f_y}\right) \qquad 4\text{-}90$$

where

r_{avail} = dynamic resistance available

r_{DL} = uniform dead load

The total deflection of the flat slab includes deflections due to dead load X_{DL} and blast X_m, so that the maximum support rotation Θ_m is given by

$$\Theta_m = \tan^{-1} \left(\frac{X_m + X_{DL}}{L_s}\right) \qquad 4\text{-}91$$

where L_s is the length of the shortest sector.

The blast load is defined in terms of its peak pressure P and its duration T which are determined from Chapter 2. Chapter 3 contains the procedures to determine the dynamic response of a slab which include the maximum dynamic deflection X_m and the time to reach that deflection t_m. It must be remembered that using the equivalent resistance-deflection curve to include tension membrane action, deflections between 2 degrees and incipient failure cannot be accurately predicted.

The required rebound resistance of the flat slab is calculated in accordance with Chapter 3 and the reinforcement necessary to attain this capacity must be provided. Note that while the dead load reduces the available resistance for the dynamic loading, this load increases the available resistance for rebound.

4-31. Dynamic Design

4-31.1. Flexural Capacity

The ultimate moment capacity of a flat slab is usually based upon a type I or type III cross section depending on the magnitude of the maximum allowable deflection. The distribution of reinforcement is critical in flat slab design. The actual moment capacity provided must be as close as possible to the unit moments required for an elastic distribution of stresses. The quantity of flexural reinforcement which is made continuous provides the tension membrane resistance.

If the amount of continuous reinforcement provided is inadequate for tension membrane action, care must be taken in furnishing additional reinforcement. Any additional reinforcement must be placed to maintain the elastic distribution of reinforcement and the new moment capacities and ultimate resistance must be re-evaluated. The ultimate moment capacity will not be altered if the additional reinforcement is provided by increasing the compression reinforcement.

4-31.2. Shear Capacity

Unlike continuously supported two-way slabs where shear stresses are "checked" after the flexural design is completed, the design for shear of a flat slab must be considered during the flexural design. Due to the nature of the support system, flat slabs will usually generate large shear stresses. Flat slabs with high percentages of flexural reinforcement and/or long spans should be avoided.

The shear forces acting at a support are a function of the tributary area of the sectors formed by the yield lines. The shears at the columns should be checked first, since design for these forces can drastically effect the flexural design of the slab. Two types of shear action must be considered; punching shear along a truncated cone around the column and beam shear across the width of the yield lines. These conditions are illustrated in Figure 4-27.

Shears at the columns may require the use of column capitals and/or drop panels. Punching shear can occur around the periphery of the columns or column capitals and drop panels. The critical section is taken at $d_e/2$ from the face of the support. The total load is calculated based on the area enclosed by the positive yield lines and is then distributed uniformly along the critical perimeter. Figure 4-27a illustrates the critical sections for punching shear. Beam shear, as a measure of diagonal tension, is taken as one-way action between supports where the width of the beam is taken as the spacing between the positive yield lines. The critical section is taken as d_e away from the face of the column or column capital and from the face of the drop panel (Fig. 4-27b). The total load is uniformly distributed along the critical section.

The slab at the exterior walls must be evaluated for diagonal tension capacity. Due to the assumed uniform distribution of load at the exterior walls, a unit width of loaded area may be considered between the positive yield line and the critical section. The critical section is taken at d_e from the face of the exterior wall or, if a haunch is used, from the face of the haunch.

The ultimate shear capacity of slabs has been previously presented. Using these procedures, the capacity of the slab is evaluated at the locations described above. If required, stirrups may be furnished. However, it is more cost effective to revise the design to incorporate the use of column capitals, drop panels and/or increased slab thickness to reduce shear stresses. As previously stated, the use of stirrups is mandatory in the flexural design of flat slabs between 2 and 4 degrees support rotation.

Diagonal bars must be provided at the face of all supports due to the cracking caused by the plastic moments formed. For slabs designed for small support rotations, minimum diagonal bars must be furnished. However, for slabs designed for large support rotations where the cracking at the supports is severe, diagonal bars must be designed to resist the total support shear but not less than the minimum required. The diagonal bars furnished at the column supports should extend from the slab into the column. In slabs where shear stresses are high, it may be impractical to place the required diagonal bars.

If column capitals were not initially used, their addition would reduce the required quantity of diagonal bars. In the case where column capitals are furnished, at least one-half of the diagonal bars should extend into the column with the remainder cut-off in the column capital. Procedures for the design of diagonal bars have been previously presented while the required construction details are illustrated in subsequent sections.

4-31.3. Columns

The interior columns of a flat slab/shear wall structure are not subjected to lateral loads nor the moments they induce. These columns are designed to resist the axial loads and unbalanced shears generated by the ultimate resistance of the flat slab. The axial load and moments at the top of the column are obtained from the flat slab shear forces acting on the perimeter of the column capital plus the load on the tributary area of the column capital. As can be seen from Figure 4-28, the axial load is:

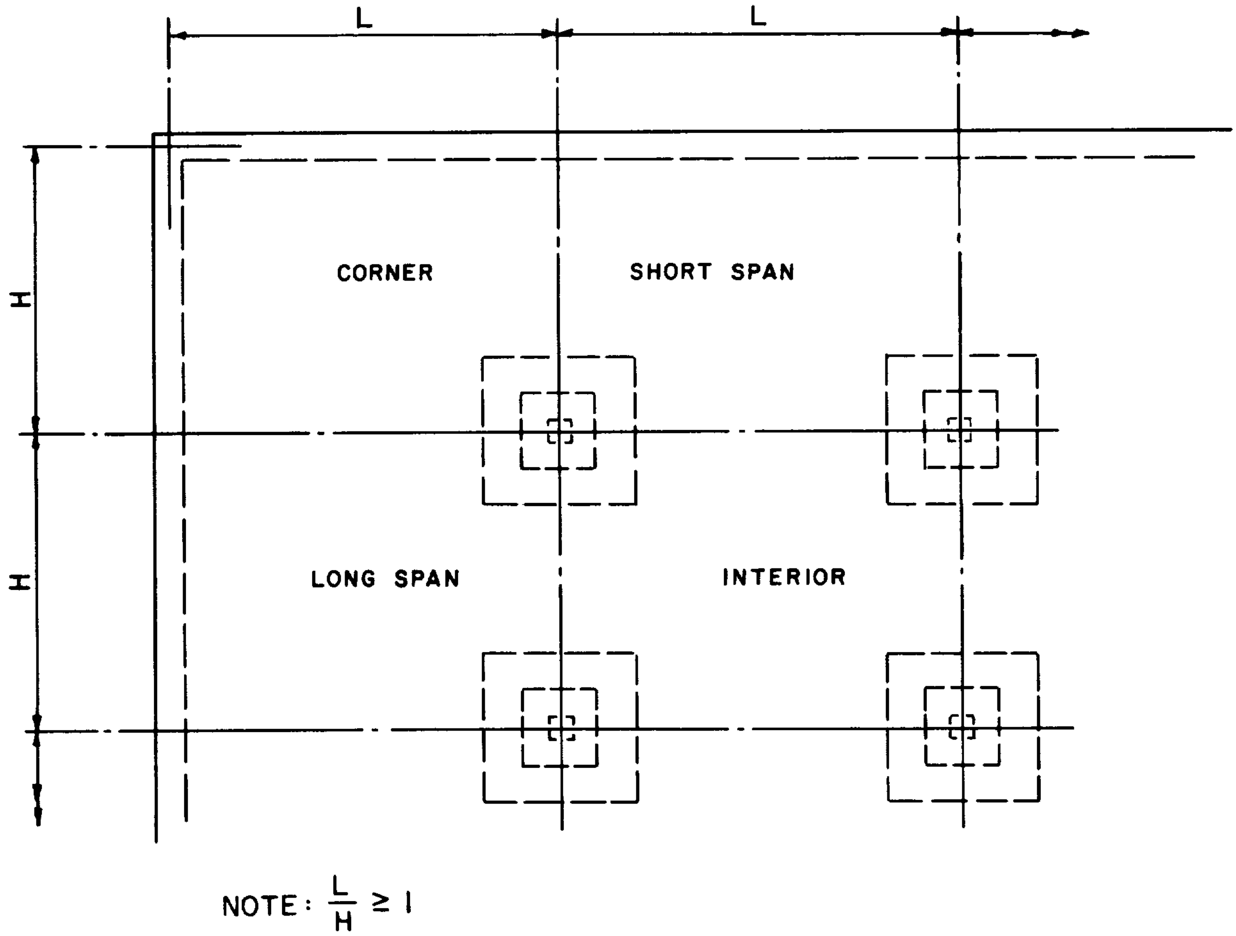
$$P = r_u \Sigma \ A + r_u c^2 \qquad 4\text{-}92$$

and the unbalanced moments are:

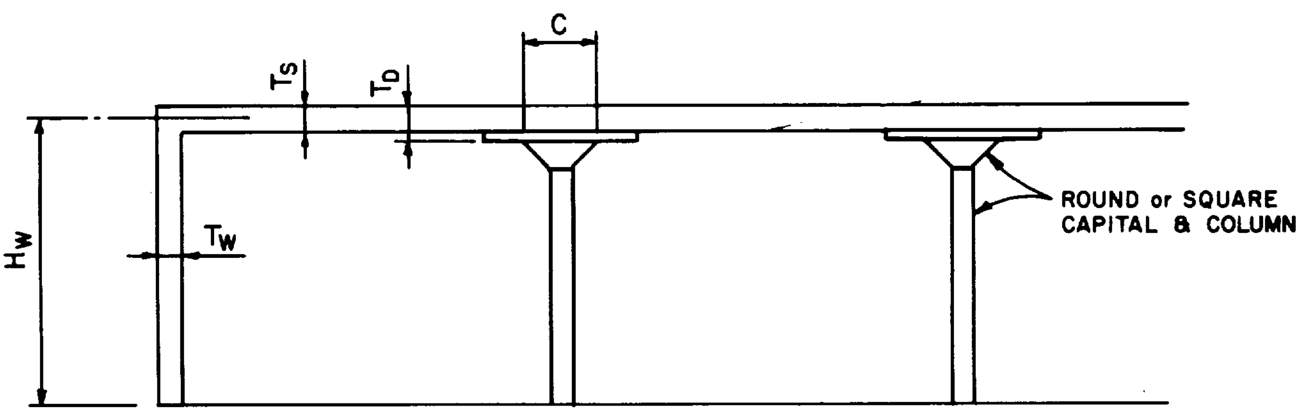
$$M_x = (V_4 - V_2)\ (c/2) \qquad 4\text{-}93$$

$$M_y = (V_1 - V_3)\ (c/2) \qquad 4\text{-}94$$

The procedures for the design of columns is presented in Section 4-49. When using these procedures, the unsupported length of the column is from the top of the floor to the bottom of the column capital.



PLAN



SECTION

Figure 4-20 Typical flat slab structure

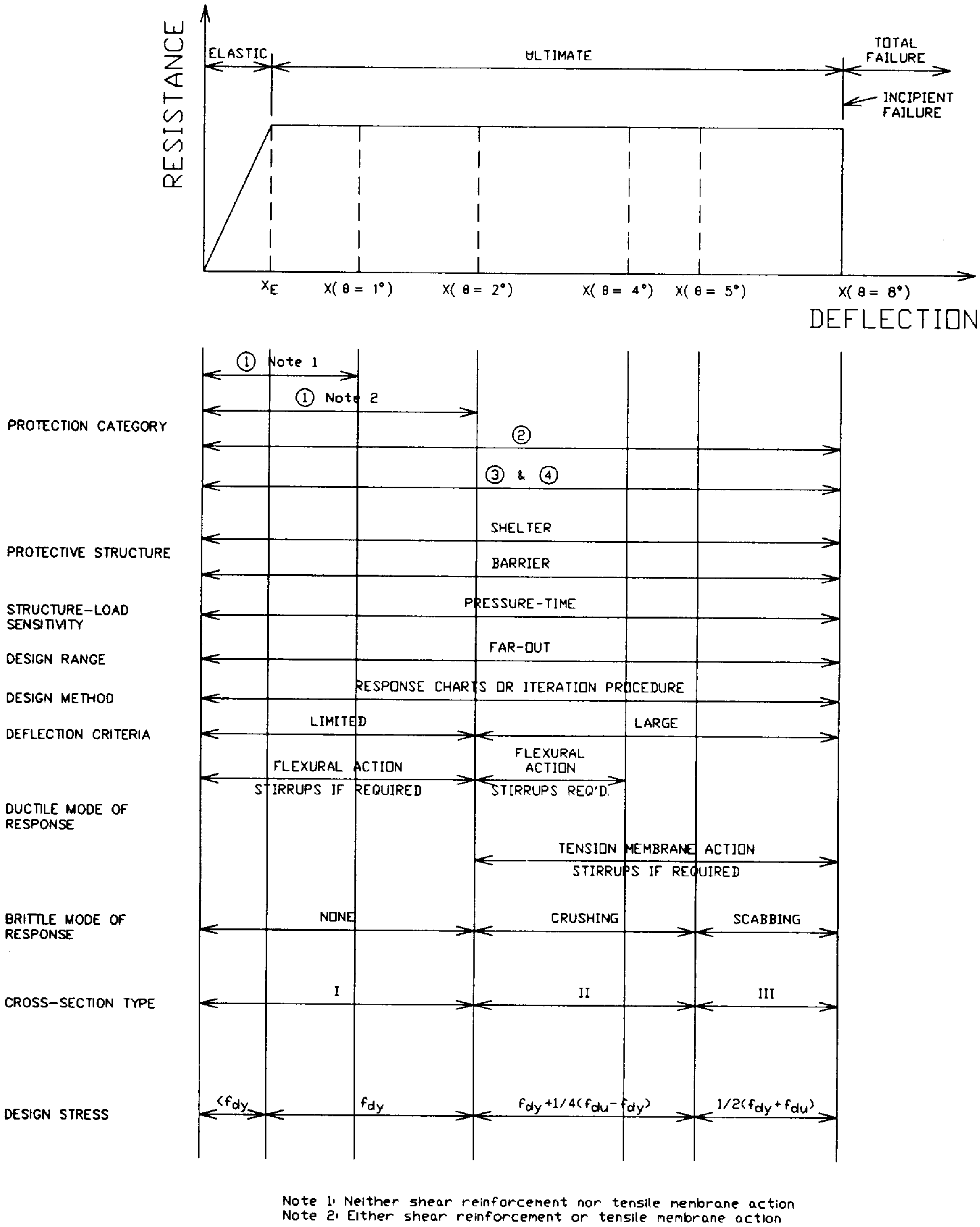


Figure 4-21 Relationship between design parameters for flat slabs

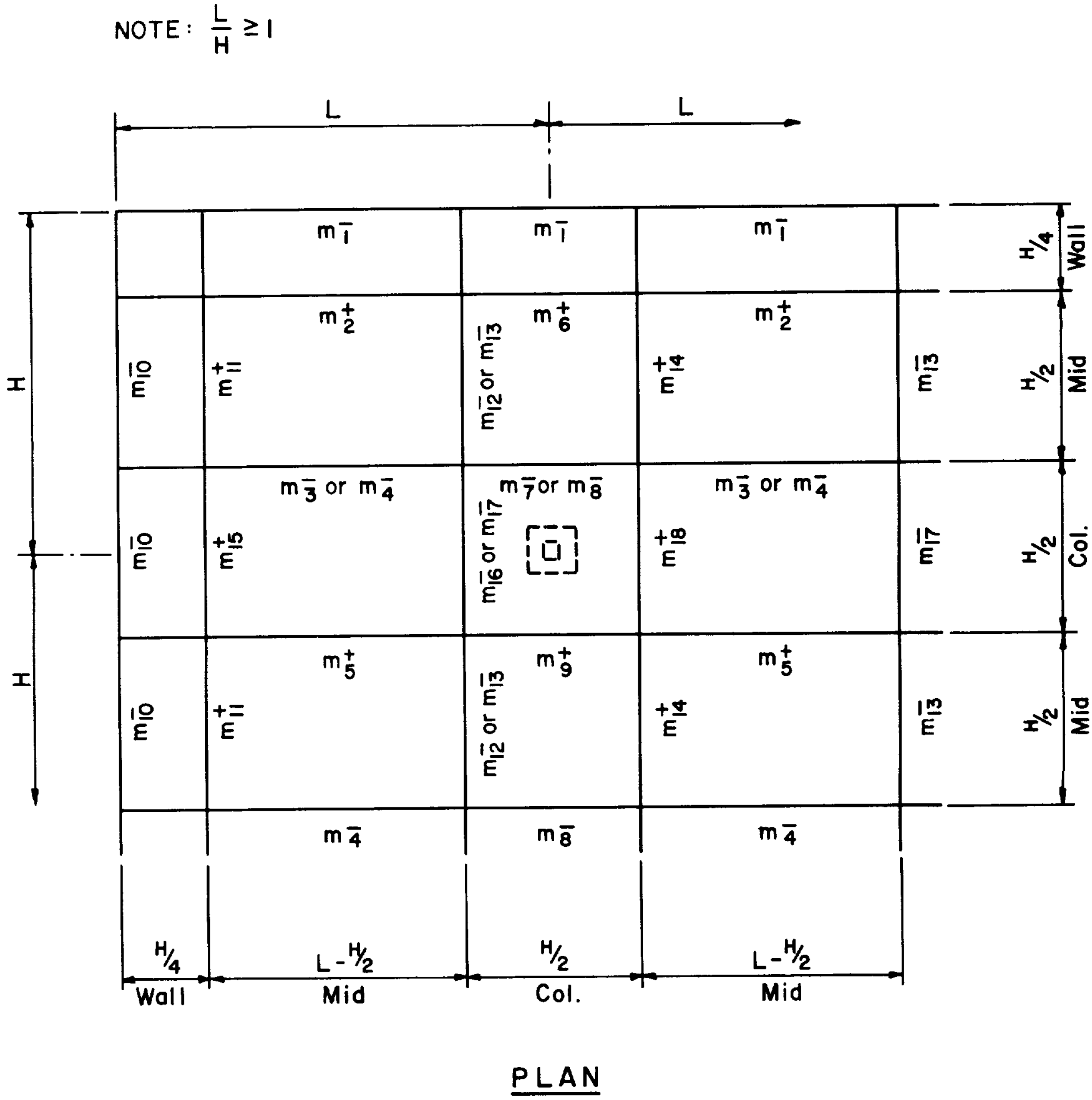
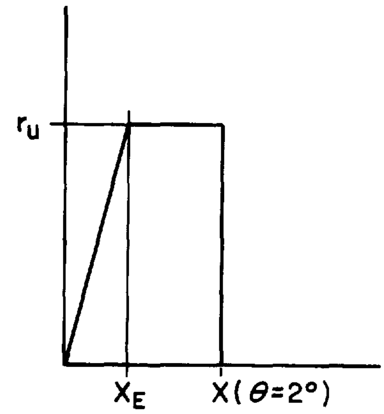
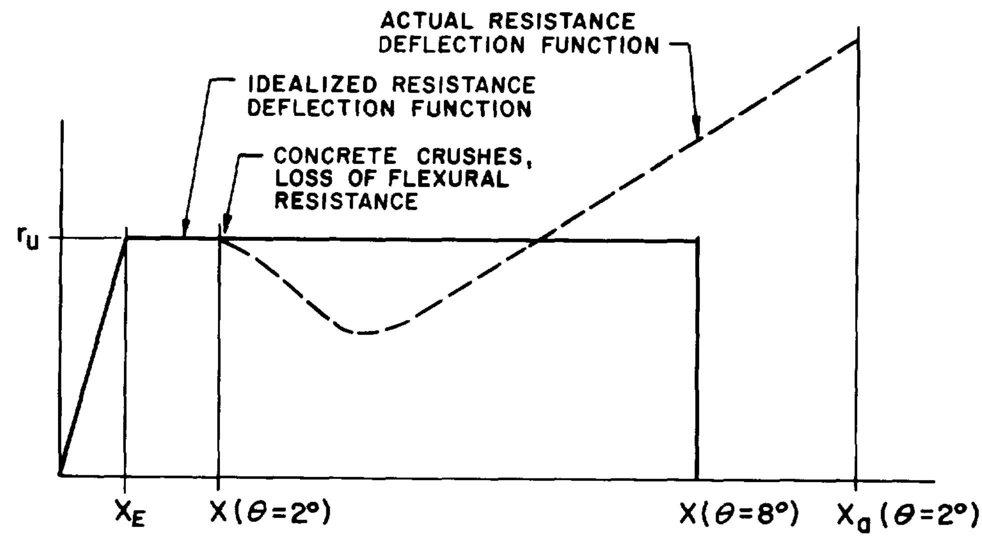


Figure 4-22 Unit moments



a) Small Deflection



b) Large Deflection

Figure 4-23 Typical resistance-deflection functions for flat slabs

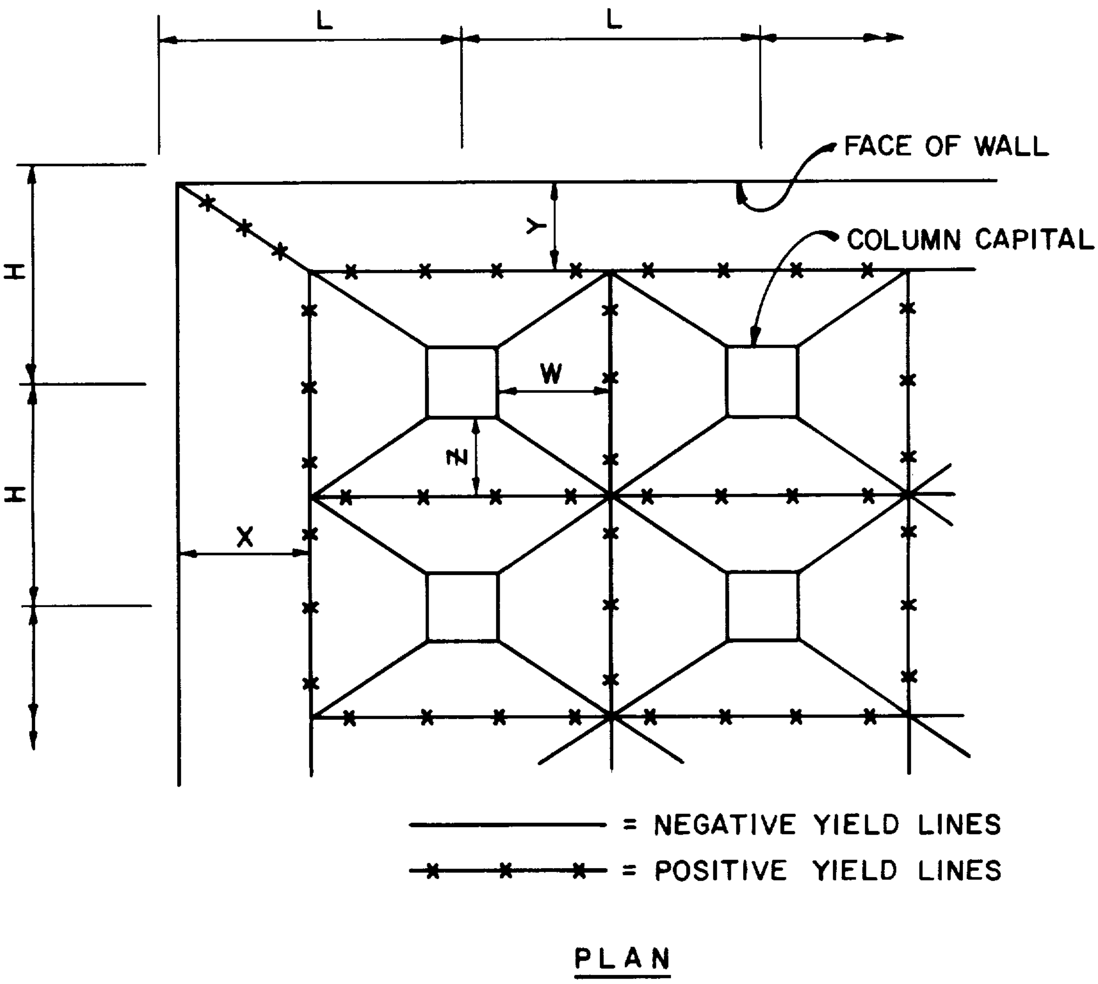
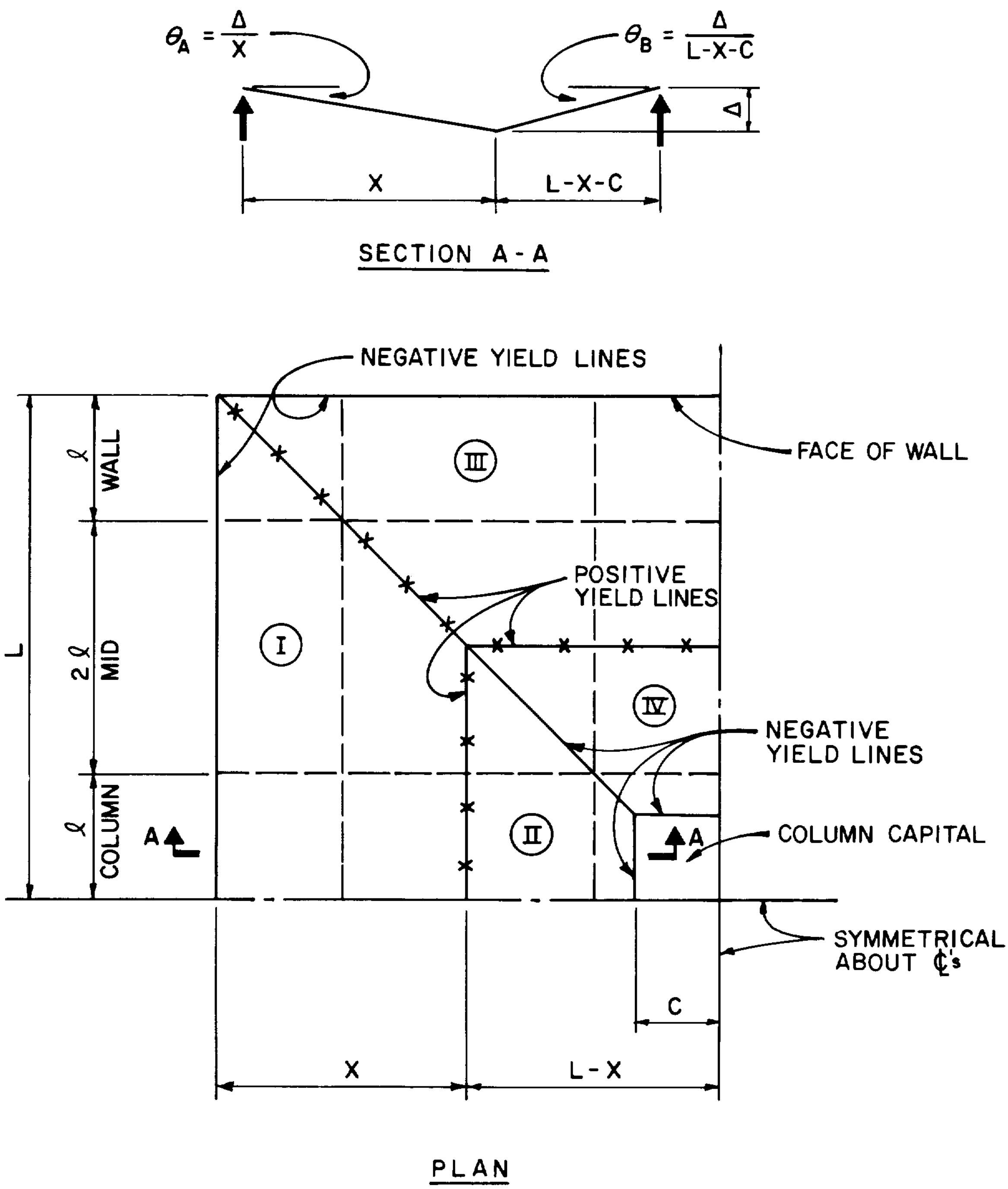


Figure 4-24 Yield line pattern for multi-panel flat slab



UNIT MOMENT CAPACITIES:

	EXT. NEG.	POS.	INT. NEG.
WALL STRIP	m*	2m*	1.5m
MID STRIP	m*	2m*	1.5m
COLUMN STRIP	m	3m	4.5m

* 2/3 OF MOMENT CAPACITY EFFECTIVE AT CORNERS

Figure 4-25 Quarter panel of flat slab

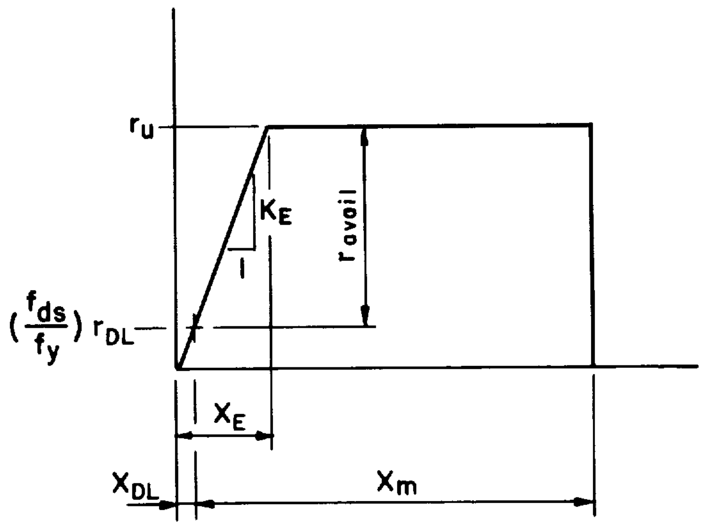


Figure 4-26 Dynamic resistance-deflection curve

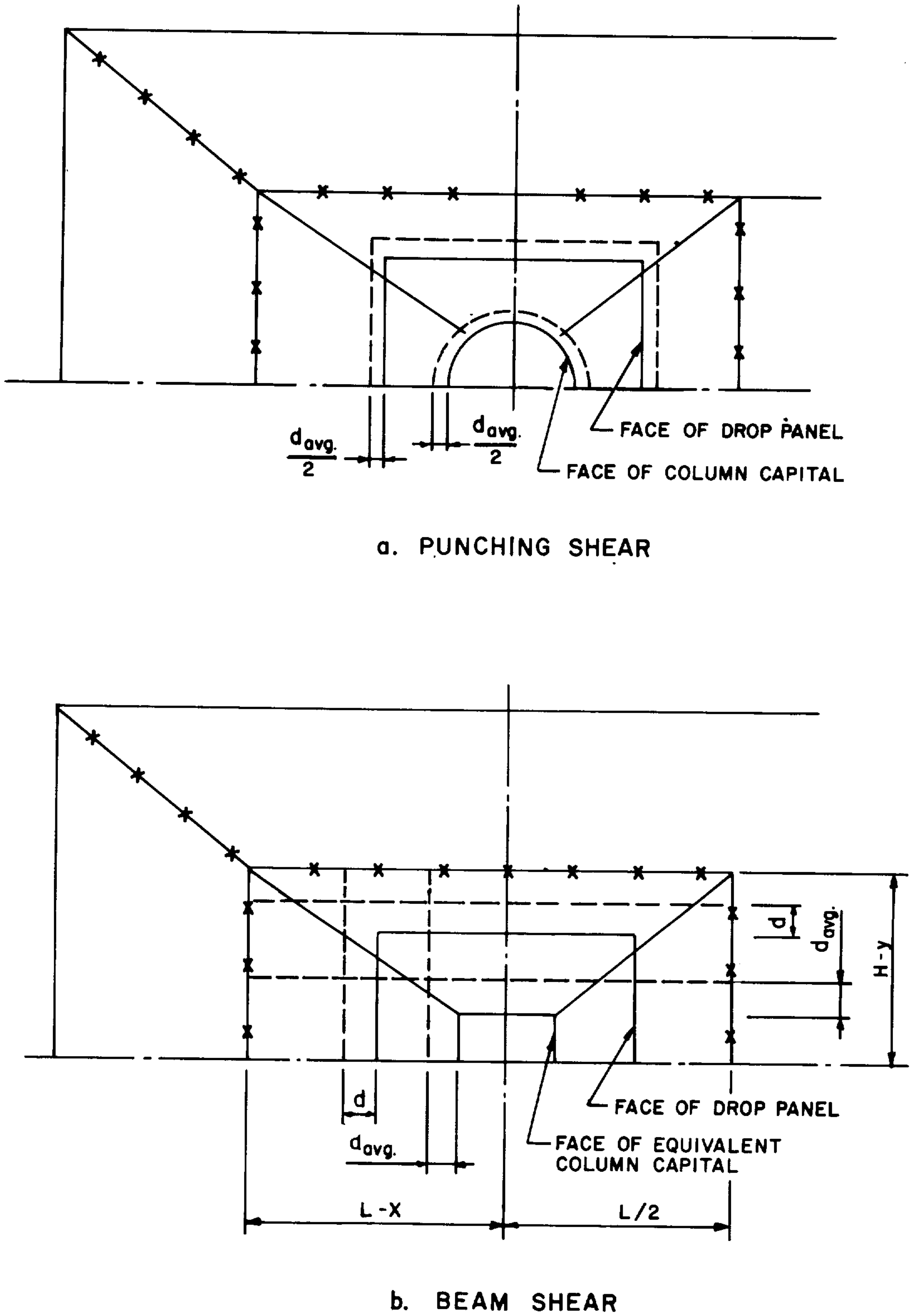


Figure 4-27 Critical locations for shear stresses

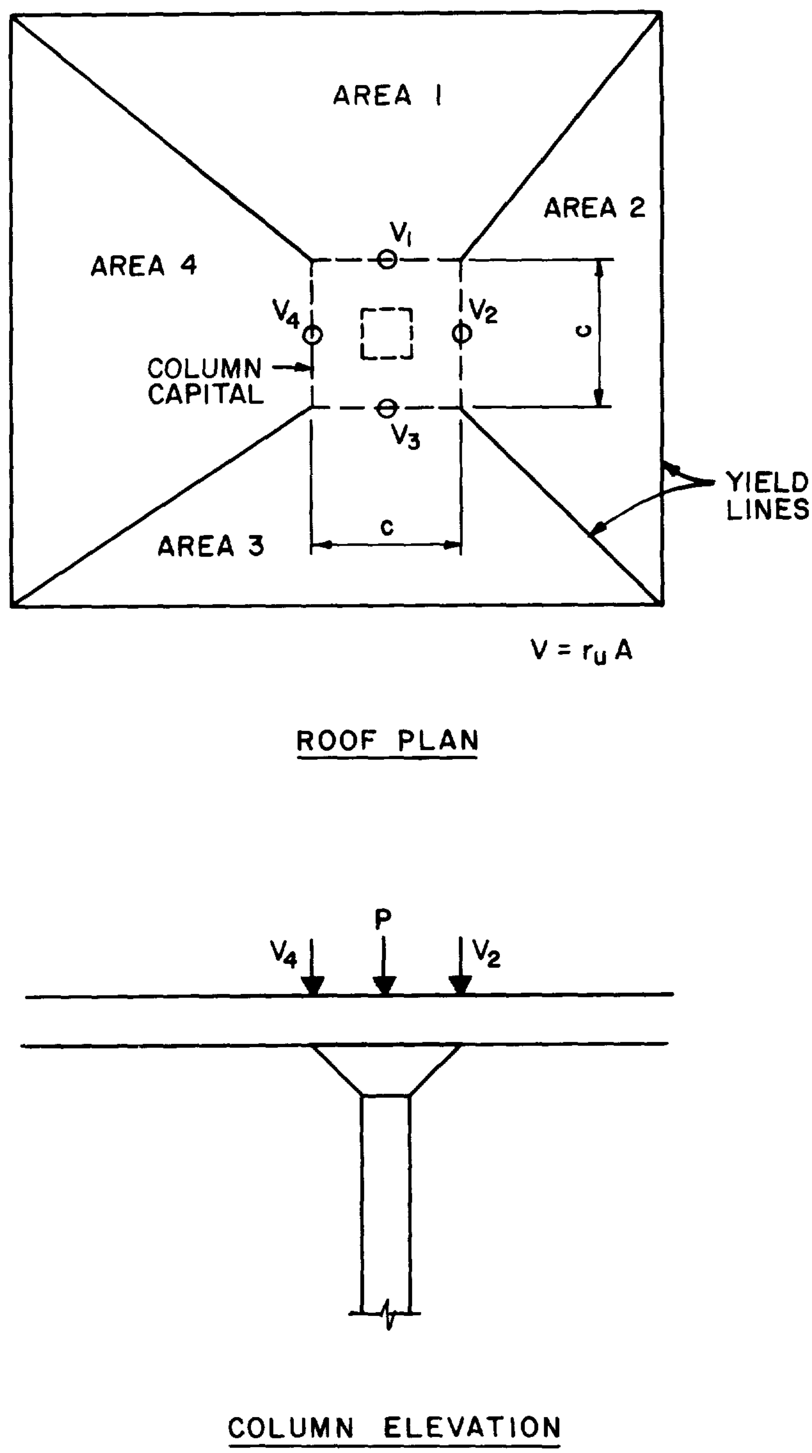
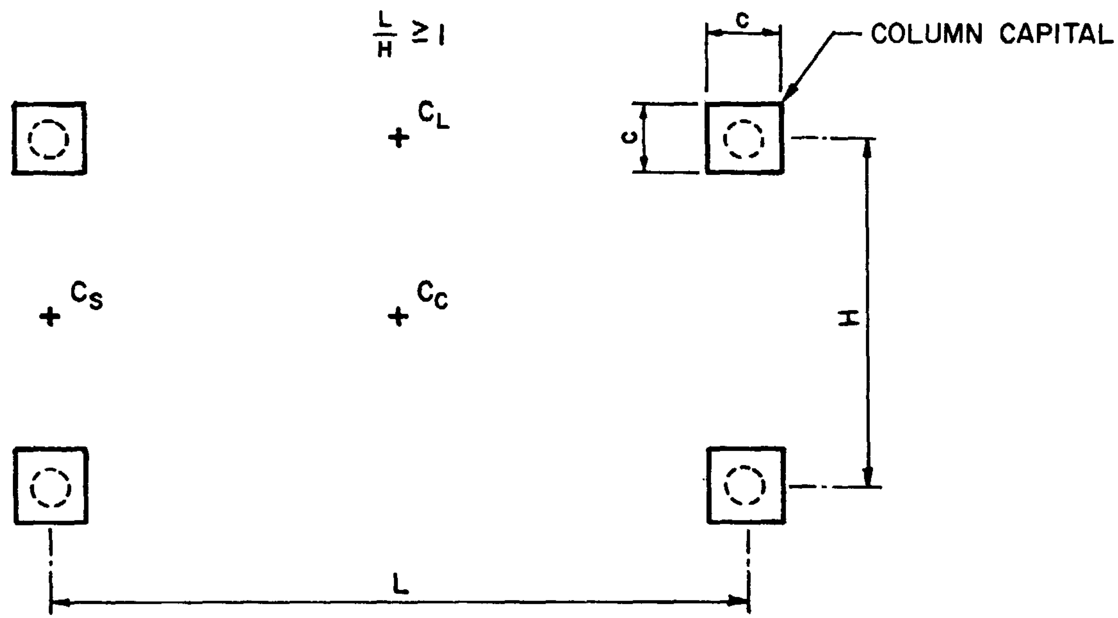


Figure 4-28 Typical column loads

Table 4-8 Deflection Coefficients for Interior Panels of Flat Slabs

	Center of Panel C_c				Midspan of Long Side C_L				Midspan of Short Side C_s			
L/H \ c/L	0	0.1	0.2	0.3*	0	0.1	0.2	0.3*	0	0.1	0.2	0.3*
1.00	0.00581	0.00441	0.00289	0.00200	0.00435	0.00304	0.00173	0.00100	0.00435	0.00304	0.00173	0.00100
1.25	0.00420	0.0301	0.00189	0.00120	0.0378	0.00262	0.00155	0.00085	0.00230	0.00131	0.00057	0.00020
1.67	0.00327	0.00234	0.00143	0.00080	0.00321	0.00228	0.00137	0.00075	0.00099	0.00040	0.00008	0.00005
2.00	0.00301	0.00216	0.00129	0.00071	0.00299	0.00214	0.00127	0.00069	0.00058	0.00018	0.00004	-

* Values for c/L = 0.3 are extrapolated



DESIGN OF LACED ELEMENTS

4-32. Introduction

The detonation of an explosive charge close to a barrier produces a non-uniform, high intensity blast load which acts on the barrier for a comparatively short period of time. The concept of lacing reinforcement (Fig. 4-4 and 4-5) has been developed for use in protective structures subjected to such loads. Lacing maintains the structural integrity of a barrier and permits it to attain large plastic deflections.

Extremely high pressure concentrations are caused by close-in detonations. These concentrations can produce local (punching) failure of an element. However, with the use of lacing, the high shears produced in the vicinity of these pressure concentrations are transferred to other areas of the element where the applied blast loads are less severe. In effect, the lacing tends to spread out the effects of the non-uniformity of the loading and permits the use of an average blast load over the entire surface area of the element. In addition, lacing is required in those elements where large deflections are desirable. In these cases, the lacing not only resists the high shears produced but also maintains the integrity of the severely cracked concrete between the tension and compression reinforcement during the latter stages of deflection.

The primary use of laced elements is to resist the effects of explosive charges located close to barriers. The minimum separation distance between the charge and the laced element is given in Section 2-14.2.1 of Chapter 2. It should be emphasized that these separation distances are the minimum clear distance from the surface of the charge to the surface of the laced element. The normal scaled distances R_A (center of charge to surface of barrier) corresponding to these minimum clear separation distances are equal to approximately 0.25 $ft/lb^{1/3}$.

A laced element may be designed for limited deflections (less than 5 degrees support rotation), large deflections (up to 12 degrees support rotation) or controlled post-failure fragments depending upon the protection requirements of the receiver system. The stresses developed in the reinforcement is a function of the deflection attained by the element. The type of cross-section which determines the ultimate moment capacity of the reinforced section is also a function of the deflection but, more importantly, is a function of the elements brittle mode response. High intensity blast pressures cause direct spalling during the initial phase of an element's response. Therefore, a type III cross-section will usually be available to provide moment capacity as well as the available mass to resist motion.

Single leg stirrups may be somewhat more economical than lacing as shear reinforcement. However, in many design situations, the use of lacing reinforcement is mandatory. When explosives are located at scaled distances less than 1.0, lacing must be used; single leg stirrups are not effective for such close charge locations. Also, the blast capacity of laced elements are greater than corresponding (same concrete thickness and quantity of reinforcement) elements with single leg stirrups. Laced elements may attain deflections corresponding to 12 degrees support rotation whereas elements with single leg stirrups are designed for a maximum rotation of 8 degrees. These non-laced elements must develop tension membrane action in order to develop this large support rotation. If support conditions do not permit tension membrane action, lacing reinforcement must be used to achieve large deflections.

The design of concrete elements subjected to blast loads involves an iterative (trial and error) design procedure in which the element is assumed and then its adequacy is verified through a dynamic analysis (Chapter 3). The design of laced elements for limited deflections is performed in much the same manner. However, the design of laced elements for large deflections has unique features which permit the formulation of design equations. Since a laced element is subjected to very short duration blast loads, the actual pressure-time relationship of the load need not be considered. In fact, the actual duration of the load need not be considered at all. The load may be taken as an impulse (area under the pressure-time curve), that is, the entire load is applied instantaneously to the element. This assumption results in an insignificant error since the time for the element to reach the maximum deflection is large in comparison to the actual duration of the load. Secondly, the elastic portion of the element's resistance-deflection curve need not be considered. This assumption will also result in a negligible error since the plastic portion of the curve is many times that of the elastic portion. Lastly, laced elements must be symmetrically reinforced which greatly simplifies the expressions for an element's capacity. These features permit the formulation of design equations and design charts which are used to design laced elements for large deflections and for the preliminary design of laced elements for limited deflections.

This section includes the design of laced elements for ductile mode response. The brittle mode of response including the occurrence of spalling and the design for controlled post-failure fragments are presented in subsequent sections. The interrelationship of the parameters involved in the design of laced elements is illustrated in the idealized resistance-deflection curve shown in Figure 4-29.

4-33. Flexural Design for Large Deflections

4-33.1. General

The basic equations for the analysis of the impulse capacity of an element were derived in Chapter 3. For a two-way element which exhibits a post-ultimate resistance range and is designed for large deflections, the response is:

$$\frac{i_b^2}{2m_u} = r_u X_1 + \frac{m_u}{m_{up}} r_{up} (X_m - X_1) \qquad 4\text{-}95$$

The response equation for a one-way element, or a two-way element which does not exhibit a post-ultimate resistance range is:

$$\frac{i_b^2}{2m_u} = r_u X_m \qquad 4\text{-}96$$

i_b = applied blast impulse load

m_u, m_{up} - effective unit mass in the ultimate and post-ultimate ranges, respectively

r_u, r_{up} - unit resistances in the ultimate and post-ultimate ranges, respectively

X_1 - deflection at partial failure

X_m - maximum deflection

The above equations give the impulse capacity of a given structural element. Use of such equations for design purposes is not practical since the procedure would involve a tedious trial and error design.

4-33.2. **Impulse Coefficients**

Equations suitable for design are obtained by substituting the general expressions from Chapter 3 for the effective masses (m_u and m_{up}), the ultimate resistances (r_u and r_{up}) and maximum deflections (X_1, and X_u) into Equations 4-95 and 4-96 The resulting equations take the form:

$$\frac{i_b{}^2 H}{p_H\ d_c{}^3 f_{ds}} = C \qquad 4\text{-}97$$

H - height of the element

p_H - horizontal reinforcement ratio

d_c - distance from the centroid of the compression reinforcement to the centroid of the tension reinforcement

f_{ds} - dynamic design strength of the steel

C - impulse coefficient

To illustrate the method used to obtain the impulse coefficients, consider a two-way element (roof slab or wall) fixed on two adjacent edges and free on the other two. The yield line location is defined by y and $L < y < H$. The solution desired is for incipient failure (deflection X_u) of a spalled section (cross section type III).

From Chapter 3 the equations for the resistances, deflections and effective masses for this two-way element are as follows:

1. Ultimate unit resistance

$$r_u = \frac{5\ (M_{VN} + M_{VP})}{y^2} \qquad \text{(Table 3-2)}$$

where

$$M_{VN} = M_{VP} = \frac{A_s\ f_{ds}\ d_c}{b} = p_V f_{ds}\ d_c{}^2 \qquad \text{(eqs. 4-18 and 4-19)}$$

and p_V is defined as the vertical reinforcement ratio on each face.

2. Post-ultimate unit resistance

$$r_{up} = \frac{(M_{VN} + M_{VP})}{H^2} \qquad \text{(Table 3-4)}$$

3. Partial failure deflection

$$X_1 = L \tan 12° \qquad \text{(Table 3-6)}$$

4. Ultimate deflection

$$X_u = y\tan 12° + (H - y) \tan \gamma \qquad \text{(Table 3-6)}$$

where

$$\gamma = 12° - \tan^{-1}\left[\frac{\tan 12°}{y/L}\right]$$

5. Effective unit mass in the ultimate range

$$m_u = (K_{LM})_u\, m$$

$(K_{LM})_u$ is from figure 3-44

$$m = \frac{d_c}{1728}\left[\frac{150}{386\ (10^{-6})}\right] = 225d_c$$

6. Effective unit mass in the post-ultimate range

$$m_{up} = (K_{LM})_{up}\, m = (2/3)\, m$$

the units used are:

y, L, H, d_c, b, X_1, X_u	inches
i_b	psi-ms
M_{VN}, M_{VP}	in.-lbs/in.
f_{ds}, r_u, r_{up}	psi
A_s	in.^{2}
m , m_u, m_{up}	psi-ms^2/in.

Substituting into Equation 4-95

$$i_b^2 = 2 \text{ X } 225d_c\ (K_{LM})_u \left[\frac{10p_V d_c^2 f_{ds}}{y^2}\right] L\tan 12° \quad +$$

$$\frac{(K_{LM})_u}{0.66}\left[\frac{2p_V\ d_c^{\,2}f_{ds}}{H^2}\right]\left[Y\tan 12° + (H - y)\tan\gamma - L\tan 12°\right] \qquad 4\text{-}98a$$

Factoring

$$i_b^{\,2} = 2(225d_c)p_V d_c^{\,2}f_{ds}\tan 12°\left[\frac{10(K_{LM})_u L}{y^2} + \frac{3(K_{LM})_u^{\,2}}{H^2}\left(y - L + \frac{(H - y)\tan\gamma}{\tan 12°}\right)\right] \qquad 4\text{-}98b$$

Dividing each side by p_H the horizontal reinforcement ratio, and rearranging

$$\frac{i_b^{\,2}H}{p_H d_c^{\,3}f_{ds}} = 450\left(\frac{p_V}{p_H}\right)\tan 12°\left[\frac{10(K_{LM})_u\ (L/H)}{(y/H)^2} + 3(K_{LM})_u^{\,2}\left[\frac{Y}{H} - \frac{L}{H} + \left(1 - \frac{Y}{H}\right)\frac{\tan\gamma}{\tan 12°}\right]\right] \qquad 4\text{-}98c$$

$$\frac{i_b^{\,2}H}{p_H\ d_c^{\,3}f_{ds}} = 287\left(\frac{p_V}{p_H}\right)\ \frac{3.33(K_{LM})_u(L/H)}{(y/H)^2} + 4.70\ (K_{LM})_u\left[\frac{Y}{H} - \frac{L}{H} + 4.70\left(1 - \frac{Y}{H}\right)\tan\gamma\right] \qquad 4\text{-}99$$

where

$$\gamma = 12° - \tan^{-1}\left(\frac{0.2126}{y/L}\right) \qquad 4\text{-}100$$

The solution for partial failure (deflection X_1,) for the above two-way element is obtained in a similar manner. Substituting the general expressions for partial failure into Equation 4-96 yields:

$$\frac{i_b^{\,2}H}{p_H d_c^{\,3}f_{ds}} = 957\left(\frac{p_V}{p_H}\right)\ (K_{LM})_u\frac{(L/H)}{(y/H)^2} \qquad 4\text{-}101$$

Equations 4-99 and 4-101 can be rewritten as:

$$\frac{i_b^{\,2}H}{p_H\ d_c^{\,3}f_{ds}} = C_1 \qquad 4\text{-}102$$

$$\frac{i_b^2 H}{p_H d_c^3 f_{ds}} = C_u \qquad 4\text{-}103$$

where the right-hand side of the equation is designated as the impulse coefficient. The impulse coefficient C_u is used for incipient failure design (maximum deflection equals X_u) whereas C_1 is for partial failure design (maximum deflection equals X_1). These impulse coefficients are a proportional measure of the impulse capacity under the resistance-deflection curve up to the maximum deflection.

Expressions for the impulse coefficients of elements with various support conditions and yield line locations have been derived as above. Equations for C_1 and C_u for two-way elements are given in Table 4-9 and Table 4-10, respectively. For one-way elements which do not exhibit the secondary resistance range ($X_1 = X_u$), the coefficient C_1 is equal to C_u. In addition, for a given support condition, C_u for a one-way element is a constant value. Table 4-11 gives the values of C_u for one-way elements.

4-33.3. Design Equations for Deflections X_1 and X_u

For design purposes, Equations 4-102 and 4-103 can be rewritten as:

$$p_H d_c^3 = \frac{i_b^2 H}{C_1 f_{ds}} \qquad 4\text{-}104$$

$$p_H d_c^3 = \frac{i_b^2 H}{C_u f_{ds}} \qquad 4\text{-}105$$

For a two-way element C_1 and C_u are functions of support conditions, aspect ratio, yield line location, reinforcement ratios and the load-mass factors. It was shown in Chapter 3 the $(K_{LM})_u$ for a two-way element varies with the yield line location ratio y/H or x/L. Furthermore, it was shown that yield line location ratio is a function of the span ratio L/H and the moment ratio $[(M_{VN} + M_{VP})/(M_{HN} + M_{HP})]$. Since the cross sections used for large deflection design are equally reinforced on each face, the moment ratio is, in effect, the ratio of the reinforcement ratio p_V/p_H. Thus it can be seen that the impulse coefficients are solely functions of L/H and p_V/p_H for a given support condition.

To facilitate the design procedure, charts have been constructed for the impulse coefficients C_1 and C_u for two-way elements as a function of p_V/p_H and L/H. These curves for various support conditions are given in Figures 4-30 through 4-32 for C_1 and Figures 4-33 through 4-35 for C_u. For one-way elements C_u is a constant (see Table 4-11).

4-33.4. Optimum Reinforcement

A prime factor in the design of any facility is construction economy. Proper selection of section sizes and reinforcing steel will result in a design having optimum capacity and minimum cost. See discussion in paragraph 4-23.1.

To determine the optimum design of any particular two-way structural element, consideration must be given to the following:

1. There is an ideal distribution of flexural reinforcement, defined by the reinforcement ratio p_V/p_H, which is independent of section depth. This ratio will yield the maximum blast impulse capacity for a given total amount of flexural reinforcement p_T.

2. There is an ideal relationship between the quantity of reinforcement to the quantity of concrete which will result in the minimum cost of an element. This relationship is defined by the total percentage of reinforcement in one face of an element. This total percentage p_T is the sum of the vertical and horizontal reinforcement ratios, p_V and p_H, respectively.

4-33.4.1. Optimum Reinforcement Distribution

The blast impulse capacity of an element varies with the distribution of the reinforcement even though the total amount of reinforcement and the concrete thickness remains the same. This optimum reinforcement ratio varies for different support conditions as a function of the aspect ratio L/H. In addition, the optimum ratio is different for partial failure and incipient failure design.

To illustrate the determination of the optimum reinforcement distribution ratio p_V/p_H, consider a two-way panel fixed on three sides. The panel has an aspect ratio L/H equal to 3 and a total percentage of reinforcement p_T equal to 1 percent. For various values of p_V/p_H, the impulse capacity can be determined for both partial and incipient failure design from Figures 4-31 and 434, respectively.

If $(i_b{}^2H)/(d_c{}^3f_{ds}$ is plotted versus p_Vp_H the resulting curves are shown in Figure 4-36. The ideal p_V/p_H occurs at the maximum value of $i_b{}^2H/d_c{}^3f_{ds}$ and is indicated on the illustration as 1.58 for incipient failure and 1.93 for partial failure design. Increasing or decreasing the total amount of steel p_T, will shift the curves up or down but not effect the optimum p_V/p_H ratio. This optimum p_V/p_H ratio for other L/H ratios and support conditions are determined from similarly constructed curves.

The optimum values of p_V/p_H for various support conditions are plotted as a function of the aspect ratio. Figure 4-37 gives the optimum reinforcement for partial failure design, while Figure 4-38 gives the optimum ratio for incipient failure design.

The optimum reinforcement ratio for partial failure design always results in positive yield lines which bisect the 90 degree angle at the corners of the element (45 degree yield lines) for all support conditions. Consequently, all supports reach the maximum rotation of 12 degrees simultaneously and they are all on the verge of failure. Therefore, the optimum condition for partial failure is a particular case of incipient failure. This condition is evident from the common point on Figure 4-36. It can also be seen from this figure that at p_V/p_H ratios other than the common point, partial failure design is more conservative than incipient failure design which includes the post ultimate range. The optimum p_V/p_H ratio for partial failure design maximizes the impulse capacity up to X_1 leaving no reserve capacity (post ultimate range). Therefore, at this ratio, the capacity is numerically equal to that for incipient failure design. While there is no quantitative advantage to optimum partial failure design over incipient failure design, there is a qualitative advantage. The elements remain intact since all supports are on the verge of failure as opposed to optimum incipient failure where some supports have failed and the remaining supports are on the verge of failure. In this latter case, there is unknown secondary cracking which is not accounted for in the design.

As previously explained, incipient failure design includes the capacity from two-way action of an element up to partial failure X_1, and the capacity of one-way action up to incipient failure X_u. Except as explained below, the optimum reinforcement ratio for incipient failure design results from maximizing the capacity due to one-way action after partial failure (post-ultimate range). The resulting optimum reinforcement ratios for incipient failure design produce various yield line configurations depending upon the support conditions. For four edges fixed, the optimum reinforcement ratio is 0.25 and 4.0 for aspect ratios less than and greater than one, respectively. This distribution maximizes the post ultimate one-way action in the shorter direction. For two edges fixed, the increase in capacity due to cantilever action in the post ultimate range is less than the decrease in capacity of the ultimate range. Thus, for these elements, the capacity cannot be increased above that for partial failure, and the optimum ratio for incipient failure design is the same as for partial failure design (45 degree yield lines). For three edges fixed, the post ultimate range capacity is due to either cantilever action in the vertical direction or fixed-fixed beam action in the horizontal direction. In regions where the post ultimate range consists of cantilever action (L/H ratio in the immediate vicinity of 2 and L/H ratio greater than 4) the optimum ratio is the same as for partial failure. For L/H ratios less than 1.5, the post ultimate range consists of fixed-fixed beam action and, therefore, the optimum ratio is equal to 0.25. Between these L/H regions, neither behavior dominates and the resulting optimum p_V/p_H ratios maximizes the combination of ultimate and post ultimate range capacities.

4-33.4.2. **Optimum Total Percentage of Reinforcement**

The optimum total percentage of reinforcement p_T gives the relationship between the quantity of reinforcement to the quantity of concrete which results in the minimum cost of an element. The total percentage of reinforcement in one face of the element is defined as:

$$p_T = p_V + p_H \qquad 4\text{-}106$$

The optimum percentage of reinforcement depends upon the relative costs of the concrete and reinforcing steel. Based on the average costs of concrete and steel, the optimum percentage of reinforcement p_T has been determined to be between 0.6 and 0.8 percent, with 0.7 being a reasonable value to be used for design. However, for large projects, a detailed cost analysis may result in a more economical design.

In the usual design situation, the optimum p_V/p_H ratio is first determined based on the support conditions and aspect ratio. Knowing this ratio, C_1 or C_u is determined and along with the given values of i_b H, f_{ds}, Equation 4-104 or 4-105 results in:

$$p_H d_c^3 = \text{constant} \qquad 4\text{-}107$$

With the known values of p_V/p_H and the optimum total percentage of reinforcement equal to 0.7, the required quantity of horizontal reinforcement p_H is calculated. The required thickness of the element is then calculated from Equation 4-107.

In some design cases, it may be desirable to reduce the concrete thickness below the optimum thickness. The quantity of reinforcement in excess of the optimum p_T must be provided to obtain the necessary impulse capacity. The cost increase is small for total percentages of steel in the vicinity of the optimum value of p_T. In fact, the use of p_T equal to 1 percent will result in a cost increase of less than 10 percent. Beyond 1 percent reinforcement, the cost increase is more rapid. However, except for very thin elements, the use of reinforcement in excess of 1 percent is impractical since the required details cannot be maintained with such large quantities of reinforcing steel. For thick walls providing even the optimum p_T of 0.7 percent may be impractical and p_T may have to be reduced to as low as 0.3 percent (minimum reinforcement of 0.15 percent in each direction) in order to permit placement of the reinforcing steel. The total reinforcement p_T may also be less than optimum if a minimum concrete thickness is required to prevent fragment penetration. When the minimum quantity of reinforcement is provided whether for strength or to satisfy minimum requirements, the resulting cost may be far in excess of optimum.

In some cases of incipient failure design, the optimum reinforcement ratio p_V/p_H is equal to 0.25 or 4.0. However, in most cases, it is impractical to provide four times as much reinforcement in one direction as in the other direction. Since the minimum required percentage of reinforcement in a given direction is 0.15, the orthogonal direction would require 0.6 percent for a total percentage of 0.75. Although this percentage is approximately equal to the optimum percentage of 0.7. it may still be impractical in all but thin walls. Consequently, in such design situations, a trade off between optimum reinforcement ratio p_V/p_H and the optimum total percentage reinforcement p_T must be made for an economical design.

4-33.5 Design Equation for Deflections Less than X_1 or X_u

For certain conditions, it is sometimes desired to design a structural element for maximum deflections other than partial failure deflection X_1 or incipient failure deflection X_u. For those cases, the impulse coefficients can be scaled relative to the deflections.

For a maximum deflection X_m in the deflection range $X_1 < X_m < X_u$, Equation 4-107 becomes

$$p_H d_c^3 = \frac{i_b^2 H}{C'_u f_{ds}} \qquad 4\text{-}108$$

where

$$C'_u = C_1 + \frac{X_m - X_1}{X_u - X_1}(C_u - C_1) \qquad 4\text{-}109$$

For a maximum deflection corresponding to a support rotation greater than 5 degrees, but less than X_1, Equation 4-108 becomes

$$p_H\, d_c^3 = \frac{i_b^2 H}{C'_1 f_{ds}} \qquad 4\text{-}110$$

where

$$C'_1 = \left(\frac{X_m}{X_1}\right) C_1 \qquad 4\text{-}111$$

The optimum p_V/p_H ratio for a given element is a constant for any deflection less than partial failure deflection X_1, and is determined from Figure 4-37. In the deflection range $X_1 < X_m < X_u$ the optimum p_V/p_H ratio varies with the maximum deflection. However, for design purposes, the values from Figure 4-38 for incipient failure may be used.

4-33.6. **Design Equations for Unspalled Cross Sections**

The impulse coefficients derived above may also be used for type II or unspalled cross sections. However, the general form of the equation is slightly modified to account for the change in the physical properties of the cross section. For a type II cross section, the full thickness of concrete element is included in calculating the effective mass. Thus, the design equations for the impulse coefficients of unspalled sections take the form:

$$p_H\, T_c\, d_c^2 = \frac{i_b^2 H}{C_1\, f_{ds}} \qquad 4\text{-}112$$

$$p_H\, T_c\, d_c^2 = \frac{i_b^2 H}{C_u\, f_{ds}} \qquad 4\text{-}113$$

where T_c is the total thickness of the concrete section.

The optimum reinforcement ratios and the impulse coefficients are the same for spalled and unspalled cross sections . The design procedure for unspalled cross sections is very similar to the procedure described in Section 4-33.4.2. The total thickness of concrete T_c can be expressed in terms of d_c by approximating the value of d'. The value of d' can be estimated by determining the required concrete cover and assuming the reinforcing bar sizes.

4-34. **Flexural Design for Limited Deflections**

In the design of elements for large deflections, only the plastic range behavior of the element was considered, since the capacity due to elasto-plastic behavior is relatively small. For elements where support rotations are limited to 5 degrees or less, the elasto-plastic range is a significant portion of the element's total capacity as well as of its deflected shape. Therefore, it must be included in the determination of the response of such elements.

The blast impulse capacity of an element whose maximum deflection is less than or equal to 5 degrees was given in Chapter 3 as

$$\frac{i_b^2 H}{2m_a} = \frac{r_u X_E}{2} + \frac{m_a}{m_u} r_u (X_m - X_E) \qquad 4\text{-}114$$

where

m_a = average of the effective elastic and plastic unit masses

X_E = equivalent elastic deflection

This is an equation which is suitable for analysis rather than design. Impulse coefficients could theoretically be derived in a similar manner as that for large deflections . However, the equivalent elastic deflection cannot be defined by a mathematical expression making the determination of impulse coefficients for the various support conditions impractical.

The design of an element subjected to an impulse load (short duration pressure-time load) for limited deflections is accomplished using a trial and error procedure. An element would be assumed (concrete thickness and reinforcement) and its response determined from the response charts of Chapter 3. A preliminary estimate of the size of the element can be obtained using the equations for partial failure design where the impulse coefficient is modified for reduced rotations according to Equation 4-110. It should be noted that this preliminary design will underestimate the required element.

The above procedure would be used for laced elements designed for support rotations less than 5 degrees. However, if an element is designed for support rotations less than 2 degrees and single leg stirrups are used in place of lacing reinforcement, the above preliminary estimate of the size of the element may not be used. Since the position of the flexural reinforcement is not altered for single leg stirrups , an average d_c may not be used. Two values of d_c must be determined; one for the vertical reinforcement and second for the horizontal reinforcement. Therefore, the capacity of the element (flexural and shear capacity) must be determined according to the procedures for conventional reinforced slabs.

4-35 Design for Shear

4-35.1. General

After the flexural design of an element has been completed, the required quantity of shear reinforcement must be determined. This shear reinforcement insures that the desired flexural behavior in the ductile mode will be attained. The design of the lacing reinforcement has been discussed in previous sections. This section is concerned with the determination of the shear stresses and forces to be used in the design equations.

Shear coefficients can be derived in a manner similar to that used to derive the impulse coefficients above. The equations for support shear given in Chapter 3 and for the ultimate shear stress given in Section 4-27 show that the shear reinforcement is a function of the resistance of the element and not of the applied load. The shear forces and stresses vary as the ultimate unit resistance, the geometry and yield line locations of the element, and the section depth. If r_u is evaluated and substituted into these shear expressions, it can be shown that the ultimate support shear V_s can be represented as an equation in the general form

$$V_s = C \frac{p d_c^2 f_{ds}}{L} \qquad 4\text{-}115$$

and the ultimate shear stress at distance d_c from the support as

$$v_u = C p f_{ds} \qquad 4\text{-}116$$

where

C = shear coefficient

p = flexural reinforcement ratio

f_{ds} = dynamic design stress of the flexural reinforcement

The shear coefficient is different for each case and also different for one-way and two-way elements. Specific values are indicated in the following paragraphs of this section.

4-35.2. Ultimate Shear Stress

4-35.2.1. One-Way Elements

The ultimate shear stress v_u at distance d_c from the support for a one-way element is

$$v_u = C_d p f_{ds} \qquad 4\text{-}117$$

where C_d is the shear coefficient and a function of the ratio of d_c/L. Values of C_d are shown in Table 4-12.

4-35.2.2. Two-Way Elements

The ultimate shear. stress V_{uH} in the horizontal direction (along side H) at a distance d_c from the support for a two-way element is given as

$$v_{uH} = C_H p_H f_{ds} \qquad 4\text{-}118$$

and in the vertical direction (along side L) as

$$v_{uV} = C_V p_V f_{ds} \qquad 4\text{-}119$$

where C_H and C_V are the horizontal and vertical shear coefficients, respectively. The shear coefficients, given in Table 4-13, vary as d_c/x or d_c/y for the triangular sectors and as x/L and d_c/H or y/H and d_c/L for the trapezoidal sectors. The solution for the shear coefficients is presented graphically in Figures 4-39 through 4-52.

The shear coefficients for the triangular sectors, can be read directly from either Figure 4-39 or 4-40, since the yield line location is the only variable involved. Plotting the shear coefficients for the trapezoidal sectors for a particular support condition yields a family of curves. That is, the shear coefficient is plotted versus d_c/L for various values of y/H (or d_c/H for various values of x/L). The maximum value of the shear coefficient is different for each curve of y/H or x/L and occurs at various values of d_c/L or d_c/H. Therefore, these family of curves overlap and accurate interpolation between curves is difficult.

Using a method of coordinate transformation, the family of curves has been reduced to a set of curves with a common maximum point defined (using the horizontal shear coefficient as an example) by $C_H/C_M = 1$ and $(d_c/L)/(d_c/L)_M = 1$. The quantities C_M and $(d_c/L)_M$ represent the coordinates of the maximum point on the original family of curves for y/H and x/L. The left-hand portions of the curves become identical, and accurate interpolation in the right-hand portion is now possible. This transformation results in two figures to define the shear coefficient for a particular support condition and yield line pattern.

The above sets of curves are presented in Figures 4-41 through 4-52. When using these curves, the shear parameter curve for the applicable support condition is entered first with the value of x/L or y/H to determine C_M and $(d_c/H)_M$ or $(d_c/L)_M$. The second curve is then used to determine C_H or C_V.

It should be noted that when designing two-way panels for incipient failure, the shear stresses in the post-ultimate range must also be checked using the equations for one-way elements.

4-35.3. Ultimate Support Shears

4-35.3.1. One-Way element

The ultimate support shear V_s for a one-way element is

$$V_s = C_s \frac{p\, d_c^2 f_{ds}}{L} \qquad \text{4-120}$$

where C_s is the shear coefficient and is a constant for a given support condition. Values of C_s for several one-way elements are given in Table 4-14.

4-35.3.2. Two-Way Elements

For a two-way element, the ultimate support shear V_{sH} in the horizontal direction (along side H) is represented as

$$V_{sH} = C_s \frac{p_H\, d_c^2 f_{ds}}{L} \qquad \text{4-121}$$

and V_{sV}, in the vertical direction (along side L) is

$$V_{sV} = C_{sV} \frac{p_V\, d_c^2\, f_{ds}}{H}$$

where C_{sH} and C_{sV} are the horizontal and vertical shear coefficients, respectively. For a given support condition, these coefficients vary as the yield line location ratios x/L or y/H. The shear coefficients are listed in Table 4-15 and for the trapezoidal sectors only are plotted in Figures 4-53 through 4-56 for various support conditions.

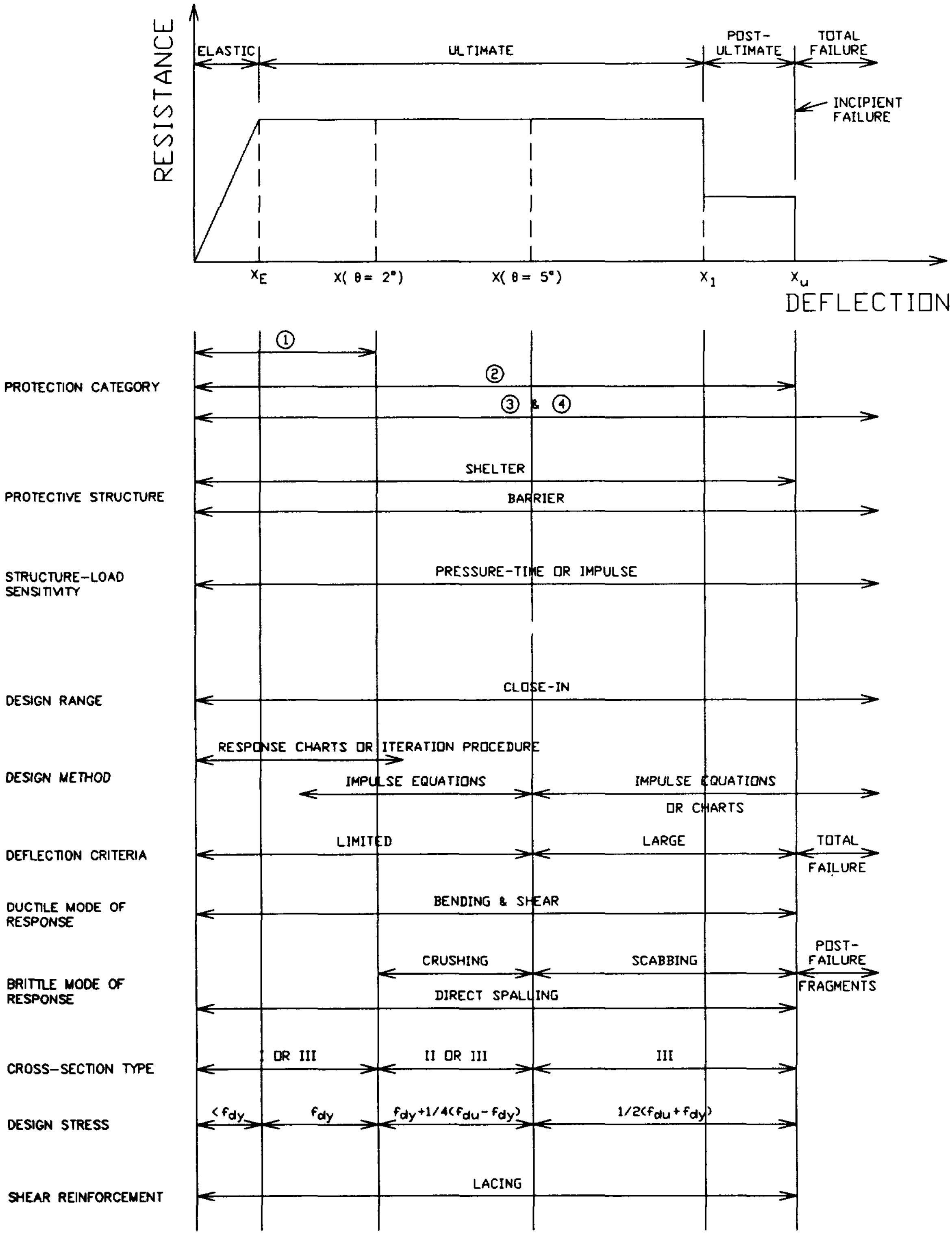


Figure 4-29 Relationship between design parameters for laced elements

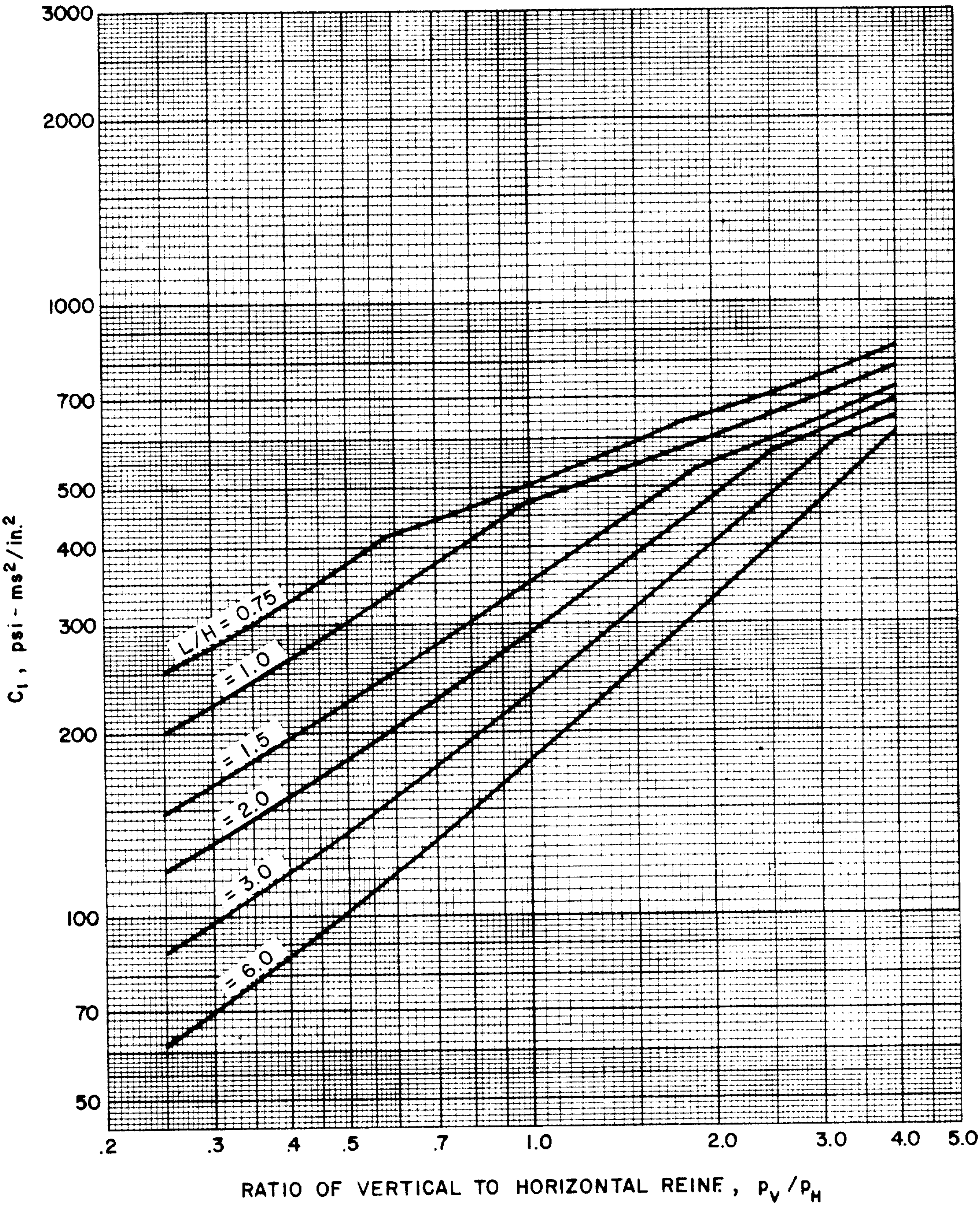


Figure 4-30 Impulse coefficient C_1 for an element with two adjacent edges fixed and two edges free

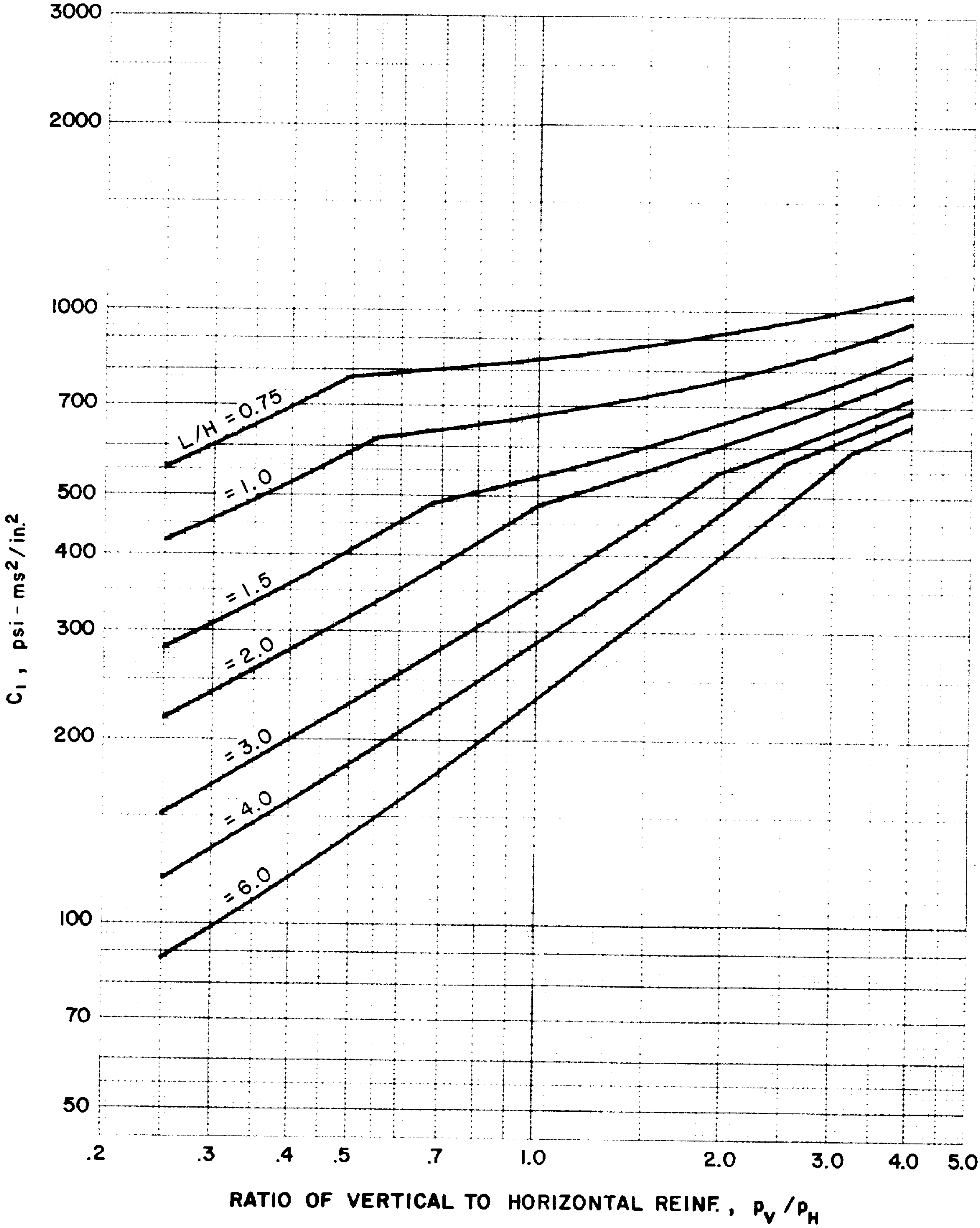


Figure 4-31 Impulse coefficient C_I for an element with three edges fixed and one edge free

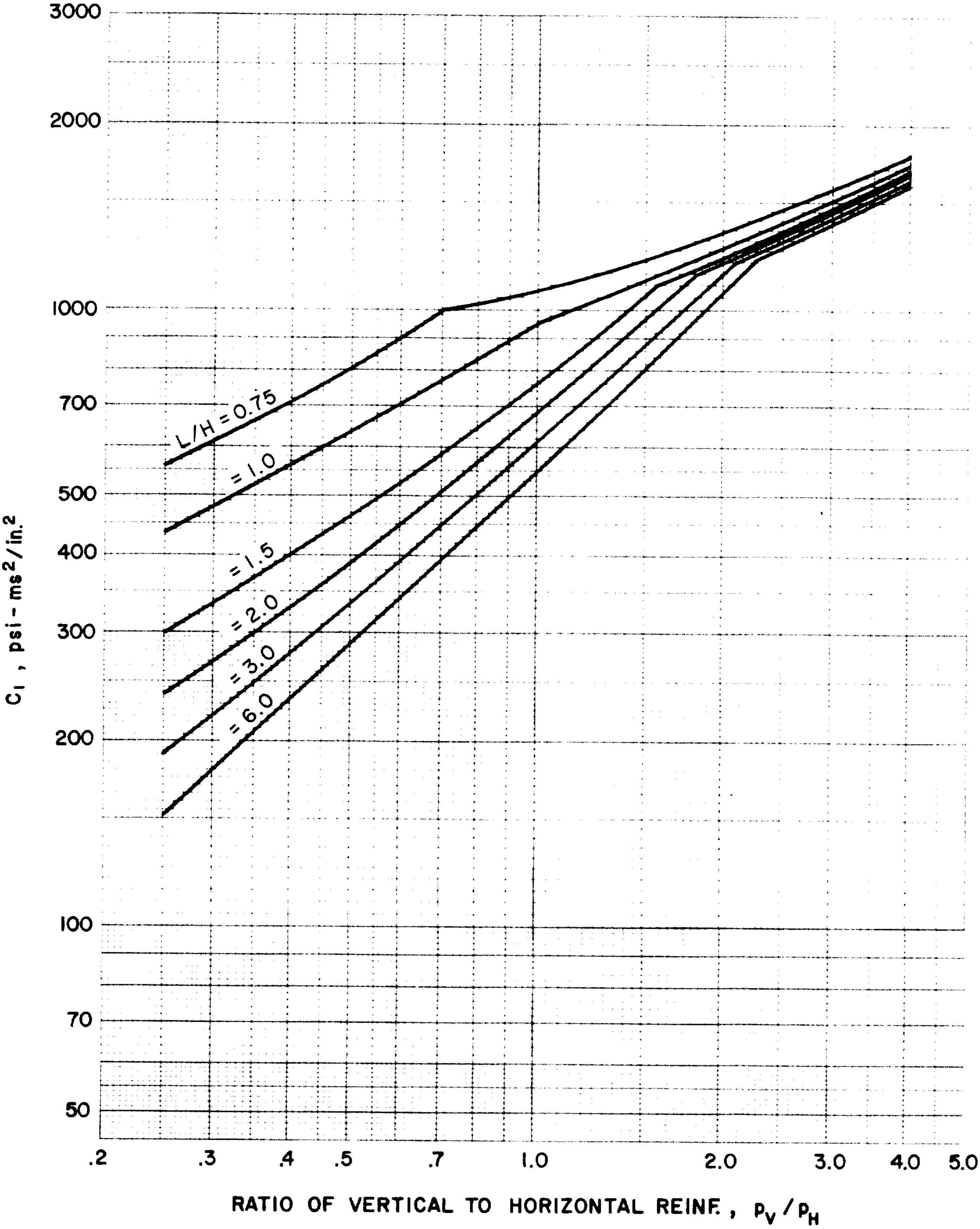


Figure 4-32 Impulse coefficient C_1 for an element with four edges fixed

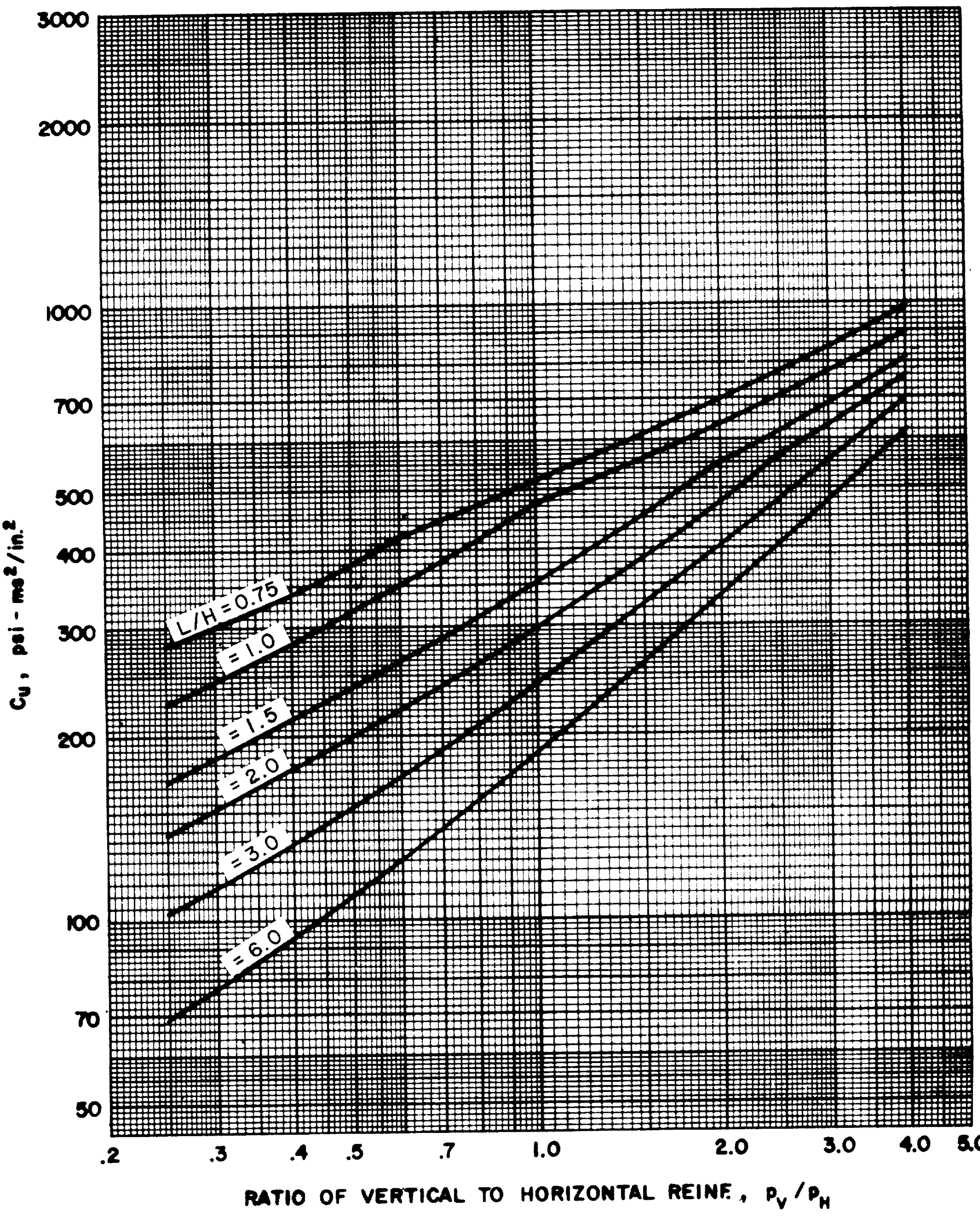


Figure 4-33 Impulse coefficient C_u for an element with two adjacent edges fixed and two edges free

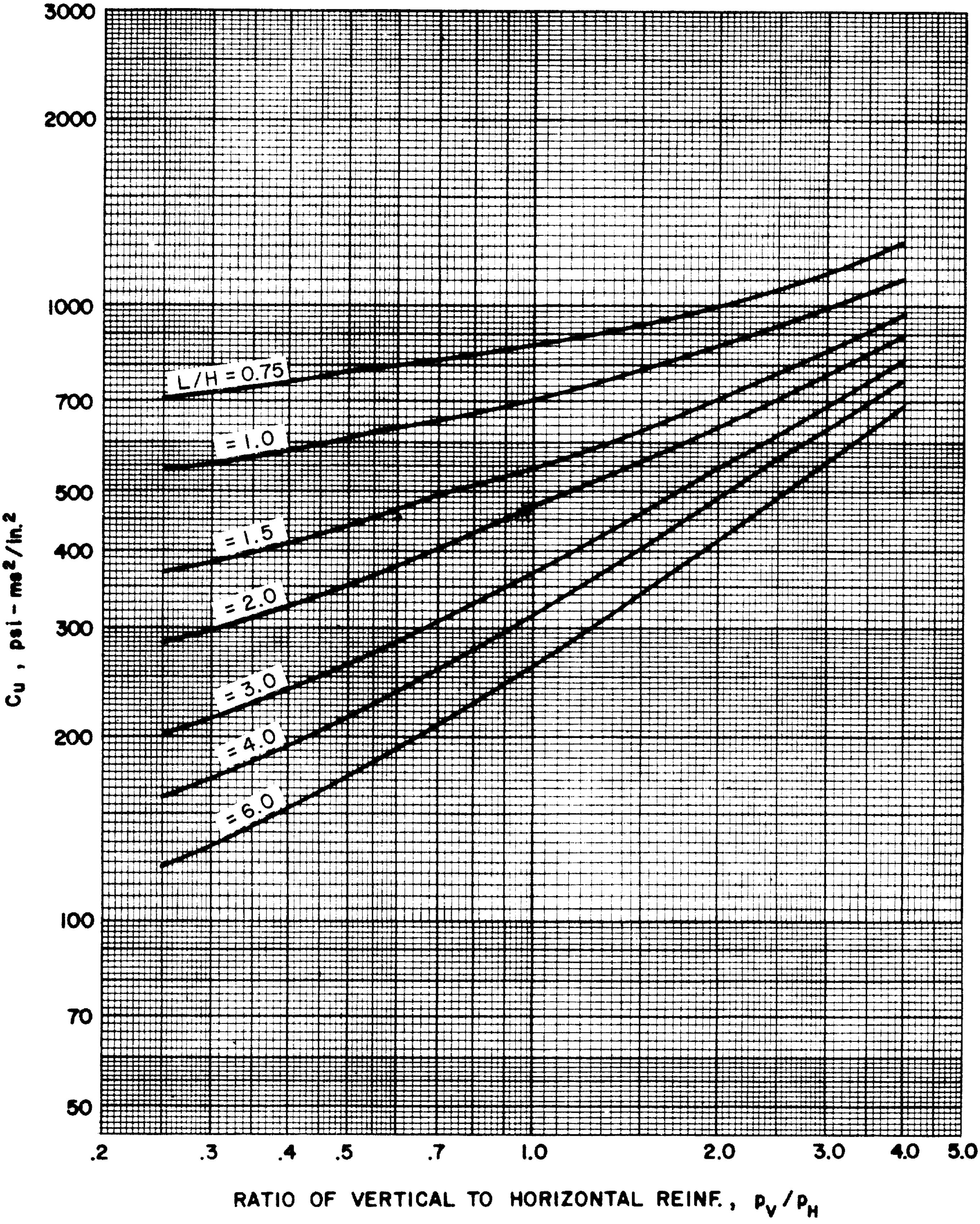


Figure 4-34 Impulse coefficient C_u for an element with three edges fixed and one edge free

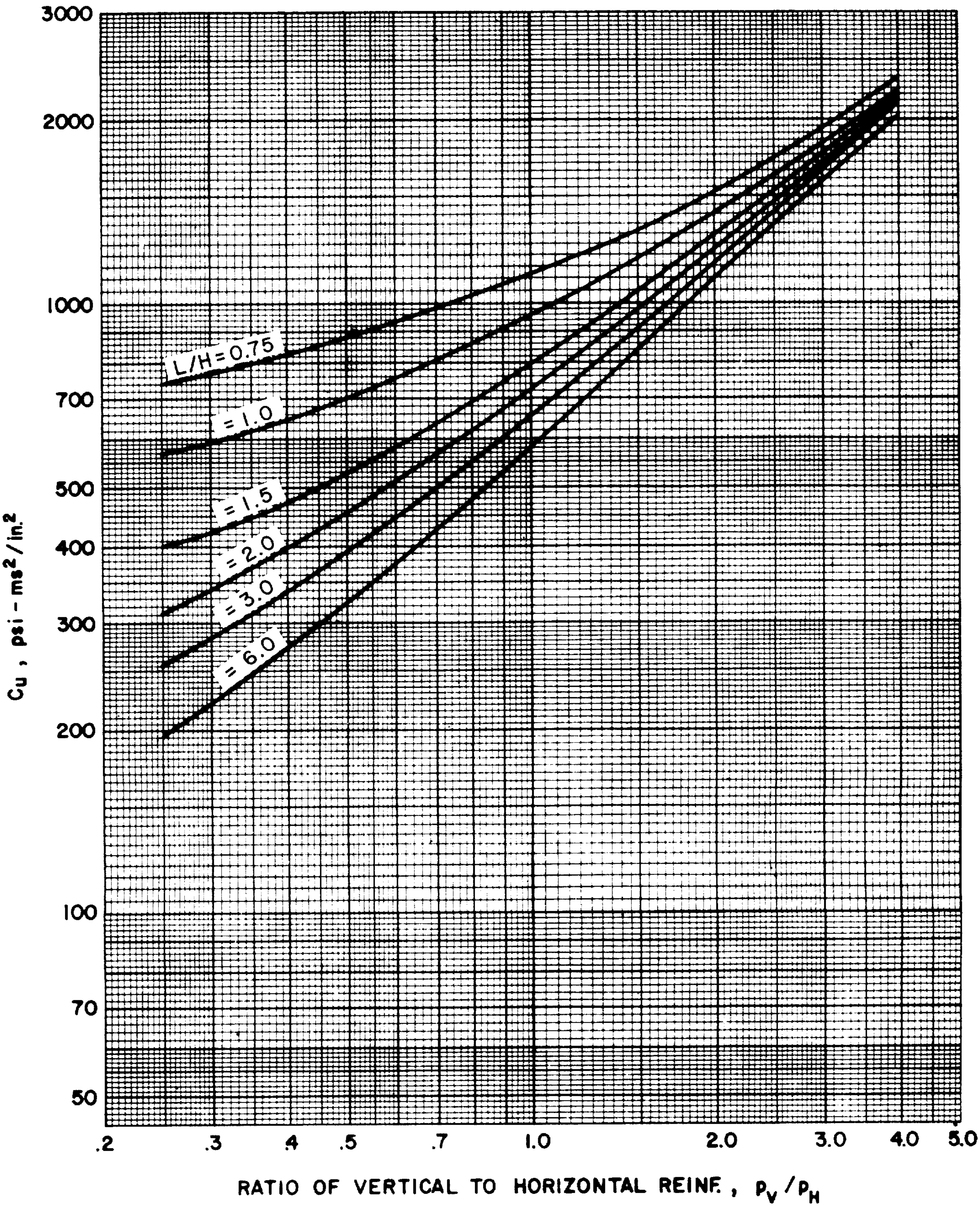


Figure 4-35 Impulse coefficient C_u for an element with four edges fixed

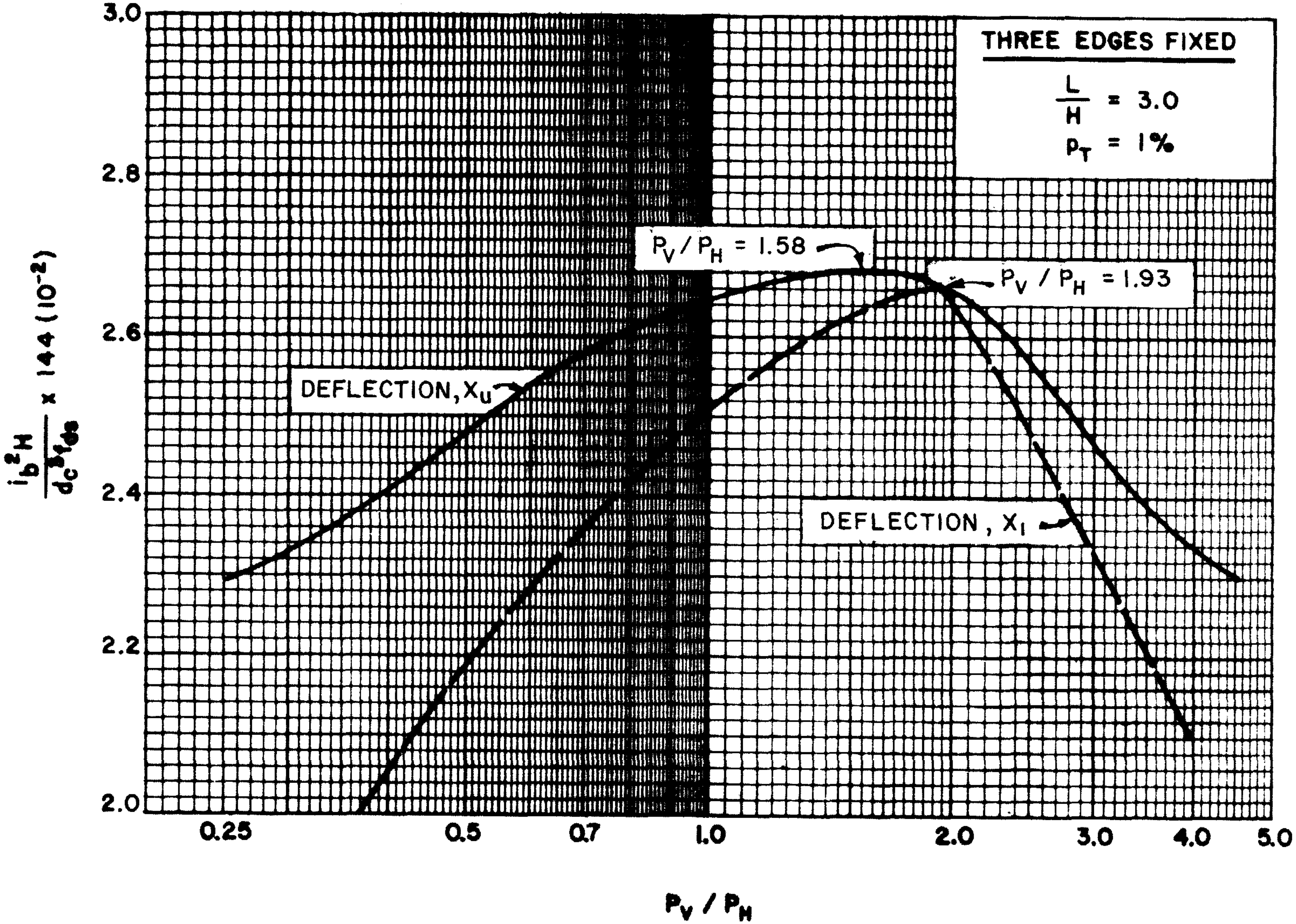


Figure 4-36 Determination of optimum ratio of p_V/p_H for maximum impulse capacity

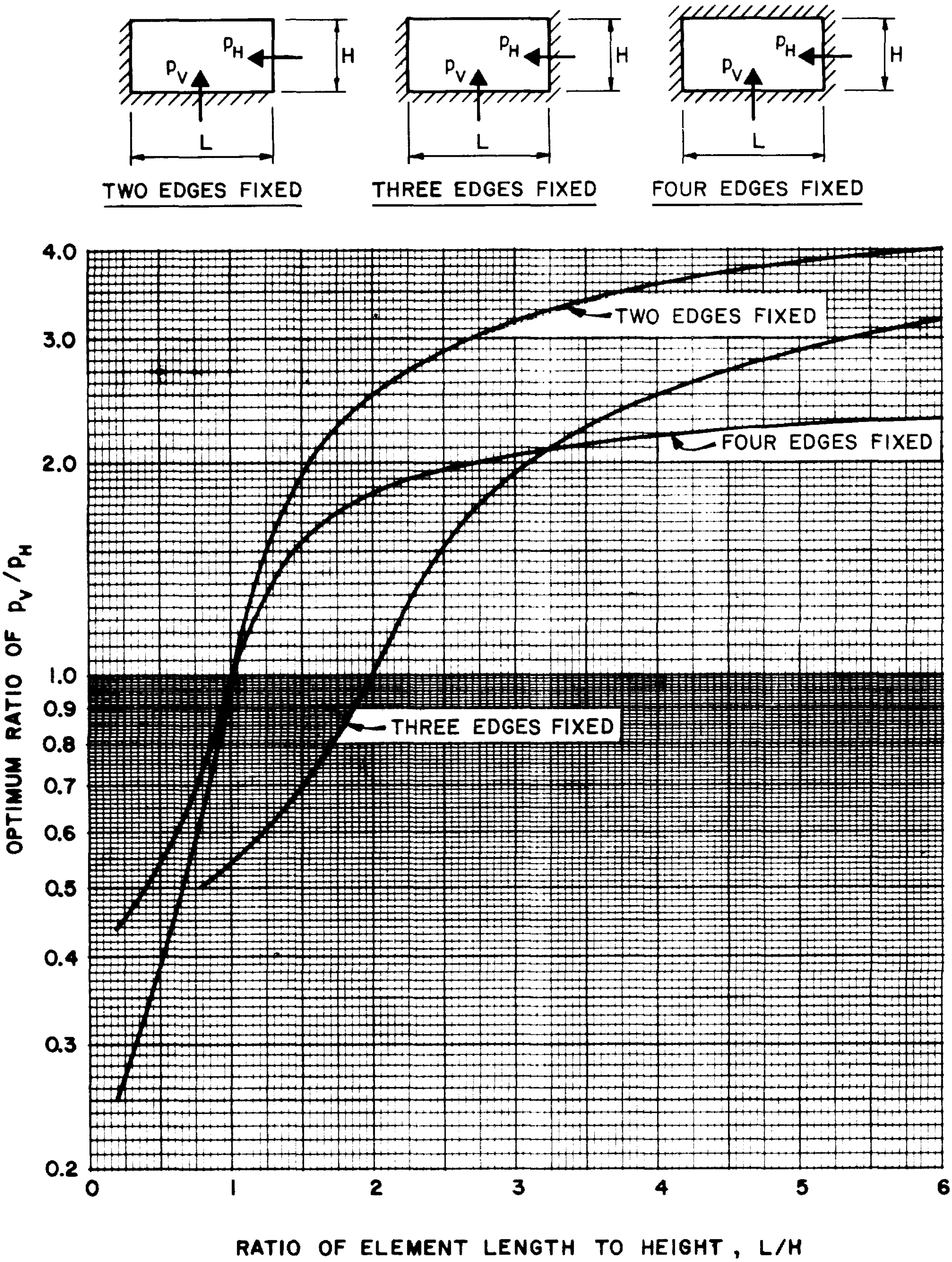


Figure 4-37 Optimum ratio of p_V/p_H for maximum capacity at partial failure deflection, X_1

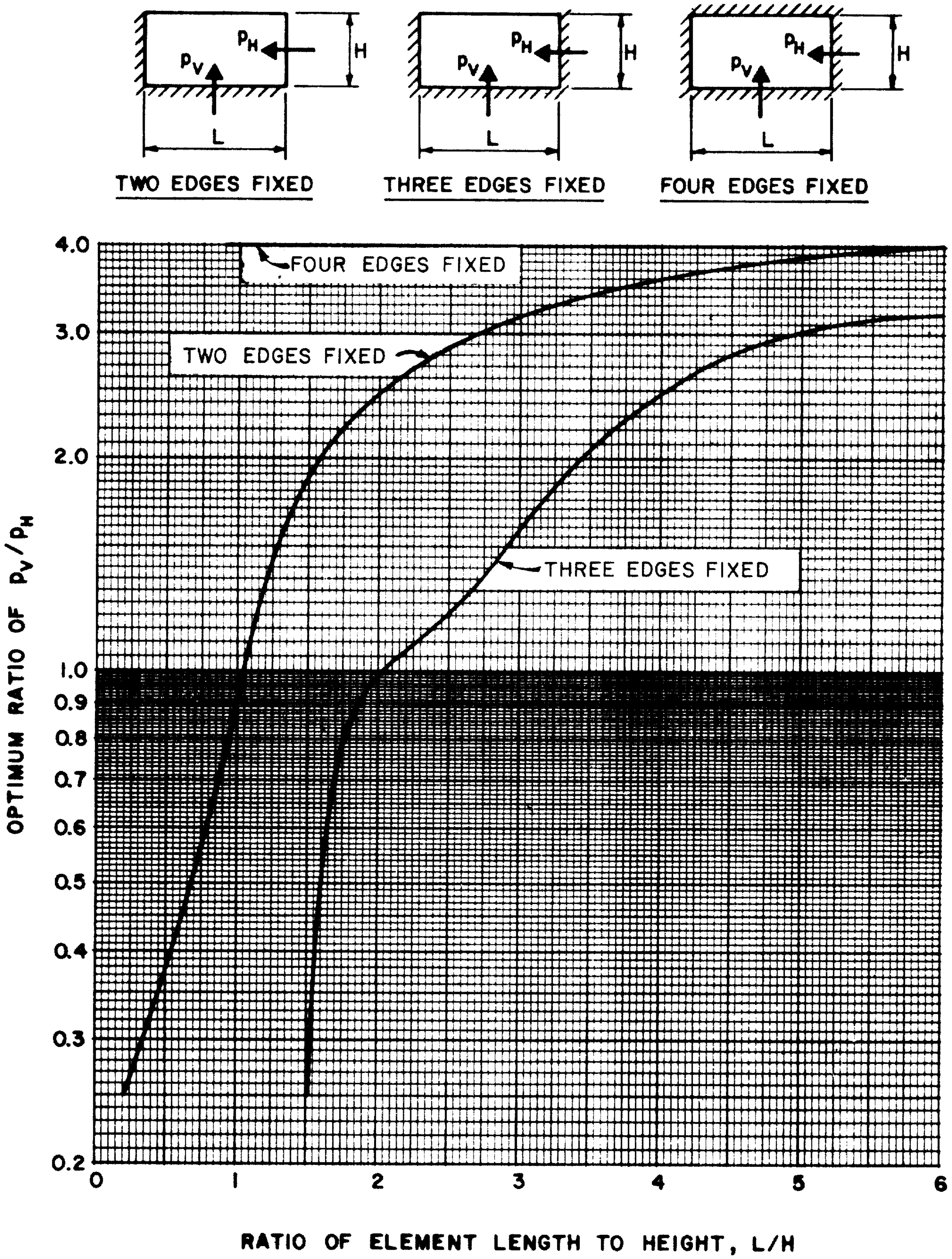


Figure 4-38 Optimum ratio of p_V/p_H for maximum capacity at incipient failure, X_u

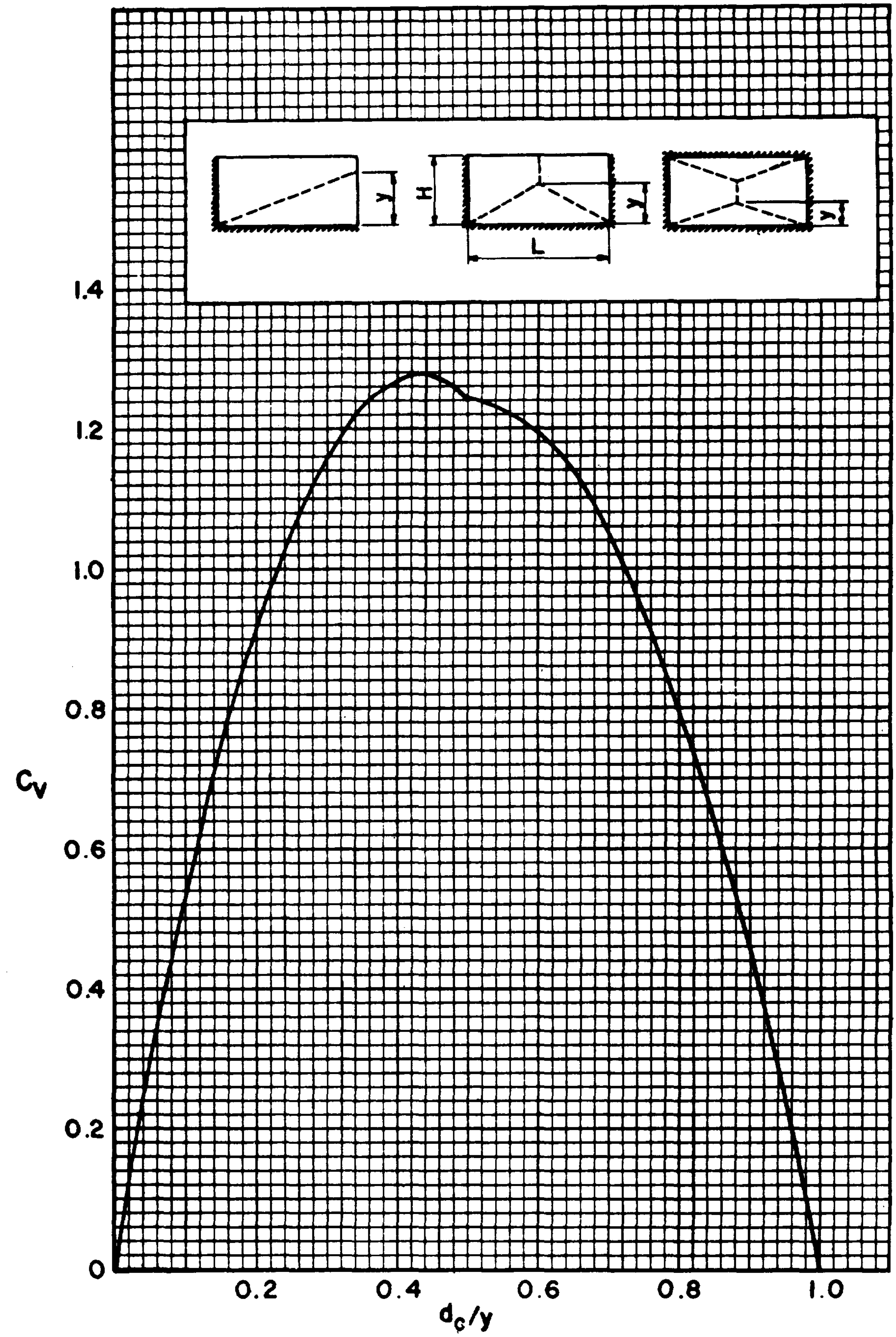


Figure 4-39 Vertical shear coefficients for ultimate shear stress at distance d_c from the support (cross section type II and III)

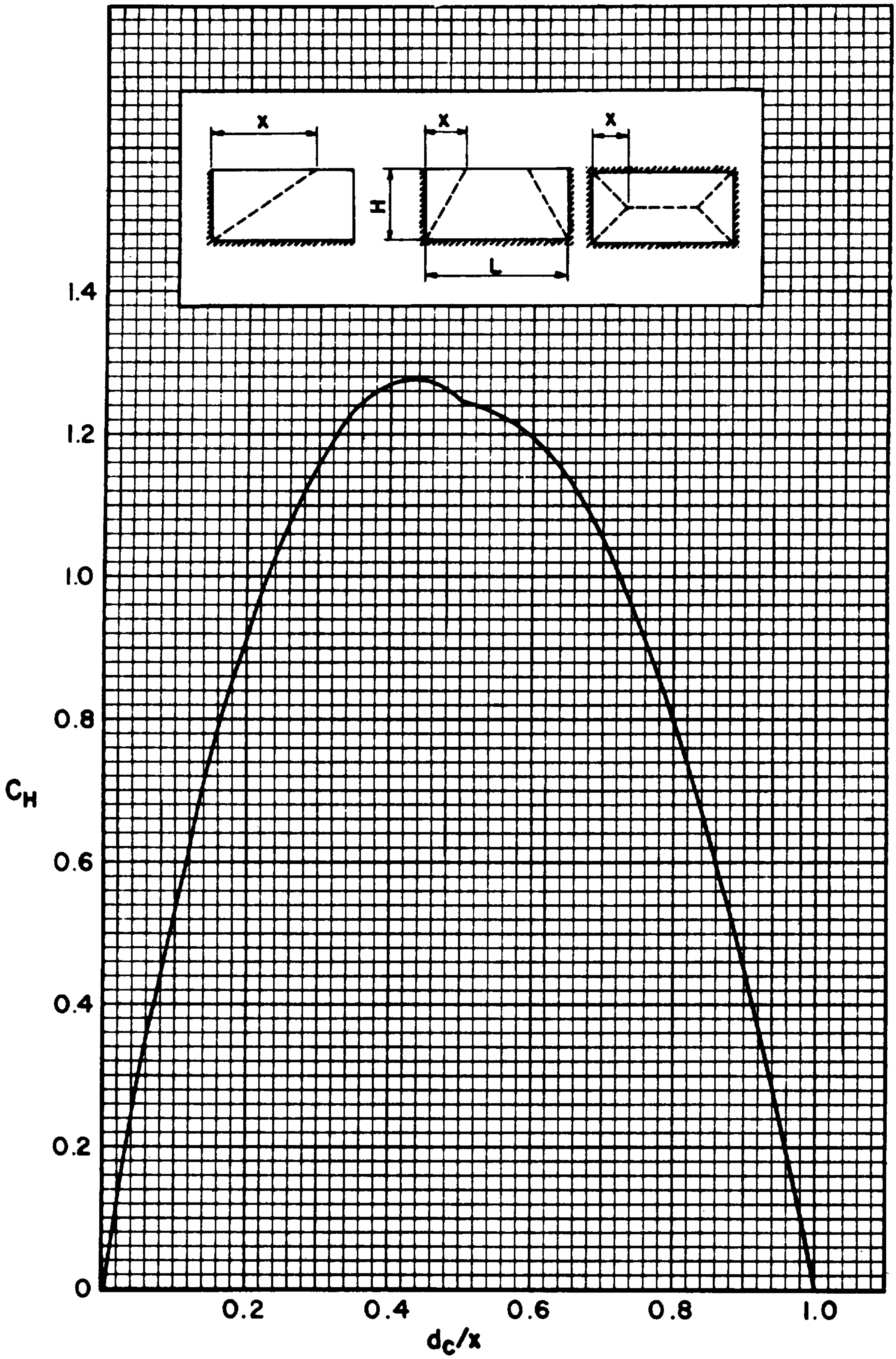


Figure 4-40 Horizontal shear coefficients for ultimate shear stress at distance d_c from the support (cross section type II and III)

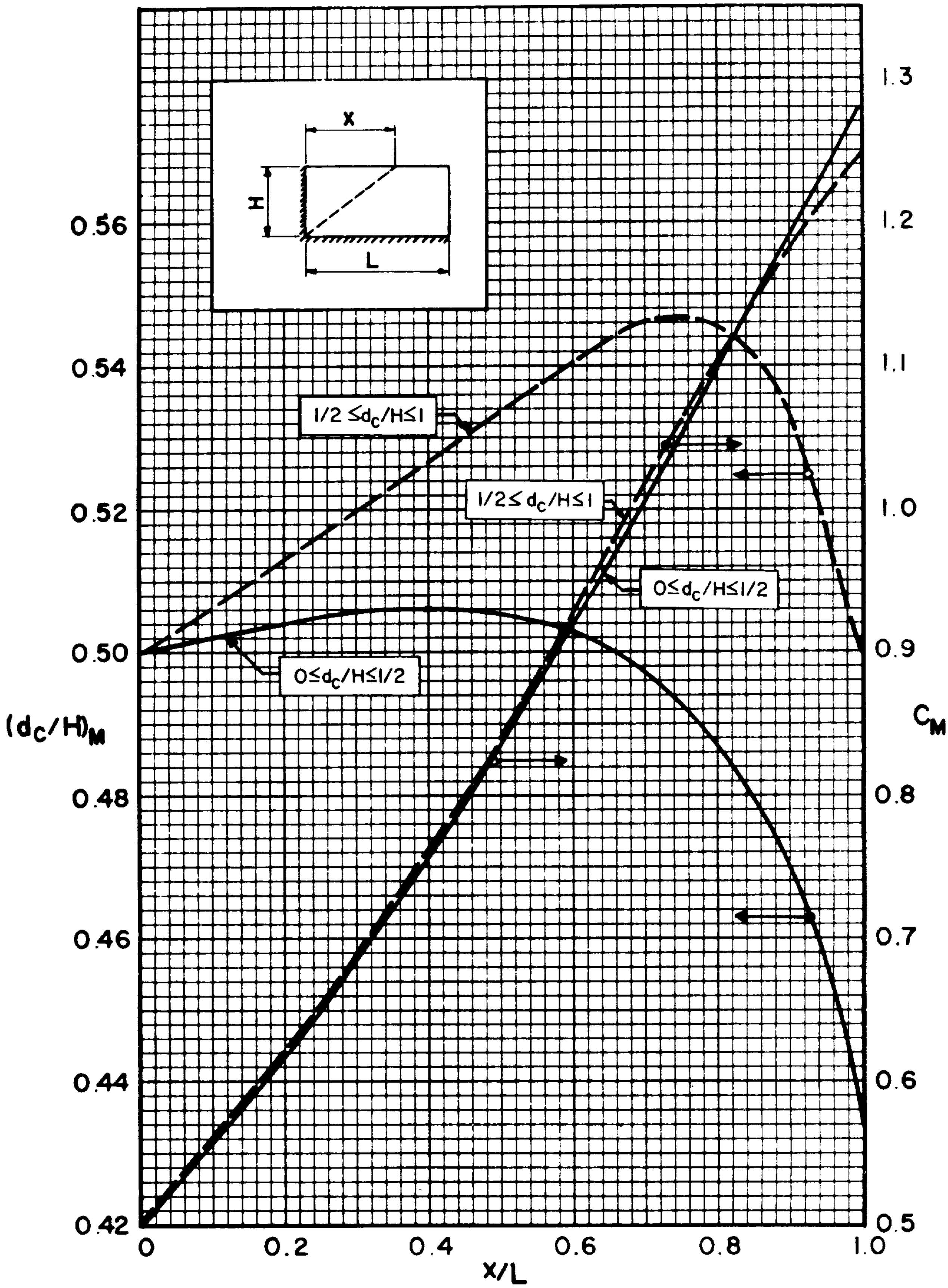


Figure 4-41 Vertical shear parameters for ultimate shear stress at distance d_c from the support (cross section type II and III)

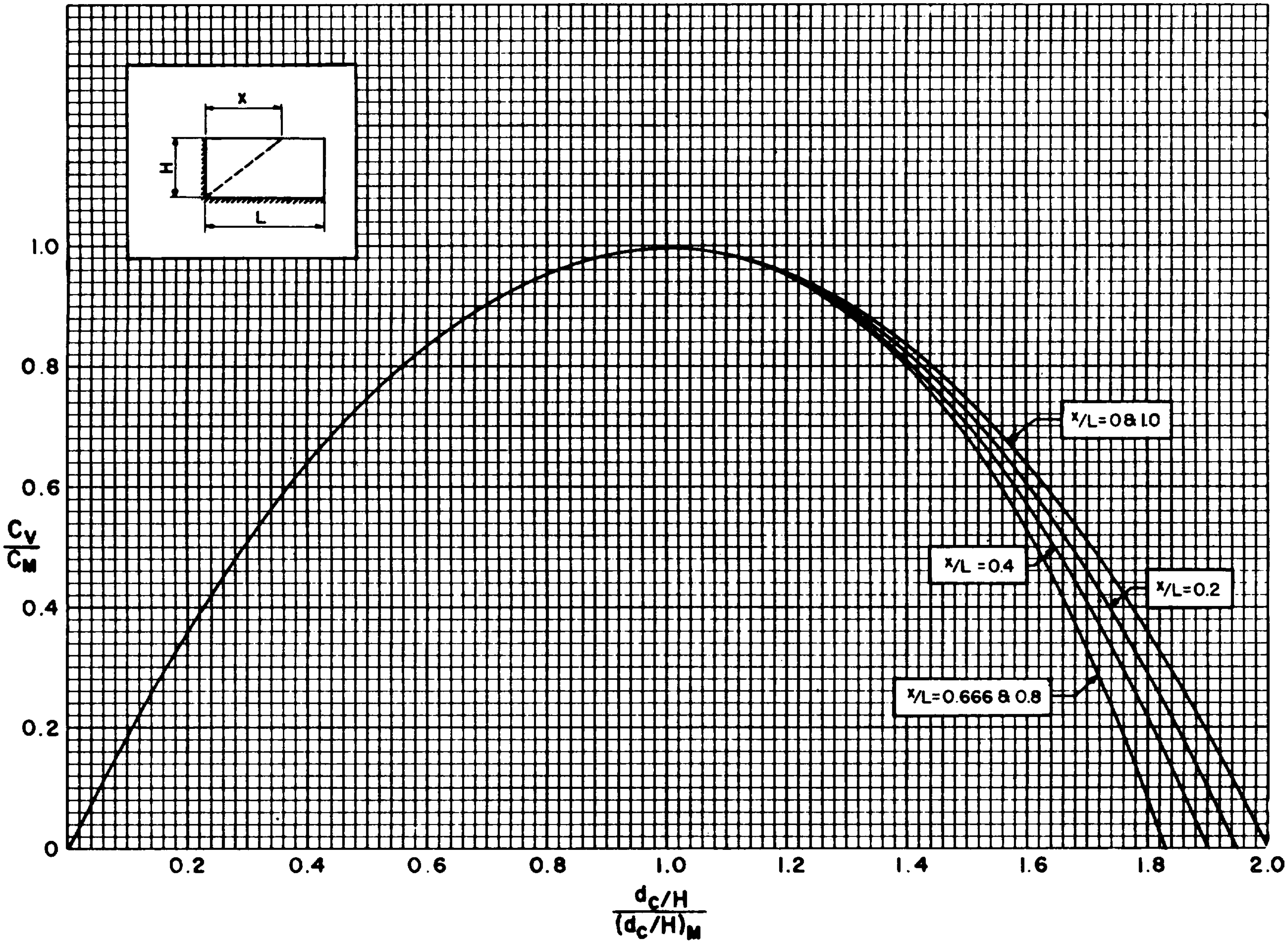


Figure 4-42 Vertical shear coefficient ratios for ultimate shear stress at distance d_c from support (cross section type II and III)

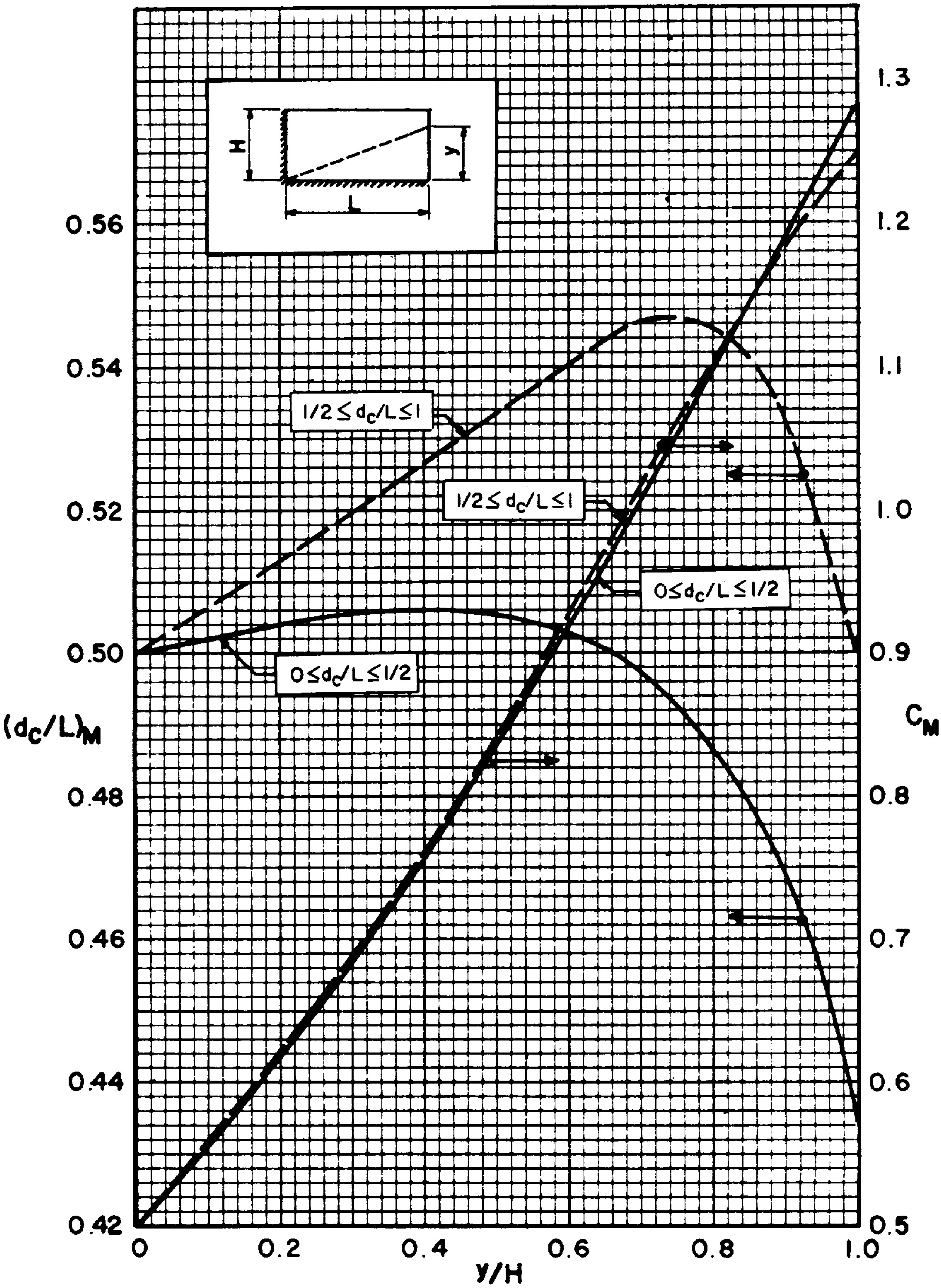


Figure 4-43 Horizontal shear parameters for ultimate shear stress at distance d_c from the support (cross section type II and III)

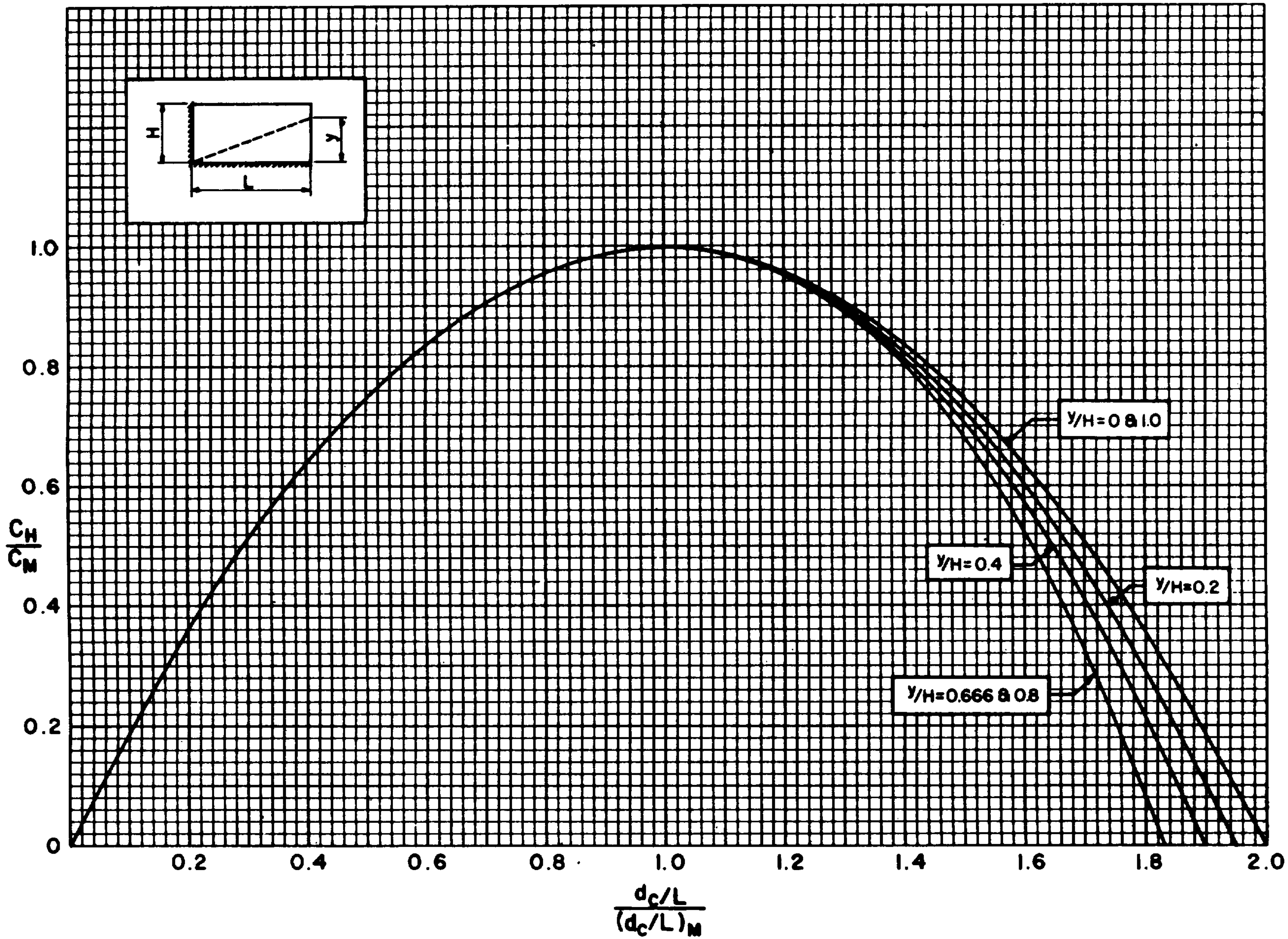


Figure 4-44 Horizontal shear coefficient ratios for ultimate shear stress at distance d_C from the support (cross section type II and III)

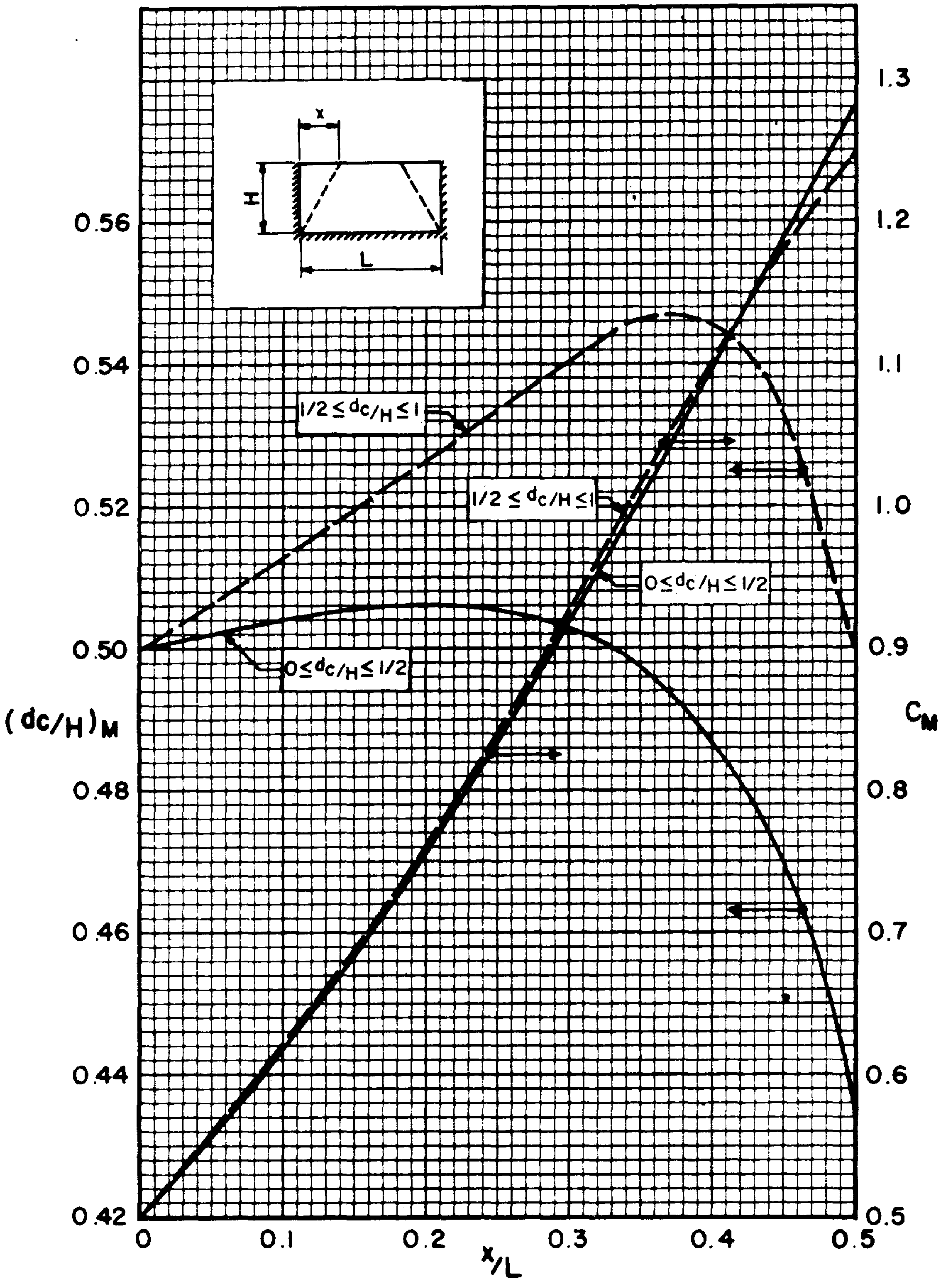


Figure 4-45 Vertical shear parameters for ultimate shear stress at distance d_c from the support (cross section type II and III)

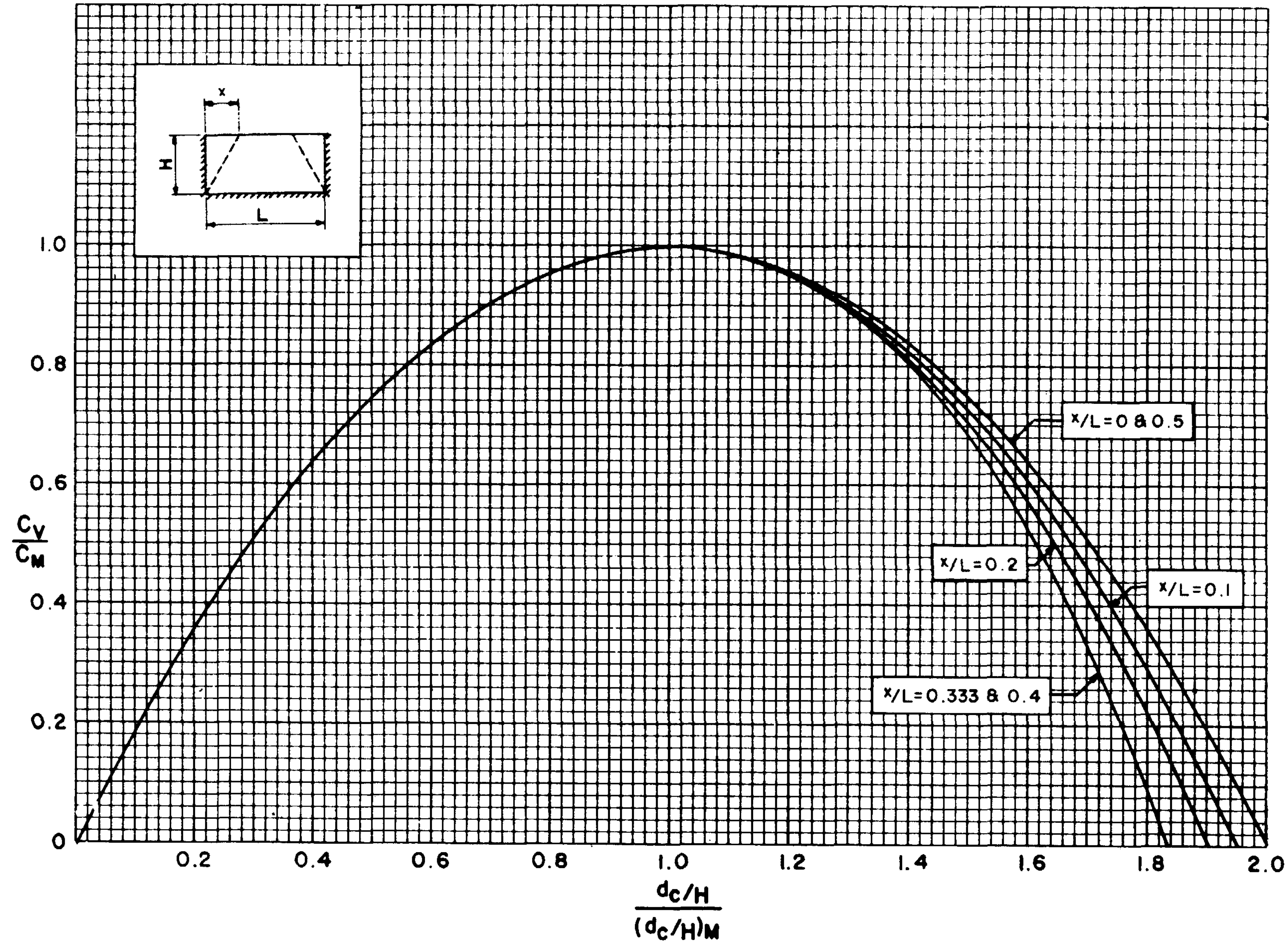


Figure 4-46 Vertical shear coefficient ratios for ultimate shear stress at distance d_c from the support (cross section type II and III)

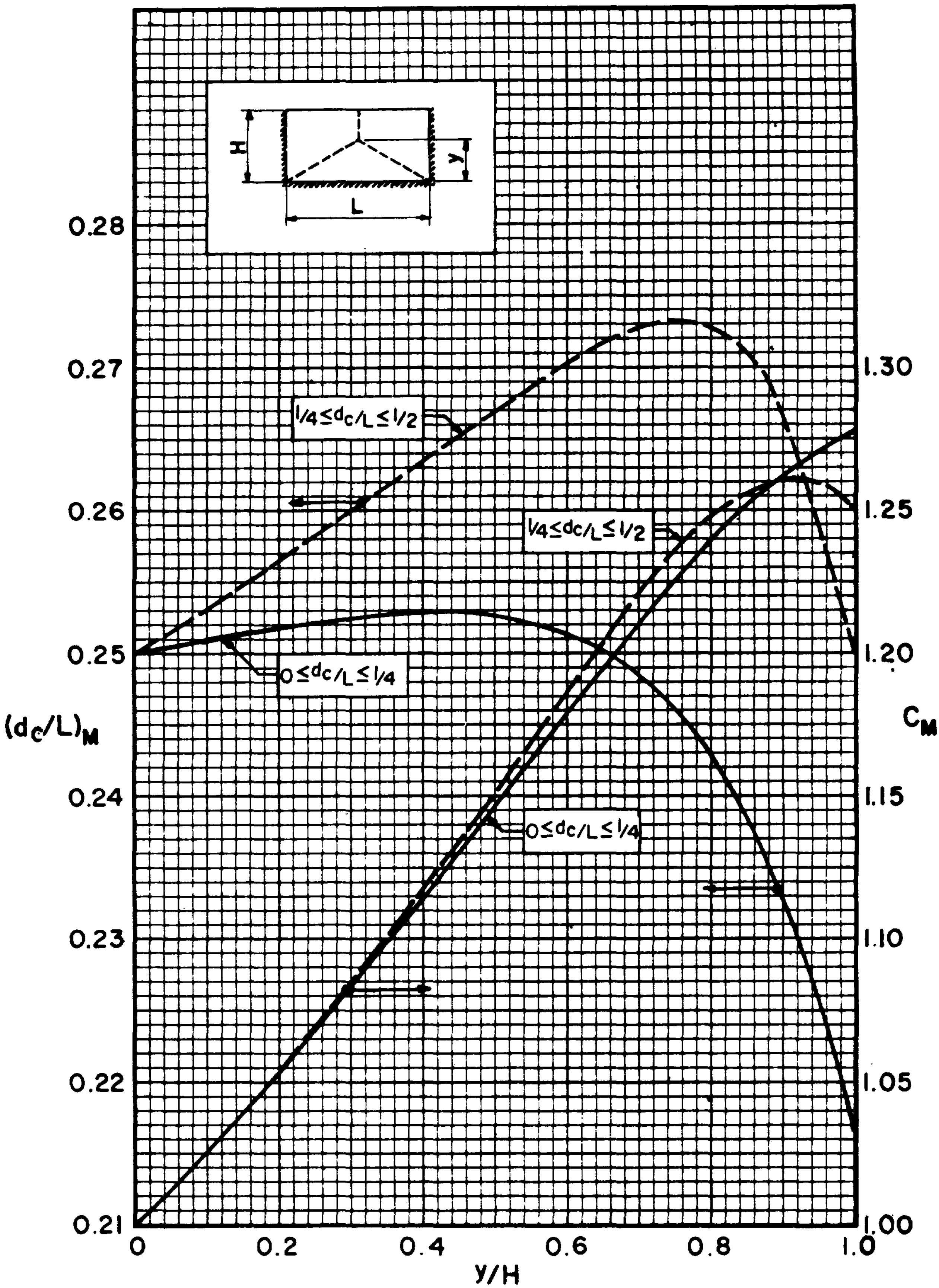


Figure 4-47 Horizontal shear parameters for ultimate shear stress at distance d_c from the support (cross section type II and III)

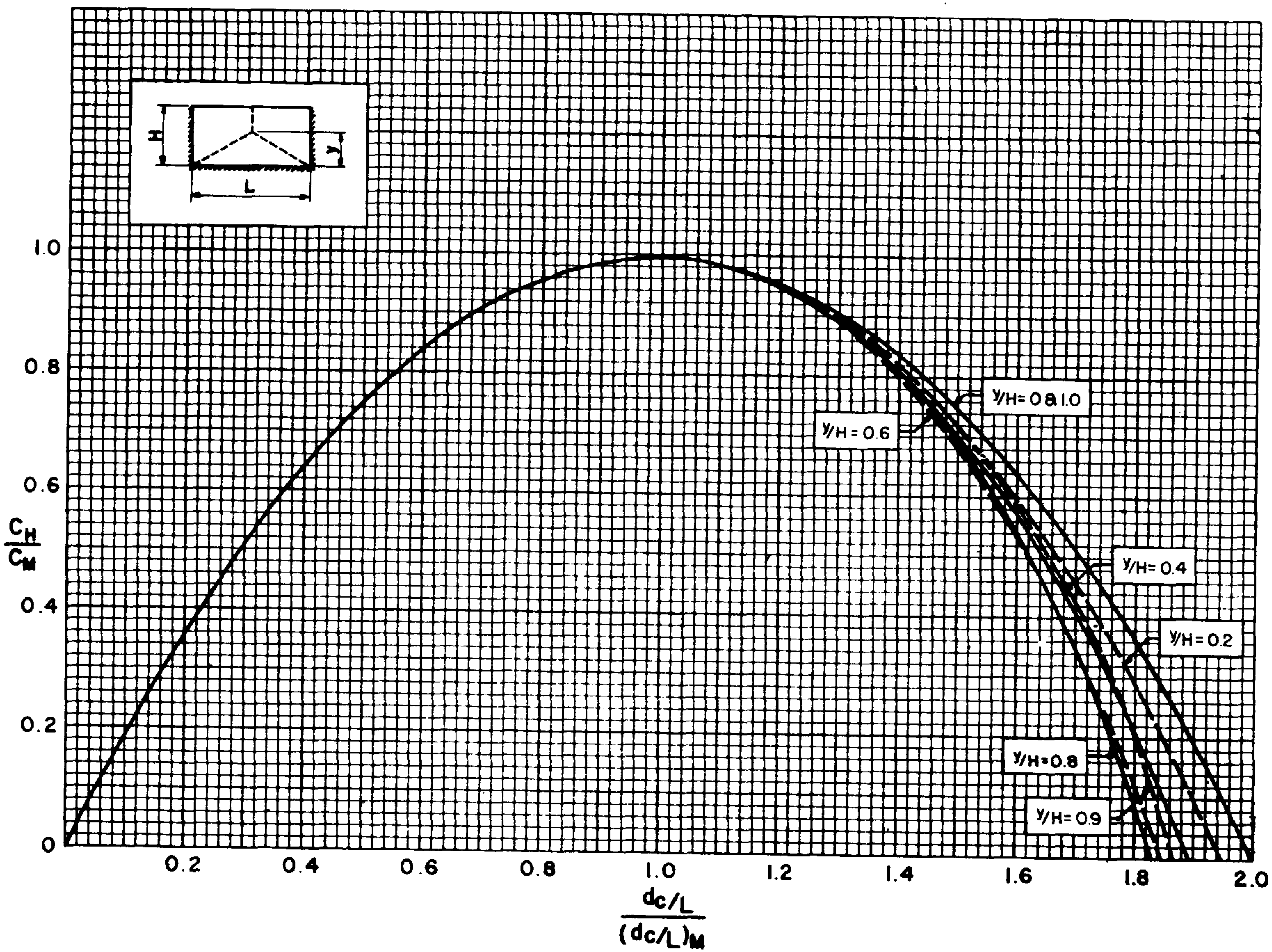


Figure 4-48 Horizontal shear coefficient ratios for ultimate shear stress at distance d_c from the support (cross section type II and III)

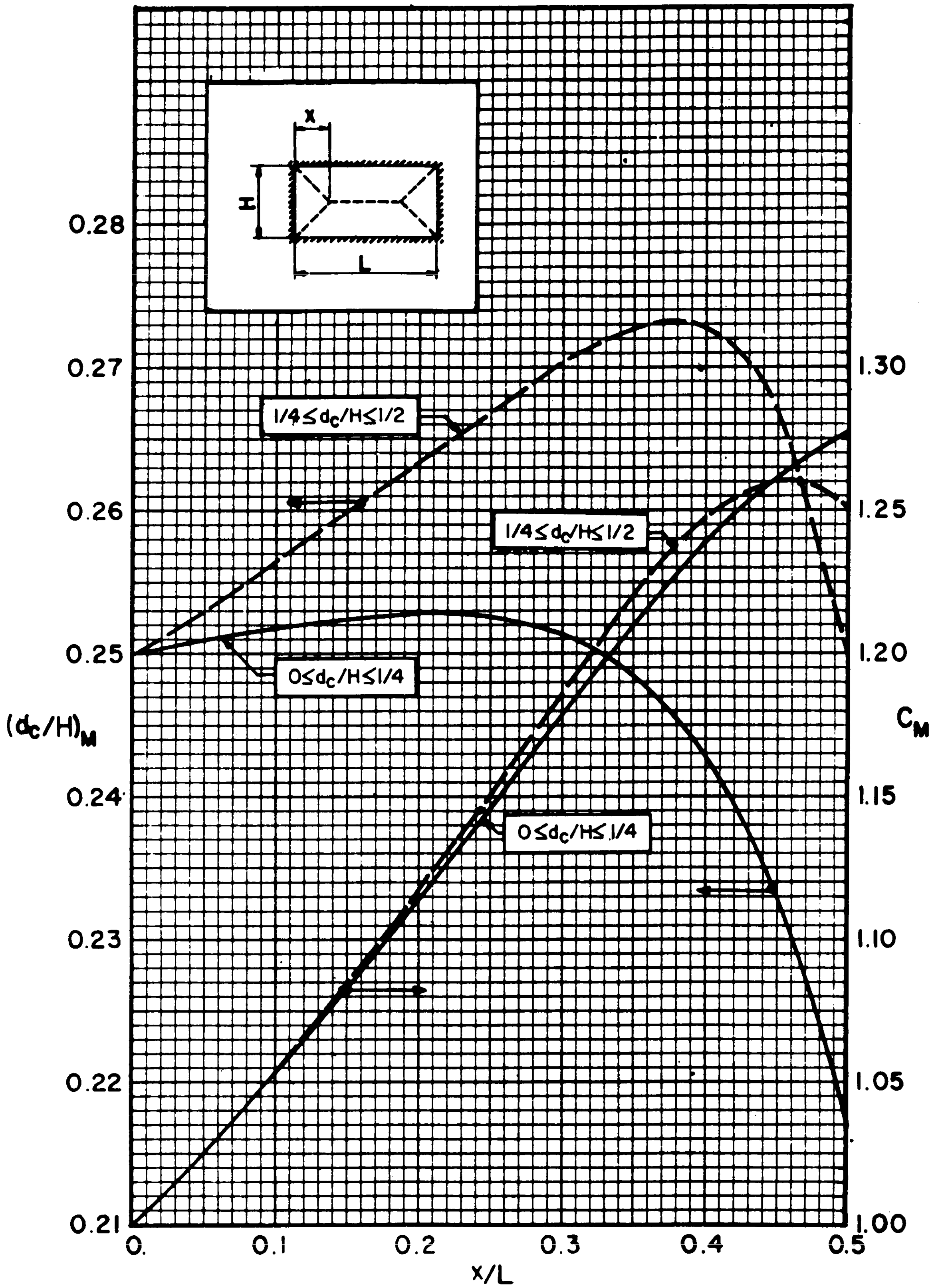


Figure 4-49 Vertical shear parameters for ultimate shear stress at distance d_c from the support (cross section type II and III)

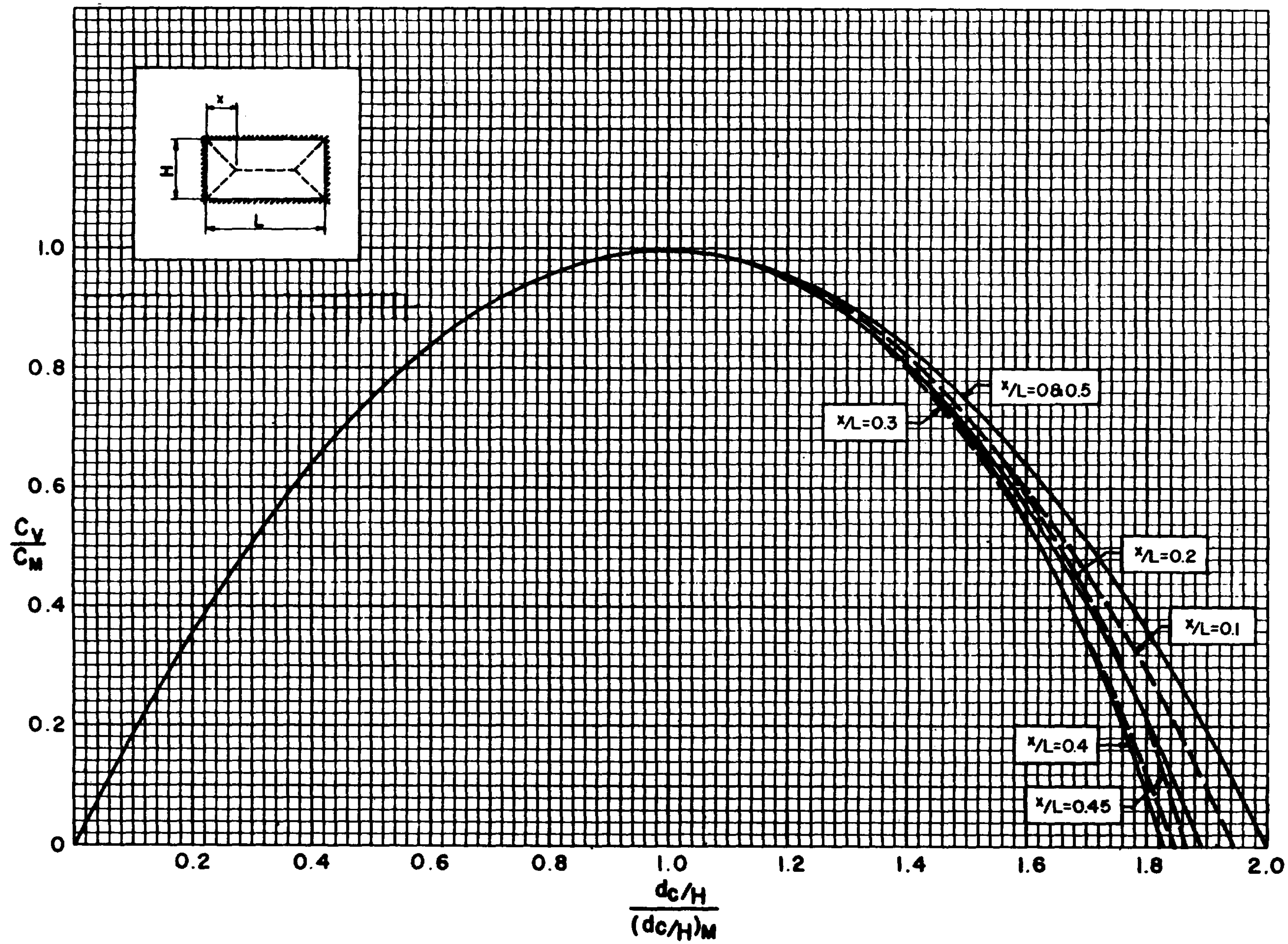


Figure 4-50 Vertical shear coefficient ratios for ultimate shear stress at distance d_c from the support (cross section type II and III)

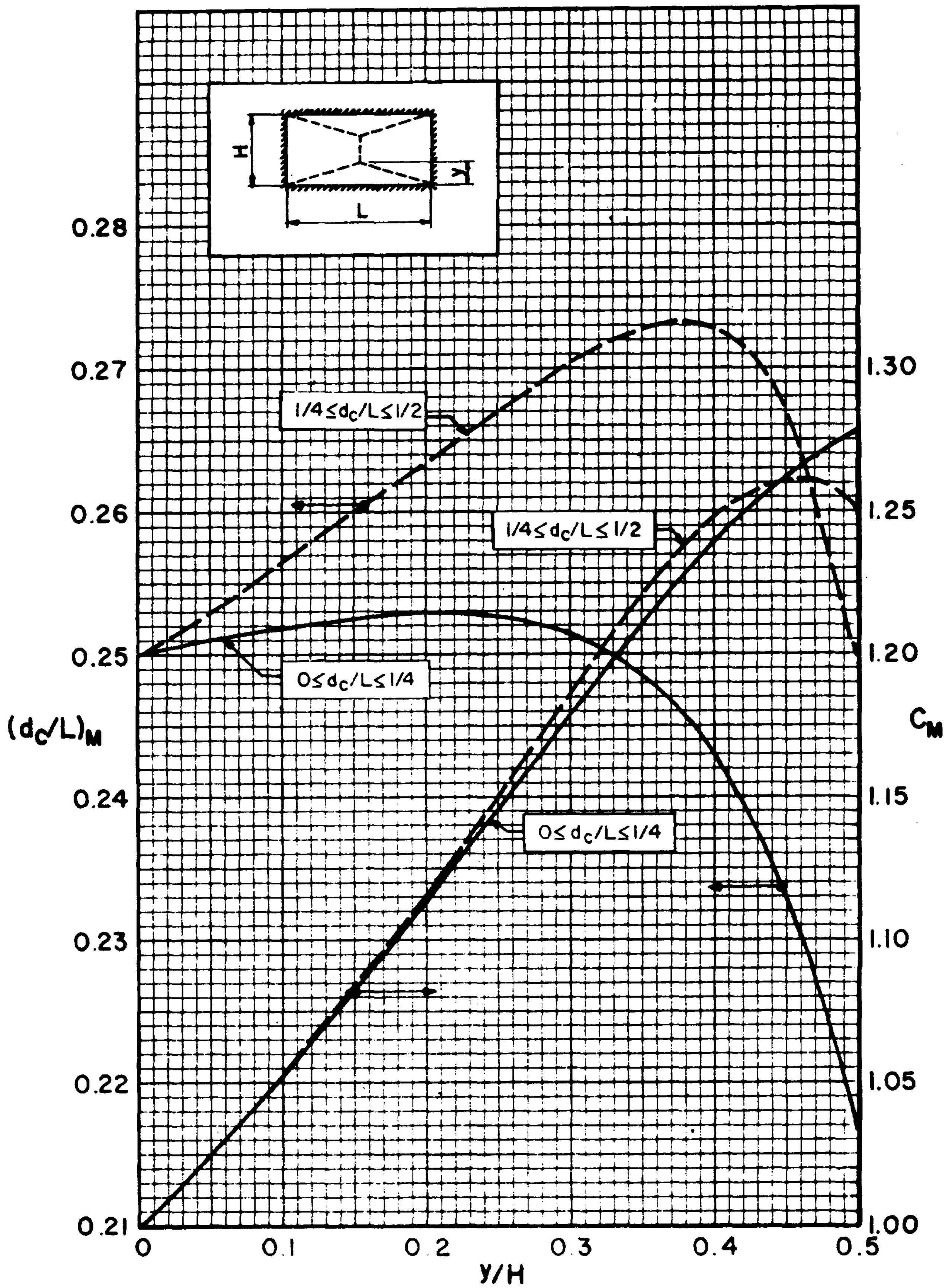


Figure 4-51 Horizontal shear parameters for ultimate shear stress at distance d_c from the support (cross section type II and III)

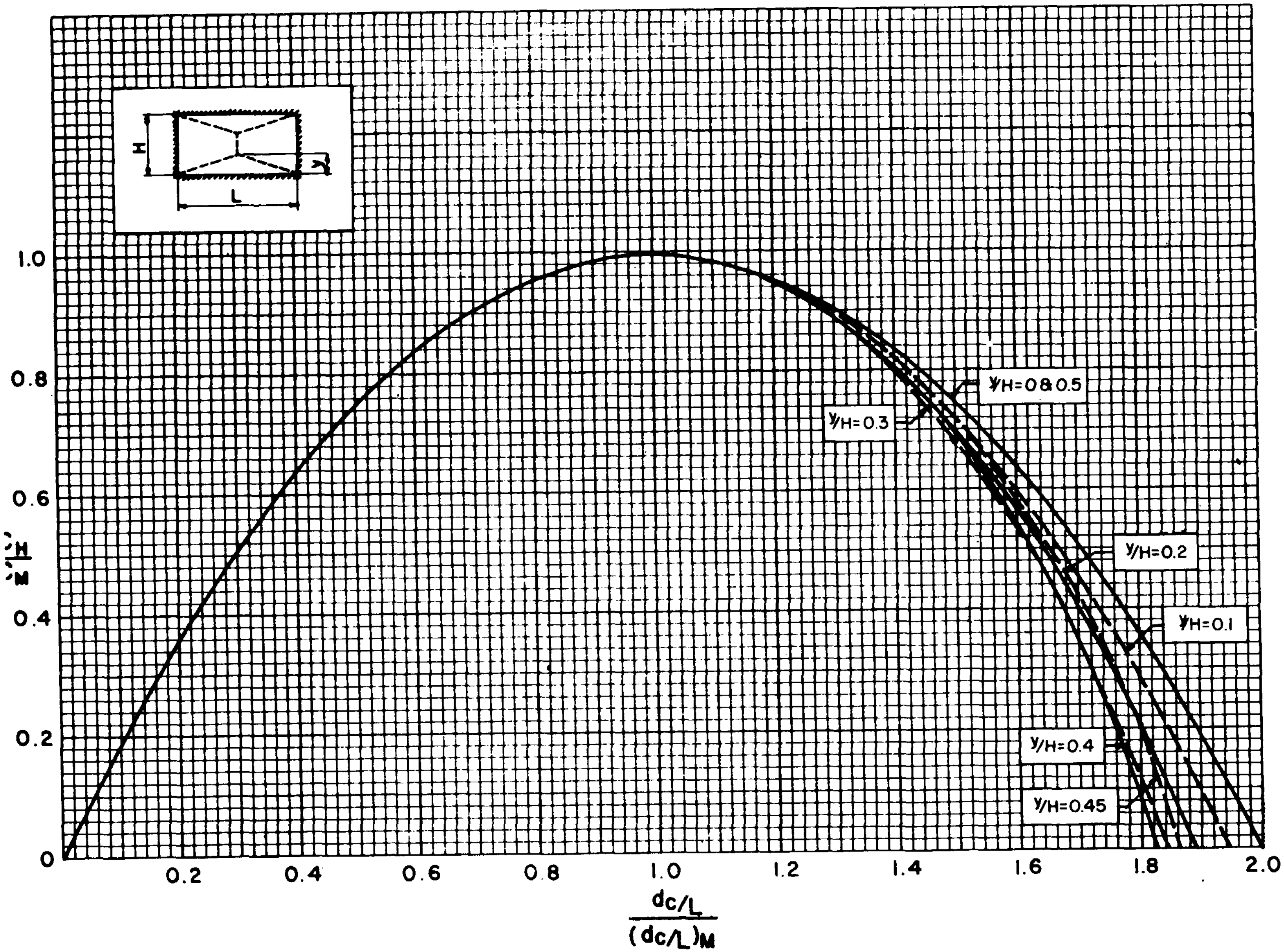


Figure 4-52 Horizontal shear coefficient ratios for ultimate shear stress at distance d_c from the support (cross section type II and III)

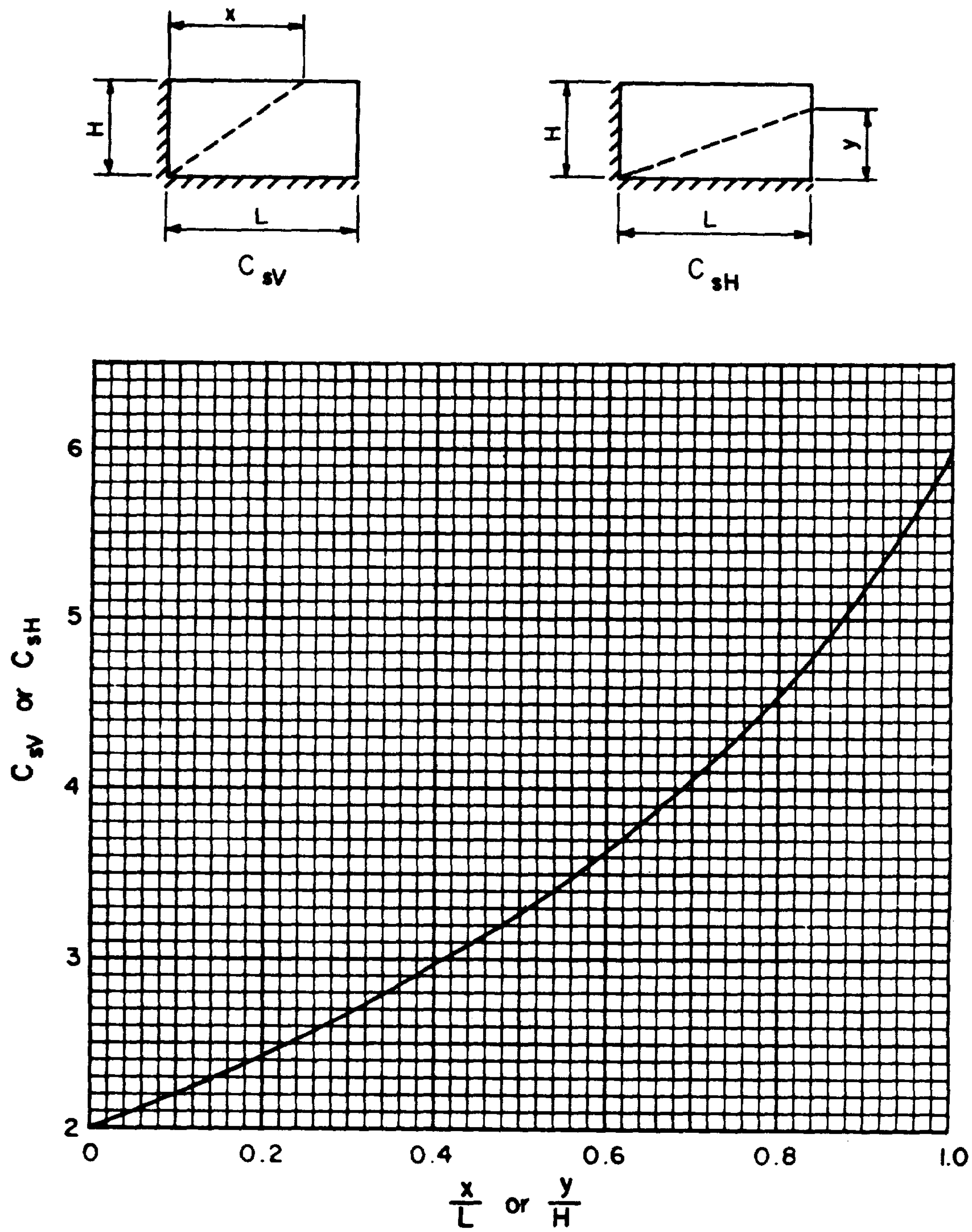


Figure 4-53 Shear coefficients for ultimate support shear (cross section type II and III)

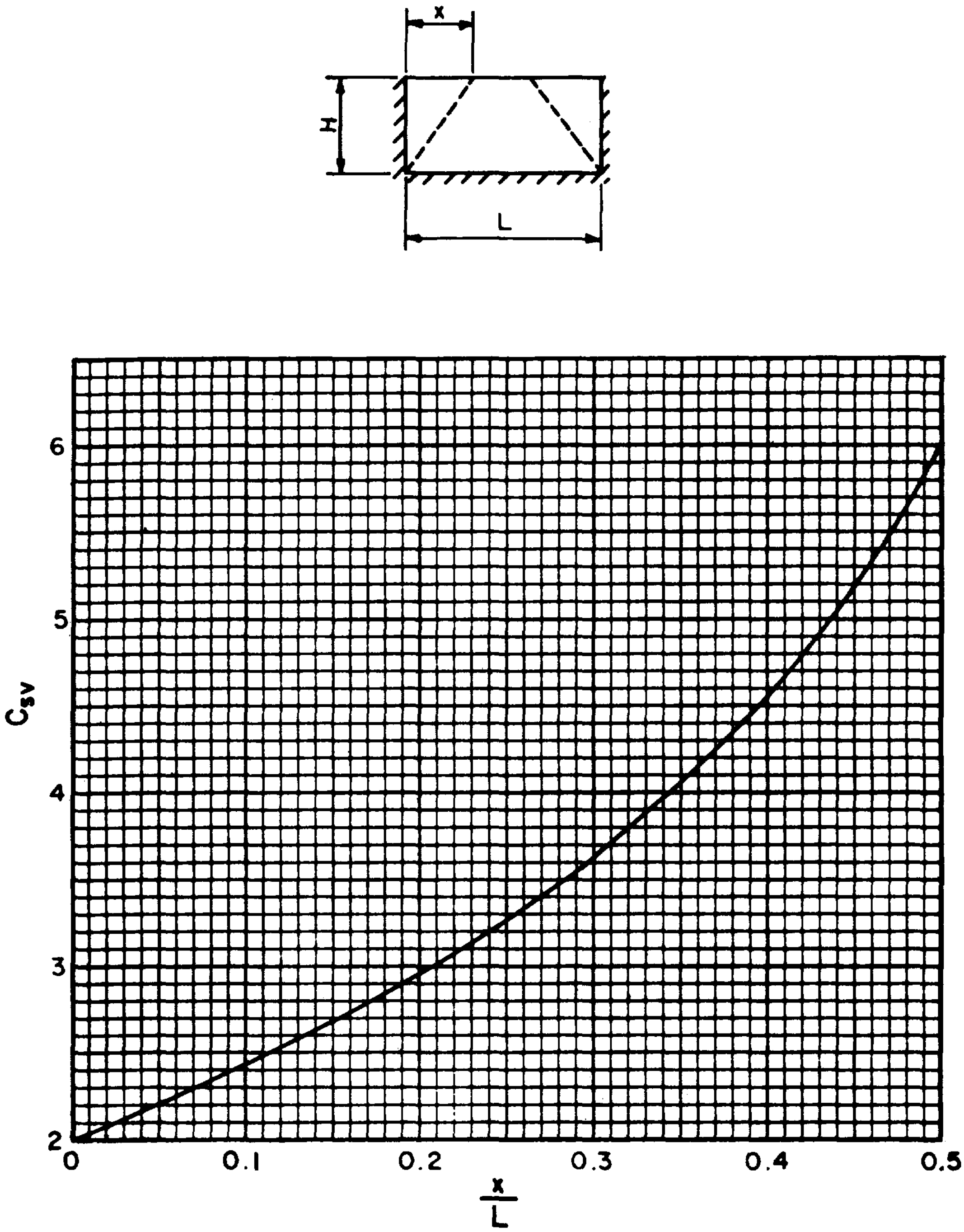
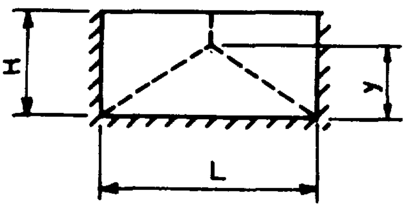


Figure 4-54   Shear coefficients for ultimate support shear
(cross section type II and III)



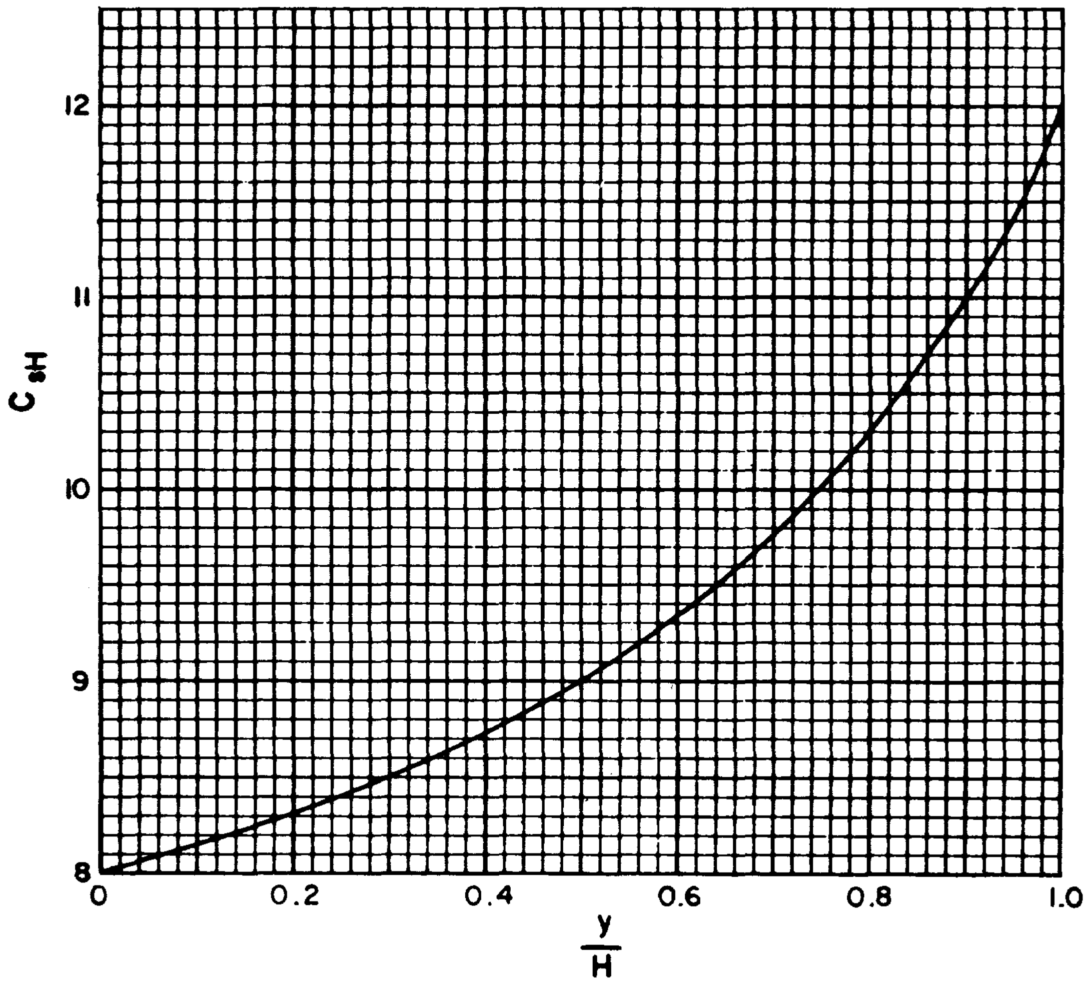


Figure 4-55 Shear coefficients for ultimate support shear (cross section type II and III)

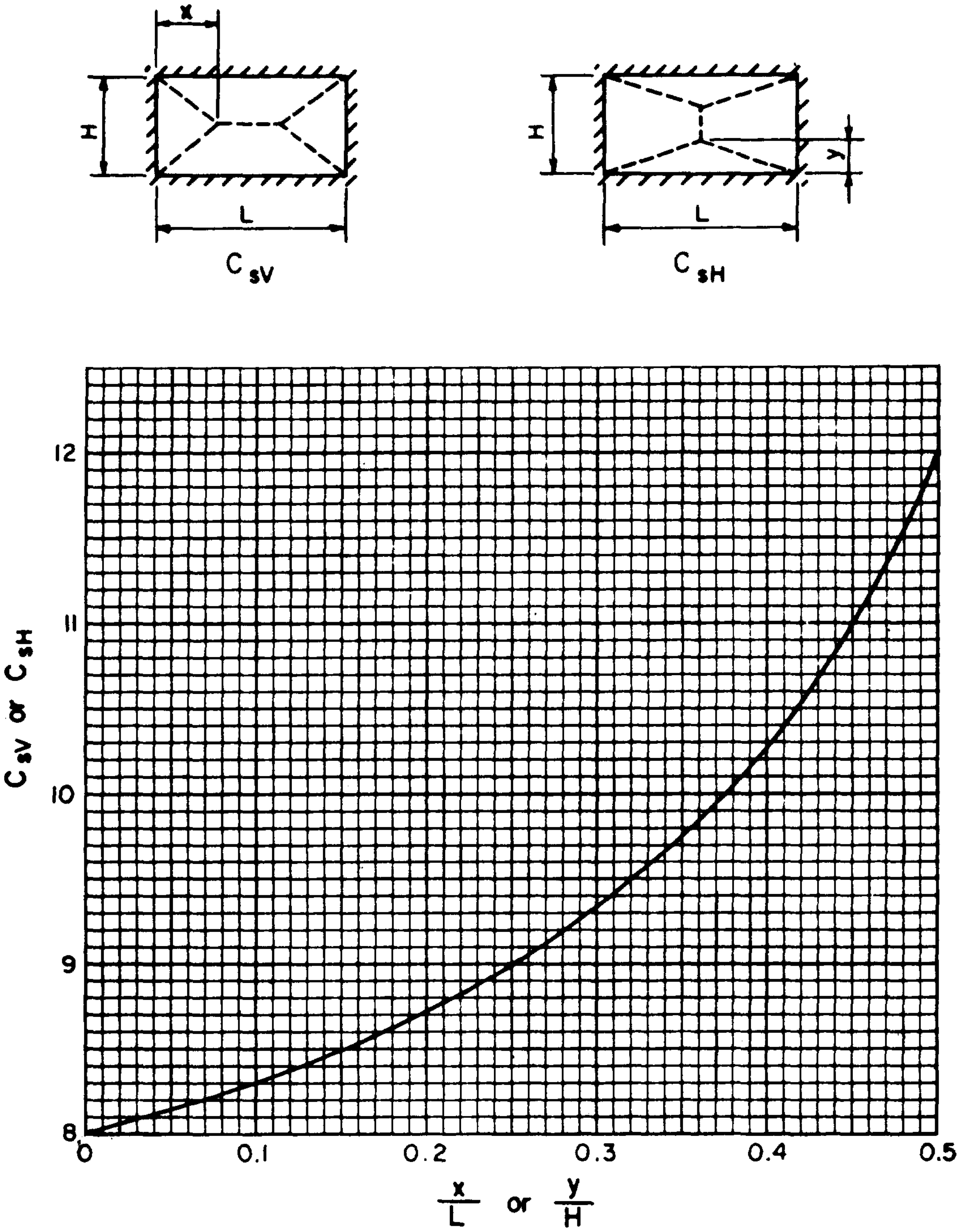


Figure 4-56 Shear coefficients for ultimate support shear (cross section type II and III)

Table 4-9 Impulse Coefficient C_1 for Two-Way Elements

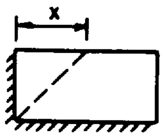
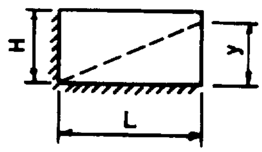
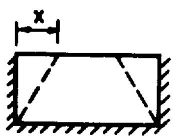
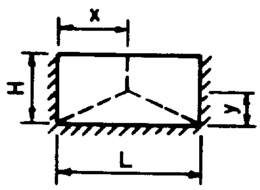
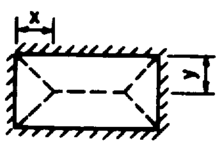
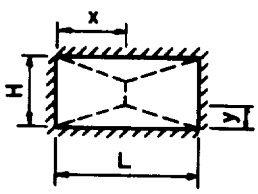
Edge Conditions	Yield Line Locations	Limits	Impulse Coefficient, C_I
Two adjacent edges supported and two edges free		$0 \leq x/H \leq 1$	$957 \frac{(K_{LM})_u}{(x/H)}$
		$H \leq x \leq L$	$957 \frac{(K_{LM})_u}{(x/H)^2}$
		$0 \leq y/L \leq 1$	$957 \frac{(K_{LM})_u (p_V/p_H)}{(y/H)}$
		$L \leq y \leq H$	$957 \frac{(K_{LM})_u (p_V/p_H)(L/H)}{(y/H)^2}$
Three edges supported and one edge free		$0 \leq x/H \leq 1$	$957 \frac{(K_{LM})_u}{(x/H)}$
		$H \leq x \leq L/2$	$957 \frac{(K_{LM})_u}{(x/H)^2}$
		$0 \leq y/L \leq \frac{1}{2}$	$957 \frac{(K_{LM})_u (p_V/p_H)}{(y/H)}$
		$L/2 \leq y \leq H$	$478 \frac{(K_{LM})_u (p_V/p_H)(L/H)}{(y/H)^2}$
Four edges supported		$0 \leq x/H \leq \frac{1}{2}$	$957 \frac{(K_{LM})_u}{(x/H)}$
		$H/2 \leq x \leq L/2$	$478 \frac{(K_{LM})_u}{(x/H)^2}$
		$0 \leq y/L \leq \frac{1}{2}$	$957 \frac{(K_{LM})_u (p_V/p_H)}{(y/H)^2}$
		$L/2 \leq y \leq H/2$	$478 \frac{(K_{LM})_u (p_V/p_H)(L/H)}{(y/H)^2}$

Table 4-10 Impulse Coefficient C_u for Two-Way Elements

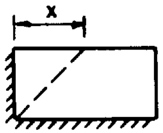
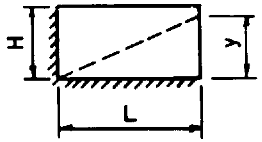
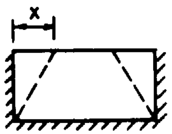
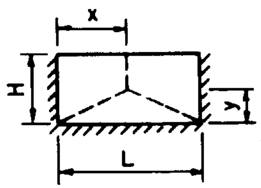
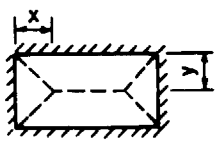
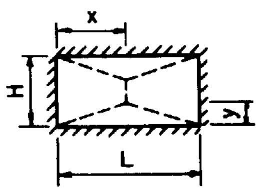
Edge Conditions	Yield Line Locations	Limits	Impulse Coefficient, C_u
Two adjacent edges supported and two edges free		$0 \le x/H \le 1$	$287\left[\frac{3.333(K_{LM})_u}{x/H}+(K_{LM})_u^2(p_V/p_H)(1-x/H)\right]$
		$H \le x \le L$	$287\left[\frac{3.333(K_{LM})_u}{(x/H)^2}+(K_{LM})_u^2\left(\frac{x/H-1}{(L/H)^2}+\frac{4.705(L/H-x/H)\tan\lambda}{(L/H)^2}\right)\right]$ Where, $\lambda=12^\circ-\tan^{-1}\left(\frac{0.2126}{x/H}\right)$
		$0 \le y/L \le 1$	$287\left[\frac{3.333(K_{LM})_u(p_V/p_H)}{y/H}+\frac{(K_{LM})_u^2(L/H-y/H)}{(L/H)^2}\right]$
		$L \le y \le H$	$287(p_V/p_H)\left[\frac{3.333(K_{LM})_u(L/H)}{(y/H)^2}+(K_{LM})_u^2(y/H-L/H+4.705(1-y/H)\tan\lambda)\right]$ Where, $\lambda=12^\circ-\tan^{-1}\left(\frac{0.2126}{y/L}\right)$
Three edges supported and one edge free		$0 \le x/H \le 1$	$287\left[\frac{3.333(K_{LM})_u}{x/H}+(K_{LM})_u^2(p_V/p_H)(1-x/H)\right]$
		$H \le x \le L/2$	$1148\left[\frac{0.833(K_{LM})_u}{(x/H)^2}+2(K_{LM})_u^2\left(\frac{x/H-1}{(L/H)^2}+\frac{4.705(L/2H-x/H)\tan\lambda}{(L/H)^2}\right)\right]$ Where, $\lambda=12^\circ-\tan^{-1}\left(\frac{0.2126}{x/H}\right)$
		$0 \le y/L \le \frac{1}{2}$	$717\left[\frac{1.333(K_{LM})_u(p_V/p_H)}{y/H}+3.2(K_{LM})_u^2\frac{(L/2H-y/H)}{(L/H)^2}\right]$
		$L/2 \le y \le H$	$287(p_V/p_H)\left[\frac{1.667(K_{LM})_u(L/H)}{(y/H)^2}+(K_{LM})_u^2(y/H-L/2H+4.705(1-y/H)\tan\lambda)\right]$ Where, $\lambda=12^\circ-\tan^{-1}\left(\frac{0.2126}{2y/L}\right)$
Four edges supported		$0 \le x/H \le \frac{1}{2}$	$1148\left[\frac{0.833(K_{LM})_u}{x/H}+(K_{LM})_u^2(p_V/p_H)\left(1-2\frac{x}{H}\right)\right]$
		$H/2 \le x \le L/2$	$1148\left[\frac{0.417(K_{LM})_u}{(x/H)^2}+2(K_{LM})_u^2\left(\frac{x/H-\frac{1}{2}}{(L/H)^2}+\frac{4.705(L/2H-x/H)\tan\lambda}{(L/H)^2}\right)\right]$ Where, $\lambda=12^\circ-\tan^{-1}\left(\frac{0.2126}{2x/H}\right)$
		$0 \le y/L \le \frac{1}{2}$	$1148\left[\frac{0.833(K_{LM})_u(p_V/p_H)}{y/H}+(K_{LM})_u^2\frac{(L/H-2y/H)}{(L/H)^2}\right]$
		$L/2 \le y \le H/2$	$1148(p_V/p_H)\left[\frac{0.417(K_{LM})_u(L/H)}{(y/H)^2}+(K_{LM})_u^2(2y/H-L/H+4.705(1-2y/H)\tan\lambda)\right]$ Where, $\lambda=12^\circ-\tan^{-1}\left(\frac{0.2126}{2y/L}\right)$

Table 4-11 Impulse Coefficient C_u for One-Way Elements

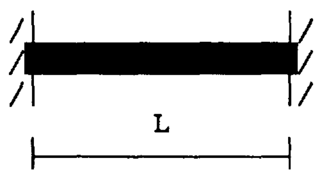
EDGE CONDITIONS		IMPULSE COEFFICIENTS C_u
CANTILEVER		127
FIXED SUPPORTS		510

Table 4-12 Shear Coefficients for Ultimate Shear Stress at Distance d_c from the Support for One-Way Elements (Cross Section Type II and III)

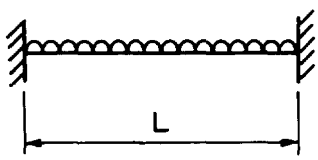
EDGE CONDITIONS		ULTIMATE SHEAR STRESS COEFFICIENTS C_d
CANTILEVER		$2\left(\frac{d_c}{L}\right)\left(1-\frac{d_c}{L}\right)$
FIXED SUPPORTS		$16\left(\frac{d_c}{L}\right)\left(\frac{1}{2}-\frac{d_c}{L}\right)$

Table 4-13 Shear Coefficients for Ultimate Shear Stress at Distance d_c from the Support for Two-Way Elements (Cross Section Type II and III)

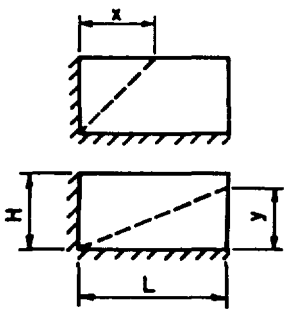
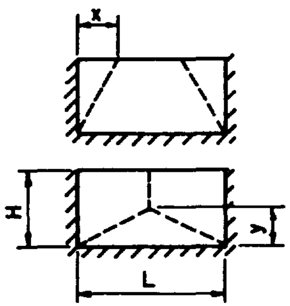
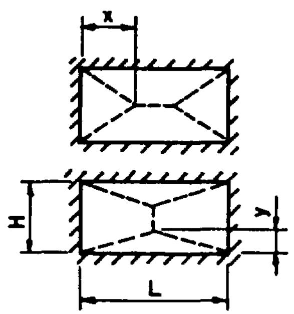
Edge conditions	Yield line location	Limits	Horizontal ultimate shear stress coefficient C_H	Limits	Vertical ultimate shear stress coefficient C_V
Two adjacent edges fixed and two edges free	 	$0 \le d_c/x \le \frac{1}{2}$	$\frac{30(d_c/x)(1-d_c/x)^2}{(5-4d_c/x)}$	$0 \le d_c/H \le \frac{1}{2}$	$\frac{6(d_c/H)(3+2x/L)(1-d_c/H)(2-x/L-d_cx/HL)}{(3-2x/L)(6-x/L-4d_cx/HL)}$
		$\frac{1}{2} \le d_c/x \le 1$	$5(d_c/x)(1-d_c/x)$	$\frac{1}{2} \le d_c/H \le 1$	$\frac{(d_c/H)(3+2x/L)(1-d_c/H)(2-x/L-d_cx/HL)}{(3-2x/L)(1-d_cx/HL)}$
		$0 \le d_c/L \le \frac{1}{2}$	$\frac{6(d_c/L)(3+2y/H)(1-d_c/L)(2-y/H-d_cy/LH)}{(3-2y/H)(6-y/H-d_cy/LH)}$	$0 \le d_c/y \le \frac{1}{2}$	$\frac{30(d_c/y)(1-d_c/y)^2}{(5-4d_c/y)}$
		$\frac{1}{2} \le d_c/L \le 1$	$\frac{(d_c/L)(3+2y/H)(1-d_c/L)(2-y/H-d_cy/LH)}{(3-2y/H)(1-d_cy/LH)}$	$\frac{1}{2} \le d_c/y \le 1$	$5(d_c/y)(1-d_c/y)$
Three edges fixed and one edge free	 	$0 \le d_c/x \le \frac{1}{2}$	$\frac{30(d_c/x)(1-d_c/x)^2}{(5-4d_c/x)}$	$0 \le d_c/H \le \frac{1}{2}$	$\frac{6(d_c/H)(3+4x/L)(1-d_c/H)(1-x/L-d_cx/HL)}{(3-4x/L)(3-x/L-4d_cx/HL)}$
		$\frac{1}{2} \le d_c/x \le 1$	$5(d_c/x)(1-d_c/x)$	$\frac{1}{2} \le d_c/H \le 1$	$\frac{2(d_c/H)(3+4x/L)(1-d_c/H)(1-x/L-d_cx/HL)}{(3-4x/L)(1-2d_cx/HL)}$
		$0 \le d_c/L \le \frac{1}{4}$	$\frac{12(d_c/L)(6-y/H)(1-2d_c/L)(2-y/H-2d_cy/LH)}{(3-2y/H)(6-y/H-8d_cy/LH)}$	$0 \le d_c/y \le \frac{1}{2}$	$\frac{30(d_c/y)(1-d_c/y)^2}{(5-4d_c/y)}$
		$\frac{1}{4} \le d_c/L \le \frac{1}{2}$	$\frac{2(d_c/L)(6-y/H)(1-2d_c/L)(2-y/H-2d_cy/LH)}{(3-2y/H)(1-2d_cy/LH)}$	$\frac{1}{2} \le d_c/y \le 1$	$5(d_c/y)(1-d_c/y)$
Four edges fixed	 	$0 \le d_c/x \le \frac{1}{2}$	$\frac{30(d_c/x)(1-d_c/x)^2}{(5-4d_c/x)}$	$0 \le d_c/H \le \frac{1}{4}$	$\frac{24(d_c/H)(3-x/L)(1-2d_c/H)(1-x/L-2d_cx/HL)}{(3-4x/L)(3-x/L-8d_cx/HL)}$
		$\frac{1}{2} \le d_c/x \le 1$	$5(d_c/x)(1-d_c/x)$	$\frac{1}{4} \le d_c/H \le \frac{1}{2}$	$\frac{8(d_c/H)(3-x/L)(1-2d_c/H)(1-x/L-2d_cx/HL)}{(3-4x/L)(1-4d_cx/HL)}$
		$0 \le d_c/L \le \frac{1}{4}$	$\frac{24(d_c/L)(3-y/H)(1-2d_c/L)(1-y/H-2d_cy/LH)}{(3-4y/H)(3-y/H-8d_cy/LH)}$	$0 \le d_c/y \le \frac{1}{2}$	$\frac{30(d_c/y)(1-d_c/y)^2}{(5-4d_c/y)}$
		$\frac{1}{4} \le d_c/L \le \frac{1}{2}$	$\frac{8(d_c/L)(3-y/H)(1-2d_c/L)(1-y/H-2d_cy/LH)}{(3-4y/H)(1-4d_cy/LH)}$	$\frac{1}{2} \le d_c/y \le 1$	$5(d_c/y)(1-d_c/y)$

Table 4-14 Shear Coefficients for Ultimate Support Shear for One-Way Elements (Cross Section Type II and III)

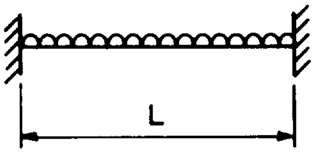
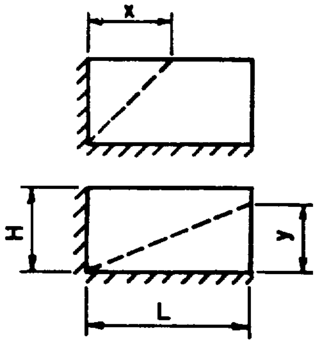
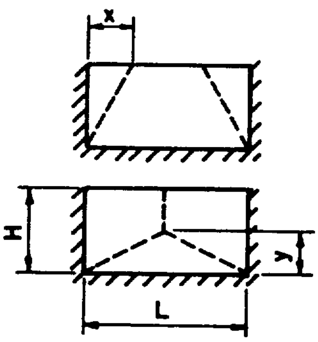
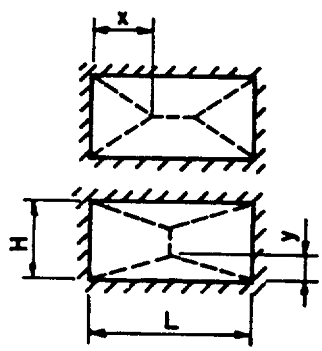
EDGE CONDITIONS		ULTIMATE SUPPORT SHEAR COEFFICIENTS C_s
CANTILEVER		2
FIXED SUPPORTS		8

Table 4-15 Shear Coefficients for Ultimate Support Shear for Two-Way Elements (Cross Section Type II and III)

Edge conditions	Yield line location	Horizontal ultimate support shear coefficient C_{sH}	Vertical ultimate support shear coefficient C_{sV}
Two adjacent edges fixed and two edges free		$\frac{6}{x/L}$ $\frac{6(2-y/H)(3+2y/H)}{(6-y/H)(3-2y/H)}$	$\frac{6(2-x/L)(3+2x/L)}{(6-x/L)(3-2x/L)}$ $\frac{6}{y/H}$
Three edges fixed and one edge free		$\frac{6}{x/L}$ $\frac{12(2-y/H)}{(3-2y/H)}$	$\frac{6(1-x/L)(3+4x/L)}{(3-x/L)(3-4x/L)}$ $\frac{6}{y/H}$
Four edges fixed		$\frac{6}{x/L}$ $\frac{24(1-y/H)}{(3-4y/H)}$	$\frac{24(1-x/L)}{(3-4x/L)}$ $\frac{6}{y/H}$

COMPOSITE CONSTRUCTION

4-36. Composite Construction

4-36.1. General

Composite elements are composed of two concrete panels (donor and acceptor) separated by a sand-filled cavity. They have characteristics which are useful in the blast resistant design of structures located close-in to a detonation. For a large quantity of explosives, replacing a single concrete panel with a composite element can result in a considerable cost savings. It is not usually cost effective to use a composite element for smaller quantities of explosives where a single concrete panel would be three feet thick or less. Where a single concrete panel would be between three and five feet thick, a detailed cost analysis is required to determine whether or not a composite element would be more cost effective.

Composite walls are generally used as barricades to prevent propagation of explosion between large quantities of explosives. These structures are usually designed for incipient failure. Composite elements may be designed to provide higher degrees of protection, but the massive walls (greater than 5 feet thick) that make composite elements cost effective are generally not required in such cases. If the maximum support rotation is limited to 4 degrees or less, and a composite element is shown to be cost effective, single leg stirrups may be used instead of lacing reinforcement. Walls using single leg stirrups are somewhat more economical than laced walls.

Composite elements can also be useful for reducing the hazard due to direct spalling. Spalled fragments from the donor panel are trapped in the sand fill and, therefore, are of no concern. Spalling of the acceptor panel can be eliminated by maintaining the required minimum thickness and maximum density of the sand fill given in Section 4-56.2.

The mechanisms by which composite elements resist the blast pressures are (1) the strength and ductility of the concrete panels and (2) the blast attenuating ability of the sand fill. The attenuation of the blast by the sand is accomplished by (1) the increased mass it affords to the concrete portions of the wall, (2) the increased distance the blast wave must travel due to the increased wall thickness produced by the sand (dispersion of blast wave) and (3) the blast energy absorbed by the displacement and compression of the sand particles.

4-36.2. Blast Attenuation Ability of Sand Fill

The method for calculating the impulse capacity of composite elements is similar to that for single laced concrete elements except that the blast attenuating ability of the sand must be included in the calculation. The blast wave attenuation is partly due to the increased mass of the slab. When computing the impulse capacity of each concrete panel, the total effective mass includes both the mass of the concrete and the mass of one-half of the sand. This increased mass is taken into account by multiplying the impulse coefficients for spalled sections, by

$$\left[\frac{(T_c + d_c)}{2} + \left(\frac{w_s}{w_c} \right) \left(\frac{T_s}{2} \right) \right] / d_c \qquad 4\text{-}123$$

or for unspalled sections by

$$\left[T_c + \frac{w_s}{w_c} \left(\frac{T_s}{2} \right) \right] / T_c \qquad 4\text{-}124$$

where

w_s = weight density of sand

w_c = weight density of concrete

T_s = thickness of sand fill

The attenuating ability of the sand due to blast wave dispersion and energy absorption is a function of the thickness and density of the sand, the impulse capacity of the concrete panels and the quantity of explosive. Figures 4-57 and 4-58 have been developed to predict the impulse capacity of the concrete element for a sand density equal to 85 pcf and 100 pcf, respectively. These figures are based on identical donor and acceptor panels. The effect of the quantity of explosive is taken into account through the use of "scaled" parameters which are defined as follows:

$$\bar{T}_c = \frac{T_c}{12W^{1/3}} \qquad 4\text{-}125$$

$$\bar{T}_s = \frac{T_s}{12W^{1/3}} \qquad 4\text{-}126$$

$$\bar{i}_{ba} = \frac{i_{ba}}{W^{1/3}} \qquad 4\text{-}127$$

$$\bar{i}_a = \frac{i_a}{W^{1/3}} \qquad 4\text{-}128$$

where

$\bar{T}_c$ = scaled thickness of concrete panel

W = weight of explosive charge

$\bar{T}_s$ = scaled thickness of sand

$\bar{i}_{ba}$ = scaled blast impulse which can be resisted by acceptor panel

i_{ba} = blast impulse capacity of acceptor panel

$\bar{i}_a$ - sum of scaled blast impulse resisted by the acceptor panel and the scaled blast impulse absorbed by the sand

i_a - sum of blast impulse capacity of the acceptor panel and the blast impulse absorbed by the sand

Explosion response slab tests have indicated that the density of the sand fill affects the amount of blast energy absorbed by the sand displacement, i.e., the higher the initial sand density, the smaller amount of blast energy absorbed. Also, it was observed in the above response tests that for a unit weight of sand equal to 100 pcf, the deflection of the donor panel is approximately equal in magnitude to the deflection of the acceptor panel. On the other hand, with a unit weight of sand fill equal to 85 pcf, it was observed that the deflection of the donor panel usually was significantly larger than that of the acceptor panel. This latter phenomenon was caused by the fact that, with the lower density, the sand had more voids and, therefore, more room for movement of the sand particles. This sand movement in turn permitted larger displacements of the donor panel before the near solid state of the sand occurred.

Based on the above information, it can be seen that if near equal displacement of the donor and receiver panels are desired, then a unit weight of sand fill equal to 100 pcf should be used. A variation of the displacement of donor and receiver panels can be achieved using a unit weight of sand equal to 85 pcf, but the actual variation cannot be predicted.

Since the impulse capacity of composite elements is a function of the density of the sand, it is important to prevent the sand from compacting due to its own weight and/or water drainage. Several possible methods for maintaining the proper sand density are discussed in subsequent sections concerned with construction details of composite elements.

4-36.3. Procedure for Design of Composite Elements

The design of composite elements is a trial and error procedure. By using Figures 4-56 and 4-57 and the impulse coefficients of previous sections, the calculations are greatly simplified. The donor and acceptor slabs are identical making it necessary to design only one wall. The depth of the sand fill is usually equal to the total thickness of the two concrete panels. Using the procedures in the previous sections, each panel is designed to have a blast impulse capacity slightly less than half the required. This includes the increase in capacity due to the additional mass of the sand (Equations 4-123 and 4-124). It should be noted that the design is based on the assumption that both panels will attain the same deflection. If the density of the sand fill is 85 pcf, this will not be true. The donor panel will probably have a larger deflection than the acceptor panel. Since the actual deflection of each panel cannot be predicted, it must be assumed that the design deflection is an average of the two.

With the blast impulse capacity of the two concrete panels, Figure 4-56 or 4-57 is used to determine the total blast capacity of the composite element. The following procedure illustrates the use of these figures.

1. Using the given charge weight calculate the scaled thickness of the concrete panel and the sand, $\bar{T}_c$ and $\bar{T}_s$, respectively.

2. Calculate the scaled impulse capacity which can be resisted by the donor panel $\bar{i}_{bd}$ and the acceptor panel $\bar{i}_{ba}$.

3. Using either Figure 4-56 or 4-57

   a) Enter the ordinate at value of $\bar{i}_{ba}$.

   b) Proceed horizontally from $\bar{i}_{ba}$ to $\bar{T}_s$.

   c) Proceed vertically from $\bar{T}_s$ to $\bar{T}_c$.

   d) Proceed horizontally from $\bar{T}_c$ to $\bar{i}_a$ (the sum of the scaled unit blast impulse resisted by the acceptor panel and the scaled unit impulse absorbed by the sand).

4. Calculate the summations of $\bar{i}_{bd}$ and $\bar{i}_a$ to get the total unit impulse which can be resisted by the composite wall $\bar{i}_{bt}$.

5. Determine the scaled unit blast impulse $\bar{i}_b$ acting on the composite element.

6. Compare $\bar{i}_{bt}$ and $\bar{i}_b$. If the blast impulse which can be resisted by the composite element is not greater than or equal to the impulse produced by the blast then the impulse capacity of the walls should be increased and/or the thickness of the sand increased.

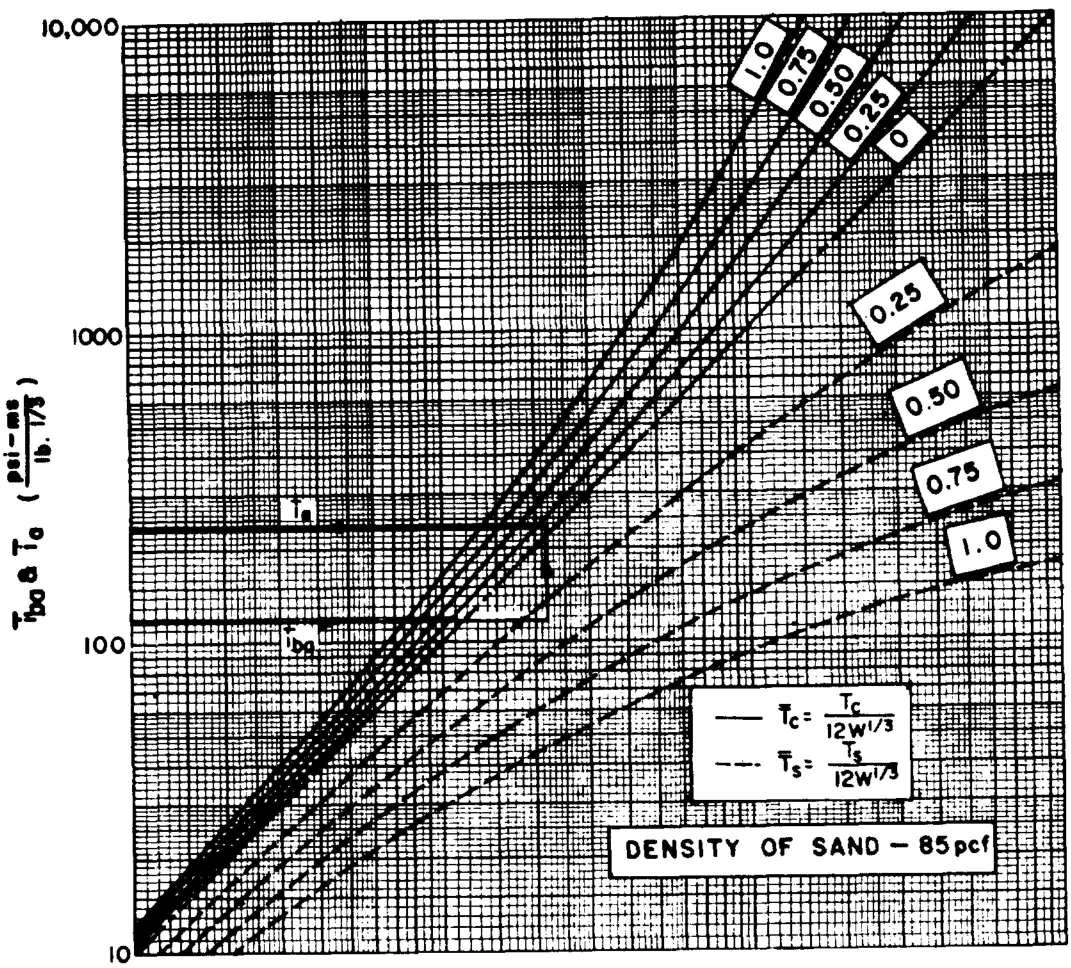


Figure 4-57 Attenuation of blast impulse in sand and concrete, w_s= 85 pcf

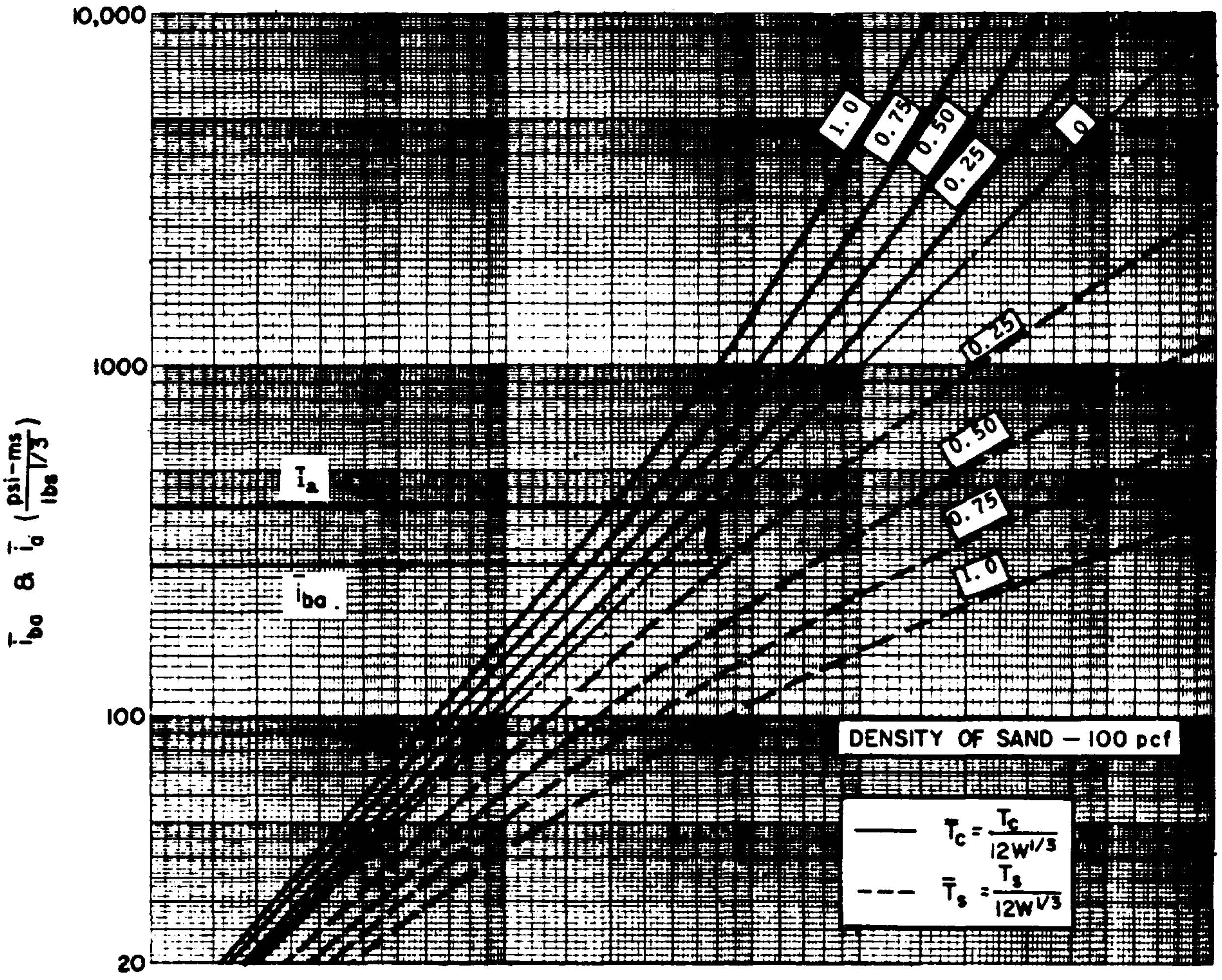


Figure 4-58 Attenuation of blast impulse in sand and concrete, w_s= 100 pcf

ULTIMATE DYNAMIC STRENGTH OF REINFORCED CONCRETE BEAMS

4-37 Introduction

Blast resistant concrete buildings subjected to external blast pressures are generally shear wall structures rather than rigid frame structures. Shear wall structures respond to lateral loads in a somewhat different manner than rigid frame structures; the basic difference being the manner in which the lateral loads are transferred to the foundation. In rigid frame structures the lateral loads are transmitted to the foundation through bending of the columns. Whereas, in shear wall structures, the lateral forces are transmitted to the foundation through both bending and shearing action of the shear walls. Shear walls are inherently strong and will resist large lateral forces. Consequently, shear wall structures are inherently capable of resisting blast loads and can be designed to resist substantially large blast loads whereas rigid frame structures cannot be economically designed to resist significant blast loads.

In shear wall structures, beams and columns are usually provided between shear walls to carry the vertical loads including blast loads on the roof and not to transmit lateral loads to the foundation. For example, blast loads applied to the front wall of a two-story shear wall structure are transmitted through the roof and intermediate floor slabs to the shear walls (perpendicular walls) and thus to the foundation. The front wall spans vertically between the foundation, the floor, and the roof slab. The upper floor and roof slabs act as deep beams, and in turn, transmit the front wall reactions to the shear walls. The roof and floor beams are not subjected to significant axial loads due to the diaphragm action of the slabs.

The design of beams as presented in the following sections applies to beams in shear wall type structures rather than rigid frame structures. The design procedure presented is for transverse loads only; axial loads are not considered. However, the procedure includes the design for torsion. The design of beams is similar to the design of slabs as described in sections 4-13 through 4-18. The most significant and yet not very important difference in the design procedure is that in the case of a slab the calculations are based on a unit area, whereas, for a beam, they are based on a unit length of beam.

Beams may be designed to attain limited or large deflections in the same manner as non-laced slabs. However, unlike non-laced slabs which in some cases do not require shear reinforcement (single leg stirrups), shear reinforcement in the form of closed ties must always be provided in beams. Under flexural action, a beam may attain deflections corresponding to 2 degrees support rotation with a type I cross-section to provide the ultimate moment capacity. The flexural action may be extended to 4 degrees support rotation if equal tension and compression reinforcement is furnished. A type II or III cross-section provides the ultimate moment capacity and the required closed ties restrain the compression reinforcement. If sufficient lateral restraint is provided, the beam may attain 8 degrees support rotation under tension membrane action. The above support rotations are incipient failure conditions for the structural configurations described.

Beams are primary support members and, as such, are generally not permitted to attain large plastic deformations. For personnel protection, the maximum deflection is limited to a support rotation of 2.0 degrees. Structures intended to protect equipment and/or explosives may be designed for deflections up to incipient failure conditions.

Beams are generally employed in structures designed to resist the effects associated with far range explosions. In these structures, beams are usually used in the roof as primary support members and as secondary support members such as pilasters around door openings. To a far lesser extent, beams are designed to resist the effects of close-in detonations in containment type structures. In these cases, they are generally used as secondary support members such as pilasters around door openings. Large tensile forces are induced in containment type structures and, therefore, these structures lend themselves to tension membrane action when the applicable design criteria permits large deformations.

The interrelationship between the various parameters involved in the design of beams is readily described with the use of the idealized resistance-deflection curve shown in Figure 4-59.

4-38. Ultimate Moment Capacity

4-38.1. Tension Reinforcement Only

The ultimate dynamic resisting moment Mu of a rectangular beam section of width b with tension reinforcement only (type I) is given by:

$$M_u = A_s f_{ds} (d - a/2) \qquad \text{4-129}$$

and:

$$a = \frac{A_s f_{ds}}{0.85 b f'_{dc}} \qquad \text{4-130}$$

where:

M_u = ultimate moment capacity

A_s = total area of tension reinforcement within the beam

f_{ds} = dynamic design stress of reinforcement

d = distance from extreme compression fiber to centroid of tension reinforcement

a = depth of equivalent rectangular stress block

b = width of beam

f'_{dc} = dynamic ultimate compressive strength of concrete

The reinforcement ratio p is defined as:

$$p = \frac{A_s}{bd} \qquad \text{4-131}$$

and to insure against sudden compression failures, the reinforcement ratio p must not exceed 0.75 of the ratio p_b which produces balanced conditions at ultimate strength and is given by:

$$p_b = \left[\frac{0.85K_1 f'_{dc}}{f_{ds}}\right]\left[\frac{87{,}000}{87{,}000 + f_{ds}}\right] \qquad 4\text{-}132$$

where:

K_1 = 0.85 for f'_{dc} up to 4,000 psi and is reduced by 0.05 for each 1,000 psi in excess of 4,000 psi

4-38.2. Tension and Compression Reinforcement

The ultimate dynamic resisting moment M_u of a rectangular beam section of width b with compression reinforcement is given by:

$$M_u = (A_s - A'_s)\, f_{ds}\, (d - a/2) + A'_s\, f_{ds}\, (d - d') \qquad 4\text{-}133$$

and:

$$a = \frac{(A_s - A'_s)\, f_{ds}}{0.85\, b\, f'_{dc}} \qquad 4\text{-}134$$

where:

A'_s = total area of compression reinforcement within the beam

d' = distance from extreme compression fiber to centroid of compression reinforcement

The compression reinforcement ratio p' is defined as:

$$p' = \frac{A'_s}{bd} \qquad 4\text{-}135$$

Equation 4-133 is valid only when the compression reinforcement yields at ultimate strength. This condition is satisfied when:

$$p - p' \le 0.85\, K_1 \left[\frac{f'_{dc}\, d'}{f_{ds}\, d}\right]\left[\frac{87{,}000}{87{,}000 - f_{ds}}\right] \qquad 4\text{-}136$$

In addition, the quantity p-p' must not exceed 0.75 of the value of p_b given in Equation 4-132 in order to insure against sudden compression failures. If p-p' is less than the value given by Equation 4-136, the ultimate resisting moment should not exceed the value given by Equation 4-129.

For the design of concrete beams subjected to far range blast loads which are to attain support rotations of 2 degrees or less, it is recommended that the ultimate resisting moment be computed using Equation 4-129 even though a considerable amount of compression reinforcement is required to resist rebound loads. It should be noted that a large amount of compression steel that does not yield due to the linear strain variation across the depth of the section, has a negligible effect on the total capacity.

For type II or III cross-sections, the ultimate resisting moment M_u of a rectangular beam section of width b is given by:

$$M_u = A_s f_{ds} d_c \qquad 4\text{-}137$$

where

A_s = area of tension or compression reinforcement within the width b

d_c = distance between the centroids of the compression and the tension reinforcement

The above moment capacity can only be obtained when the areas of the tension and compression reinforcement are equal. In addition, the support rotation must be greater than 2 degrees except for close-in designs where direct spalling may occur and result in a type III.

4-38.3. Minimum Flexural Reinforcement

To insure proper structural behavior under both conventional and blast loadings, a minimum amount of flexural reinforcement is required. The minimum reinforcement required for beams is somewhat greater than that required for slabs since an overload load in a slab would be distributed laterally and a sudden failure will be less likely. The minimum required quantity of reinforcement is given by:

$$p = 200/f_y \qquad 4\text{-}138$$

which, for 60,000 psi yield strength steel, is equal to a reinforcement ratio of 0.0033. This minimum reinforcement ratio applies to the tension steel at mid-span of simply supported beams and to the tension steel at the supports and mid-span of fixed-end beams.

Concrete beams with tension reinforcement only are not permitted. Compression reinforcement, at least equal to one-half the required tension reinforcement, must be provided. This reinforcement is required to resist the ever present rebound forces. Depending upon the magnitude of these rebound forces, the required compression reinforcement may equal the tension reinforcement.

4-39. Ultimate Shear (Diagonal Tension) Capacity

4-39.1.. Ultimate Shear Stress

The nominal shear stress vu, as a measure of diagonal tension, is computed from:

$$v_u = \frac{V_u}{bd} \qquad 4\text{-}139$$

where:

v_u = nominal shear stress

V_u = total shear at critical section

The critical section is taken at a distance d from the face of the support for those members that cause compression in their supports. The shear at sections between the face of the support and the section d therefrom need not be considered critical. For those members that cause tension in their supports, the critical section is at the face of the supports.

4-39.2. **Shear Capacity of Unreinforced Concrete**

The shear stress permitted on an unreinforced web of a beam subjected to flexure only is limited to:

$$v_c = \left[1.9\ f'_{dc}{}^{1/2} + 2{,}500\ p\right] \leq 3.5\ f'_{dc}{}^{1/2} \qquad 4\text{-}140$$

where:

v_c = maximum shear capacity of an unreinforced web

p = reinforcement ratio of the tension reinforcement at the support

4-39.3. **Design of Shear Reinforcement**

Whenever the nominal shear stress vu exceeds the shear capacity vc of the concrete, shear reinforcement must be provided to carry the excess. Closed ties placed perpendicular to the flexural reinforcement must be used to furnish the additional shear capacity. Open stirrups, either single or double leg, are not permitted. The required area of shear reinforcement is calculated using:

$$A_v = \left[\frac{(v_u - v_c)\ b\ s_s}{\phi\ f_{dy}}\right] \qquad 4\text{-}141$$

where:

A_v = total area of stirrups

v_u-v_c = excess shear stress

s_s = spacing of stirrups in the direction parallel to the longitudinal reinforcement

ϕ = capacity reduction factor equal to 0.85

4-39.4. **Minimum Shear Reinforcement**

In order to insure the full development of the flexural reinforcement in a beam, a premature shear failure must be prevented. The following limitations must be considered in the design of closed ties:

1. The design shear stress (excess shear stress v_u - v_c) used in Equation 4-140 shall be equal to or greater than the shear

capacity of unreinforced concrete v_c as obtained from equation 4-139.

2. The nominal shear stress v_u must not exceed 10 $(f'_{dc})^{1/2}$.

3. The area A_v of closed ties should not be less than 0.0015 bs_s.

4. The required area A_v of closed ties shall be determined at the critical section and this quantity and spacing of reinforcement shall be used throughout the entire member.

5. The maximum spacing of closed ties is limited to d/2 when $v_u - v_c$ is less than 4 $(f'_{dc})^{1/2}$ or 24 inches, whichever is smaller. When $v_u - v_c$ is greater than 4 $(f'_{dc})^{1/2}$ the maximum spacing is limited to d/4.

4-40. Direct Shear

Direct shear failure of a member is characterized by the rapid propagation of a vertical crack through the depth of the member. This crack is usually located at the supports where the maximum shear stresses occur. Failure of this type is possible even in members reinforced for diagonal tension.

Diagonal bars are required at supports to prevent direct shear failure: when the design support rotation exceeds 2° (unless the beam is simply supported), when the design support rotation is ≤ 2° but the direct shear capacity of the concrete is insufficient, or when the section is in tension. Diagonal reinforcement consists of inclined bars which extend from the support into the beam.

Diagonal bars are not typically recommended in beams. Therefore, beams should be designed for small rotations and with an adequate cross-sectional area for the direct shear capacity of the concrete, V_d, to exceed the ultimate direct shear force, V_s.

If the design support rotation, θ, is less than or equal to 2° ($\theta \leq 2°$), or if the section, with any rotation θ, is simply supported (total moment capacity of adjoining elements at the support must be significantly less than the moment capacity of the section being checked for direct shear), then the ultimate direct shear force, V_d, that can be resisted by the concrete in a slab is given by Equation 4-30.

If the design support rotation, θ, is greater than 2° ($\theta > 2°$), or if a section (with any support rotation) is in net tension, then the ultimate direct shear capacity of the concrete, V_d, is zero and diagonal bars are required to take all direct shear.

If diagonal bars must be used, the required cross-sectional area is:

$$A_d = (V_s b - V_d)/(f_{ds} \sin(\alpha)) \qquad \text{4-142}$$

where:

$V_d = 0.18 f'_{dc}$ bd. ($\theta \leq 2°$ or simple supports), or $V_d = 0$ ($\theta > 2°$ or section in tension).

and A_d = total area of diagonal bars at the support within a width b

V_s = shear at the support of unit width b

α = angle formed by the plane of the diagonal reinforcement and the longitudinal reinforcement.

4-41. Ultimate Torsion Capacity

4-41.1. General

In addition to the flexural effects considered above, concrete beams may be subjected to torsional moments. Torsion rarely occurs alone in reinforced concrete beams. It is present more often in combination with transverse shear and bending. Torsion may be a primary influence but more frequently it is a secondary effect. If neglected, torsional stresses can cause distress or failure.

Torsion is encountered in beams that are unsymmetrically loaded. Beams are subject to twist if the slabs on each side are not the same span or if they have different loads. Severe torsion will result on beams that are essentially loaded from one side. This condition exists for beams around an opening in a roof slab and for pilasters around a door opening.

The design for torsion presented in this Section is limited to rectangular sections. For a beam-slab system subjected to conventional loading conditions, a portion of the slab will assist the beam in resisting torsional moments. However, in blast resistant design, a plastic hinge is usually formed in the slab at the beam and, consequently, the slab is not effective in resisting torsional moments.

4-41.2. Ultimate Torsional Stress

The nominal torsional stress in a rectangular beam in the vertical direction (along h) is given by:

$$v_{(tu)V} = \frac{3T_u}{b^2 h} \qquad 4\text{-}143$$

and the nominal torsional stress in the horizontal direction (along b) is given by:

$$v_{(tu)H} = \frac{3\,T_u}{bh^2} \qquad 4\text{-}144$$

where:

v_{tu} = nominal torsional stress

T_u = total torsional moment at critical section

b = width of beam

h = overall depth of beam

The critical section for torsion is taken at the same location as diagonal tension. It should be noted that the torsion stress in the vertical face of the beam (along h) is maximum when b is less than h whereas the torsion stress along the horizontal face of the beam (along b) is maximum when b is greater than h.

4-41.3 Capacity of Unreinforced Concrete for Combined Shear and Torsion

For a beam subjected to combined shear (diagonal tension) and torsion, the shear stress and the torsion stress permitted on an unreinforced section are reduced by the presence of the other. The shear stress permitted on an unreinforced web is limited to:

$$v_c = \frac{2\,(f'_{dc})^{1/2}}{\left[1 + \left[\dfrac{v_{tu}}{1.2v_u}\right]^2\right]^{1/2}} \qquad \text{4-145}$$

while the torsion stress taken by the concrete of the same section is limited to:

$$v_{tc} = \frac{2.4\,(f'_{dc})^{1/2}}{\left[1 + \left[\dfrac{1.2v_u}{v_{tu}}\right]^2\right]^{1/2}} \qquad \text{4-146}$$

where:

v_c = maximum shear capacity of an unreinforced web

v_{tc} = maximum torsion capacity of an unreinforced web

v_u = nominal shear stress

v_{tu} = nominal torsion stress in the direction of v_u

It should be noted that the shear stress permitted on an unreinforced web of a beam subjected to shear only is given by Equation 4-139. Whereas, the torsion stress permitted on an unreinforced web of a beam subjected to torsion only is given by:

$$v_{tc} = 2.4\,(f'_{dc})^{1/2} \qquad \text{4-147}$$

Whenever the nominal shear stress vu exceeds the shear capacity vc of the concrete, shear reinforcement must be provided to carry the excess. This quantity of shear reinforcement is calculated using Equation 4-140 except the value of vc shall be obtained from Equation 4-144 which includes the effects of torsion.

4-41.4. Design of Torsion Reinforcement

4-41.4.1. Design of Closed Ties

Whenever the nominal torsion stress vtu exceeds the maximum torsion capacity of the concrete, torsion reinforcement in the shape of closed ties, shall be provided to carry the excess. The required area of the vertical leg of the closed ties is given by:

$$A_{(t)V} = \frac{(v_{(tu)V} - v_{tc})\ b^2hs}{3\phi\ \alpha_t\ b_t\ h_t\ f_{dy}} \qquad 4\text{-}148$$

and the required area of the horizontal leg of the closed ties is given by:

$$A_{(t)H} = \frac{(v_{(tu)H} - v_{tc})\ bh^2s}{3\phi\ \alpha_t\ b_t\ h_t\ f_{dy}} \qquad 4\text{-}149$$

in which:

$$\alpha_t = 0.66 + 0.33\ (h_t/b_t) \le\ 1.50 \text{ for } h_t \ge b_t \qquad 4\text{-}150a$$

$$\alpha_t = 0.66 + 0.33\ (b_t/h_t) \le\ 1.50 \text{ for } h_t \ge b_t \qquad 4\text{-}150b$$

where:

A_t = area of one leg of a closed stirrup resisting torsion within a distance s

s = spacing of torsion reinforcement in a direction parallel to the longitudinal reinforcement

ϕ = capacity reduction factor equal to 0.85

b_t = center-to-center dimension of a closed rectangular tie along b

h_t = center-to-center dimension of a closed rectangular tie along h

The size of the closed tie provided to resist torsion must be the greater of that required for the vertical (along h) and horizontal (along b) directions. For the case of b less than h, the torsion stress in the vertical direction is maximum and the horizontal direction need not be considered. However, for b greater than h, the torsion stress in the horizontal direction is maximum. In this case the required At for the vertical and horizontal directions must be obtained and the greater value used to select the closed stirrup. It should be noted that in the horizontal direction a beam, in shear wall type structures, is not subjected to lateral shear (slab resists lateral loads) and the value of v_{tc} used in Equation 4-148 is calculated from Equation 4-146 which does not include the effect of shear.

When torsion reinforcement is required, it must be provided in addition to reinforcement required to resist shear. The closed ties required for torsion may be combined with those required for shear. However, the area furnished must be the sum of the individually required areas and the most restrictive requirements for spacing and placement must be met. Figure 4-60 shows several ways to arrange web reinforcement. For low torsion and shear, it is convenient to combine shear and torsional web reinforcement in the form of a single closed stirrup whose area is equal to $A_t + A_v/2$. For high torsion and shear, it would be economical to provide torsional and shear reinforcement separately. Torsional web reinforcement consists of closed stirrups along the periphery, while the shear web reinforcement is in the form of closed stirrups distributed along the width of the member. For very high torsion, two closed stirrups along the periphery may be used. The combined area of the stirrups must equal At and they must be located as close as possible to each other, i.e., the minimum separation of the flexural reinforcement. In computing the required area of stirrups using Equation 4-147, the value of bt should be equal to the average center-to-center dimension of the closed stirrups as shown in Figure 4-60.

4-41.4.2. **Design of Longitudinal Reinforcement**

In addition to closed stirrups, longitudinal reinforcement must be provided to resist the longitudinal tension caused by the torsion. The required area of longitudinal bars Al shall be computed by:

$$A_l = 2A_t \left[\frac{b_t + h_t}{s}\right] \qquad \text{4-151a}$$

or by:

$$A_l = \left[\frac{400bs}{f_{dy}} \; \frac{v_{tu}}{v_{tu} + v_u} - 2A_t\right]\left[\frac{b_t + h_t}{s}\right] \qquad \text{4-151b}$$

whichever is greater. When using Equation 4-151b, the value of $2A_t$ shall be greater than or equal to 50 bs/f_{dy}. It should be noted that Equation 4-151a requires the volume of longitudinal reinforcement to be equal to the volume of the web reinforcement required by Equation 4-147 or 4-148 unless a greater amount of longitudinal reinforcement is required to satisfy the minimum requirements of Equation 4-151b.

Longitudinal bars should be uniformly distributed around the perimeter of the cross section with a spacing not exceeding 12 inches. At least one longitudinal bar should be placed in each corner of the closed stirrups. A typical arrangement of longitudinal bars is shown in Figure 4-60 where torsional longitudinal bars that are located in the flexural tension zone and flexural compression zone may be combined with the flexural steel.

The addition of torsional and flexural longitudinal reinforcement in the flexural compression zone is not reasonable. It is illogical to add torsional steel that is in tension to the flexural steel that is in compression. This method of adding torsional steel to flexural steel regardless of whether the latter is in tension or in compression is adopted purely for simplicity. For blast resistant design, flexural reinforcement added but not included in the calculation of the ultimate resistance could cause a shear failure. The actual ultimate resistance could be significantly greater than the calculated ultimate resistance for which the shear reinforcement is provided. Therefore, torsional longitudinal reinforcement cannot be indiscriminately placed but rather must be placed only where required.

In the design of a beam subjected to both flexure and torsion, torsional longitudinal reinforcement is first assumed to be uniformly distributed around the perimeter of the beam. The reinforcement required along the vertical face of the beam will always be provided. However, in the flexural compression zone, the reinforcement that should be used is the greater of the flexural compression steel (rebound reinforcement) or the torsional steel. In terms of the typical arrangement of reinforcement in Figure 4-60, either A'_s or A_l is used, whichever is greater, as the design steel area in the flexural compression zone. For the tension zone at the mid span of a uniformly loaded beam the torsional stress is zero and torsional longitudinal reinforcement is not added. Conversely, the tension zone at the supports is the location of peak torsional stresses and longitudinal torsional reinforcement must be added to the flexural steel.

4-41.5. Minimum Torsion Reinforcement

In the design of closed ties for beams subjected to both shear and torsion, the following limitations must be considered:

1. The minimum quantity of closed ties provided in a beam subjected to both shear and torsion shall not be less than that required for a beam subjected to shear alone.

2. The maximum nominal shear stress v_u must not exceed $10\ (f'_{dc})^{1/2}$

3. The maximum nominal torsion stress v_{tu} shall not exceed

$$\frac{12\ (f'_{dc})^{1/2}}{\left[1 + \left[(1.2\ v_u) / (v_{tu})\right]^2\right]^{1/2}}$$

4. The required spacing of closed stirrups shall not exceed $(b_t + h_t)/4$ or 12 inches nor the maximum spacing required for closed ties in beams subjected to shear only.

5. The required areas A_v and A_t shall be determined at the critical section and this quantity and spacing of reinforcement shall be used throughout the entire beam.

6. To insure the full development of the ties, they shall be closed using 135-degree hooks.

4-42. Flexural Design

4-42.1. Introduction

The flexural design of beams is very similar to the design of non-laced concrete slabs. The main difference is that in the case of a slab the calculations are performed based on a unit area, whereas for a beam, they are based on a unit length of beam. In addition, since beams are one-way members, the distribution of mutually perpendicular reinforcement does not have to be considered.

4-42.2. Small Deflections

The design range for small deflections may be divided into two regions; beams with support rotations less than 2 degrees (limited deflections) and support rotations between 2 and 4 degrees. Except for the type of cross-section available to resist moment, the design procedure is the same.

A concrete section and reinforcement are assumed. Using the equations of section 4-38 (Equation 4-129 for type I cross-sections, Equation 4-137 for type II and III cross-sections) the moment capacities of the trial section is computed. The moment capacities are required to calculate the ultimate unit resistance r_u and the equivalent elastic deflection X_E. These parameters, along with the natural period of vibration T_n, define the equivalent single-degree-of-freedom system of the beam, and are discussed in detail in Chapter 3.

A dynamic analysis (see section 4-43) is performed to check if the beam meets the allowable deflection criteria. Finally, the assumed section is designed for shear and torsion, if applicable. If the beam does not meet the allowable response criteria, the required shear reinforcement is excessive, or the beam is overdesigned, a new concrete section is selected and the entire design procedure is repeated.

4-42.3. Large Deflections

4-42.3.1. Introduction

Design of reinforced concrete beams for support rotations greater than 4 degrees depends on their ability to act as a tensile membrane. Lateral restraint of the beam must be provided to achieve this action. Thus, if lateral restraint does not exist, tensile membrane action is not developed and the beam reaches incipient failure at 4 degrees support rotation. However, if lateral restraint exists, deflection of the beam induces membrane action and axial forces. These axial tension forces provide the means for the beam to continue to develop substantial resistance up to maximum support rotations of approximately 12 degrees.

4-42.3.2. Lateral Restraint

Adequate lateral restraint of the reinforcement is mandatory in order for the beam to develop and the designer to utilize the benefits of tensile membrane behavior. Sufficient lateral restraint is provided if the reinforcement is adequately anchored into adjacent supporting members capable of resisting the axial forces induced by tensile membrane action.

Tensile membrane behavior should not be considered in the design process unless full external lateral restraint is provided. Full lateral restraint means that adjacent members can effectively resist a total lateral force equivalent to the ultimate strength of all continuous reinforcement in the beam. This external resistance is more difficult to realize for beams than for slabs due to the concentration of the end reactions.

4-42.3.3. **Resistance - Deflection Curve**

The resistance-deflection curve for a beam is the same as that for a slab which is shown in Figure 4-18. The initial portion of the curve is primarily due to flexural action (increased capacity due to possible compression forces is not shown). At 4 degrees support rotation, the beam loses flexural capacity. However, due to the presence of continuous reinforcement and adequate lateral restraint, tensile membrane action developed. The resistance due to this action increases with increasing deflection up to incipient failure at approximately 12 degrees support rotation.

In order to simplify the design calculations, the resistance is assumed to be due to flexural action throughout the entire range of behavior (same procedure for slab calculations). To approximate the energy absorbed under the actual resistance-deflection curve, the maximum support of the idealized is limited to 8 degrees. Design for this deflection would produce incipient failure conditions.

For the design of a laterally restrained beam for 8 degrees support rotation, a type III cross-section is used to compute the ultimate moment capacity of the section as well as to provide the mass to resist motion. The stress in the reinforcement f_{ds} would be equal to that corresponding to support rotations $5 \leq \theta_m \leq 12$ given in Table 4-2. At every section throughout the beam, the tension and compression reinforcement must be continuous in order to develop the tensile membrane action discussed below.

4-42.3.4. **Ultimate Tensile Membrane Capacity**

As can be seen in Figure 4-18, tensile membrane resistance is a function of deflection. It is also a function of the span length and the amount of continuous reinforcement. The tensile membrane resistance rt of a laterally restrained beam at a deflection X is expressed as:

$$r_T = X \left[\frac{8T}{L^2} \right] \qquad 4\text{-}152$$

in which

$$T = A_{sc} f_{dy} \qquad 4\text{-}153$$

where

r_t = tensile membrane resistance

X - deflection of the beam

T - force in the continuous reinforcement

L - clear span

A_{sc} - total area of continuous reinforcement

Even though the capacity of a laterally restrained beam is based on flexural action, adequate tensile membrane capacity must be provided, that is, sufficient continuous reinforcement must be provided so that the tensile membrane resistance r_t corresponding to 8 degrees support rotation must be greater than the flexural resistance r_u. The deflection is computed as a function of the plastic hinge locations. The force in the continuous reinforcement is calculated using the dynamic design stress f_{ds} corresponding to 8 degrees support rotation (Table 4-2).

4-42.3.5. Flexural Design

Since the actual tensile membrane resistance-deflection curve is replaced with an equivalent flexural curve, the design of a beam for large deflections is greatly simplified. The design is performed in a similar manner as for small deflections. However, sufficient continuous reinforcement must be provided to develop the required tensile membrane resistance. This reinforcement must be fully anchored in the lateral supports. Care must be taken to ensure that the lateral supports are capable of resisting the lateral force T as given in Equation 4-153.

4-43. Dynamic Analysis

4-43.1. Design for Shock Load

When a concrete slab supported by beams is subjected to a blast load, the slab and beams act together to resist the load. The beam-slab system is actually a two-mass system and should be treated as such. However, a reasonable design can be achieved by considering the slab and beams separately. That is, the slab and beams are transformed into single-degree-of-freedom systems completely independent of each other and are analyzed separately. The dynamic analysis of slabs is treated extensively in previous sections.

The equivalent single-degree-of-freedom system of any structural element is defined in terms of its ultimate unit resistance, r_u, equivalent elastic deflection X_E and natural period of vibration T_N. The ultimate unit resistance is obtained from the table for one-way elements in Chapter 3 for the moment capacity given above. The procedures and parameters necessary to obtain the equivalent elastic deflection and natural period are also obtained from Chapter 3.

Chapter 2 describes procedures for determining the dynamic load which is defined by its peak value P and duration T. For the ratios P/r_u and T/T_N the ductility ratio X_m/X_E and t_m/T can be obtained from the response charts of Chapter 3. These values X_m, which is the maximum deflection, and t_m, the time to reach the maximum deflection define the dynamic response of the beam.

A beam is designed to resist the blast load acting over the tributary area supported by the beam. Therefore, the peak value of the blast load P is the product of the unit peak blast pressure times the spacing of the beams, and has the unit of pounds per inch.

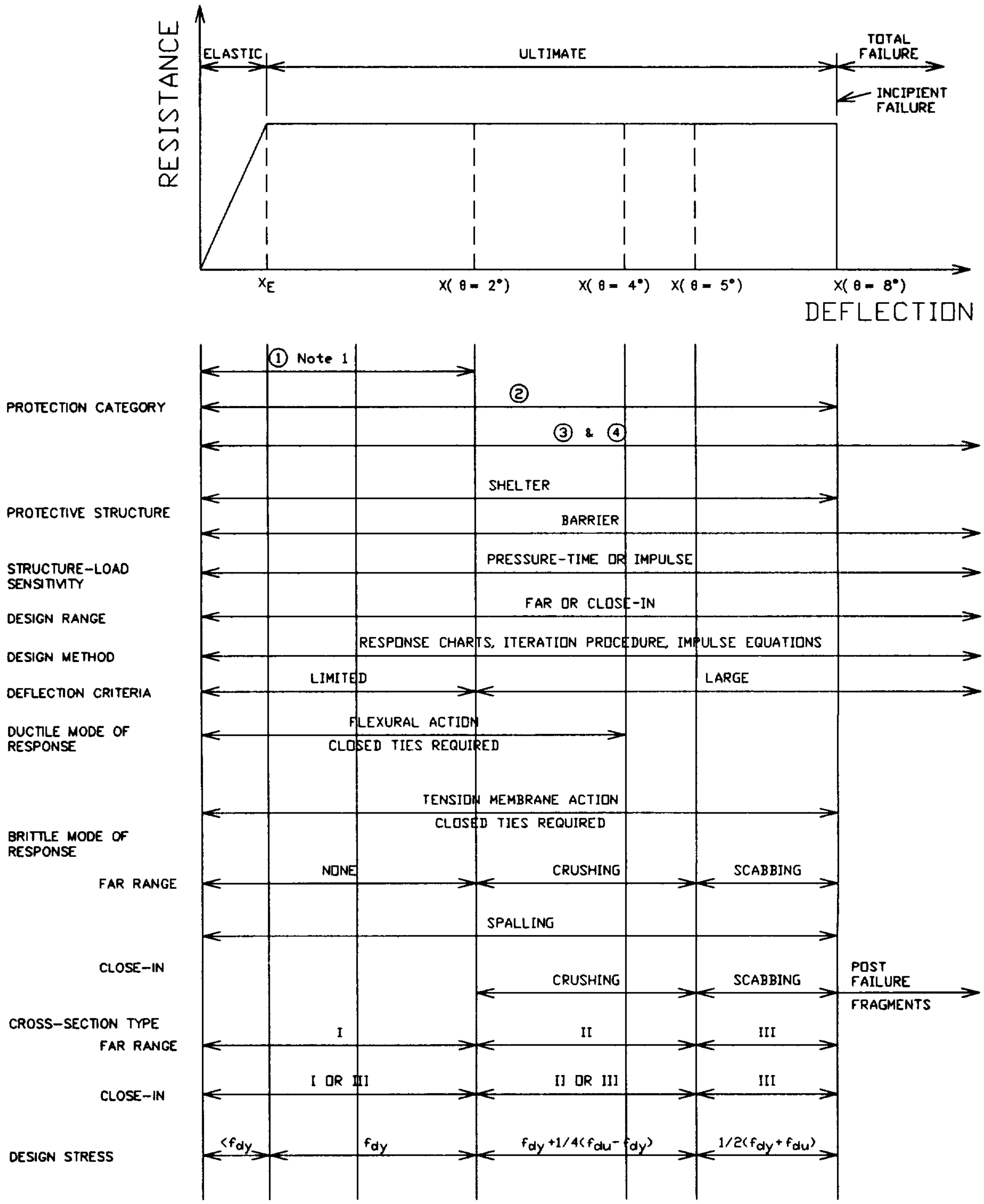
In addition to the short term effect of the blast load, a beam must be able to withstand the long term effect of the resistance of the element(s) being supported by the beam when the response time of the element(s) is equal to or greater than the duration of the blast load. To insure against premature failure, the ultimate resistance of the beam must be greater than the reaction of the supported element (slab, wall, blast door, etc.) applied to the beam as a static load.

In the case of a supported slab, the slab does, in fact, act with the beam; a portion of the mass of the slab acts with the mass of the beam to resist the dynamic load. It is, therefore, recommended that 20 percent of the mass of the slab (or blast door, wall, etc.) on each side of the beam be added to the actual mass of the beam. This increased mass is then used to compute the natural period of vibration T_N of the beam. It should be noted that in the calculation of T_N the values used for the effective mass and stiffness of the beam depends upon the allowable maximum deflection. When designing for completely elastic behavior, the elastic stiffness is used while, in other cases, the equivalent elasto-plastic stiffness K_E is used. The elastic value of the effective mass is used for the elastic range while, in the elasto-plastic range, the effective mass is the average of the elastic and elasto-plastic values. For small plastic deformations, the value of the effective mass is equal to the average of the equivalent elastic value and the plastic value while for large plastic deformations, the effective mass is equal to the plastic value.

4-43.2. Design for Rebound

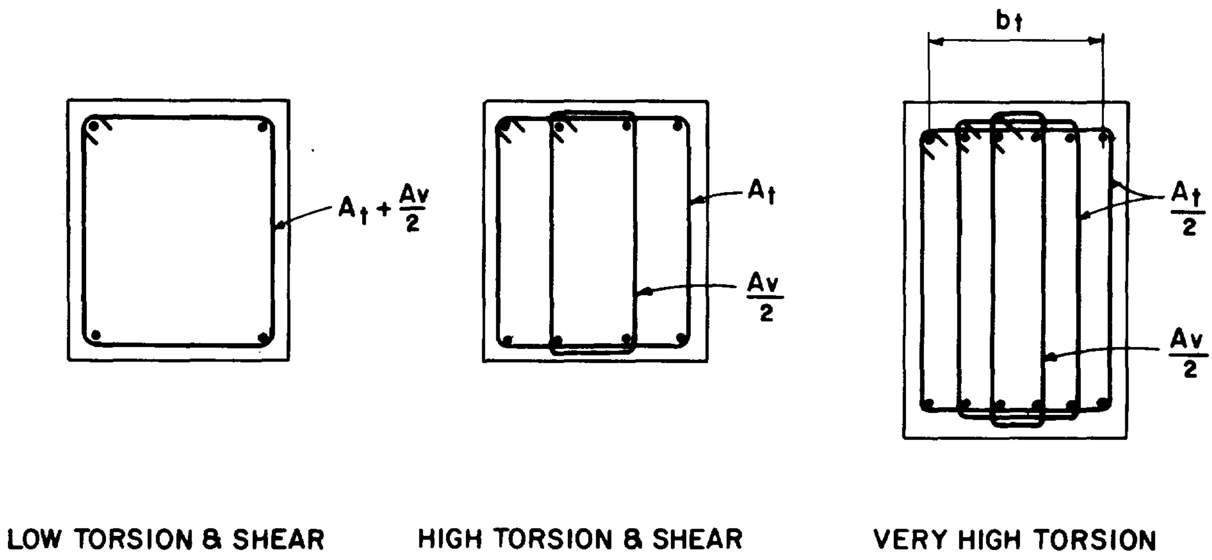
The beam must be designed to resist the negative deflection or rebound which occurs after the maximum positive deflection has been reached. The negative resistance r^-, attained by the beam when subjected to a triangular pressure-time load, is obtained from figure 3-268 in Chapter 3. Entering the figure with the ratios of X_m/X_E and T/T_N, previously determined for the positive phase of design, the ratio of the required rebound resistance to the ultimate resistance r^-/r_u is obtained. The beam must be reinforced to withstand this rebound resistance r^- to insure that the beam will remain elastic during rebound.

The tension reinforcement provided to withstand rebound forces is added to what is needed for the compression zone during the initial loading phase. To obtain this reinforcement, the beam is essentially designed for a negative load equal to the calculated value of r^-. However, in no case shall the rebound reinforcement be less than one-half of the positive phase reinforcement. The moment capacities and the rebound resistance capacity are calculated using the same equations previously presented.

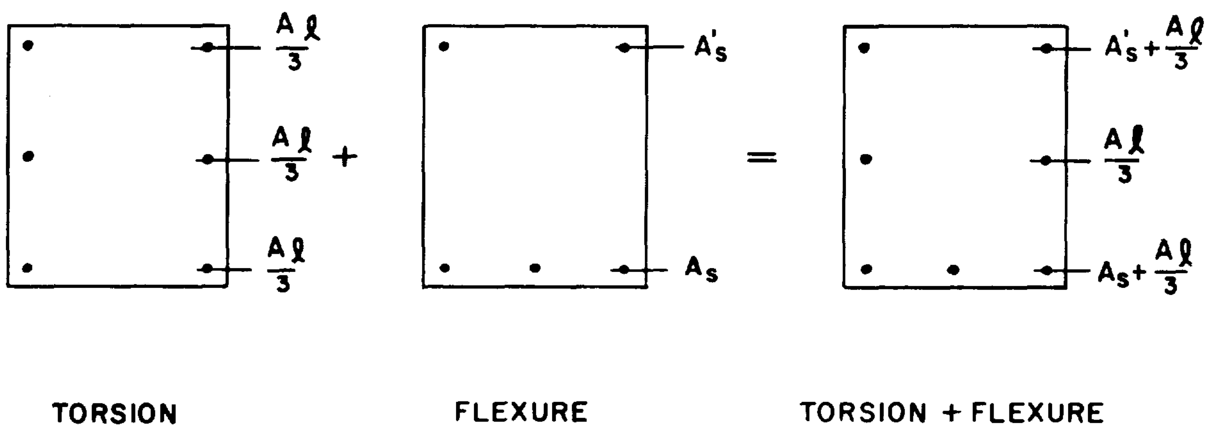


Note 1: Stirrups (closed ties) always required.

Figure 4-59 Relationship between design parameters for beams



(a) Arrangement of Shear and Torsional Web Reinforcement



(b) Arrangement of Torsional and Flexural Longitudinal Steel

Figure 4-60 Arrangement of reinforcement for combined flexure and torsion

DYNAMIC DESIGN OF INTERIOR COLUMNS

4-44. Introduction

The design of columns is limited to those in shear wall type structures where the lateral loads are transmitted through the floor and roof slabs to the exterior (and interior, if required) shear walls. Due to the extreme stiffness of the shear walls, there is negligible sidesway in the interior columns and, hence, no induced moments due to lateral loads. Therefore, interior columns are axially loaded members not subjected to the effects of lateral load. However, significant moments can result from unsymmetrical loading conditions.

4-45. Strength of Compression Members (P-M Curve)

4-45.1. General

The capacity of a short compression member is based primarily on the strength of its cross section. The behavior of the member encompasses that of both a beam and a column. The degree to which either behavior predominates depends upon the relative magnitudes of the axial load and moment. The capacity of be the column can be determined by constructing an interaction diagram as shown in Figure 4-61. This curve is a plot of the column axial load capacity versus the moment it can simultaneously withstand. Points on this diagram are calculated to satisfy both stress and strain compatibility. A single curve would be constructed for a given cross section with a specified quantity of reinforcement. The plot of a given loading condition that falls within the area represents a loading combination that the column can support, whereas, a plot that falls outside the interaction curve represents a failure combination. Three points of the interaction diagram are used to define the behavior of compression members under combined axial and flexural loads. These points are: (1) pure compression (P_o, $M = 0$), (2) pure flexure ($P = 0$, M_o), and, (3) balanced conditions (P_b, M_b). The eccentricity of the design axial load for the condition of pure compression is zero. However, under actual conditions, pure axial loads will rarely, if ever, exist. Therefore, the maximum axial load is limited by a minimum eccentricity, e_{min}. At balanced conditions, the eccentricity is defined as e_b while the eccentricity at pure flexure is infinity. The strength of a section is controlled by compression when the design eccentricity $e = M_u/P_u$, is smaller than the eccentricity under balanced conditions. The strength of the section is controlled by tension when the design eccentricity is greater than that for balanced conditions.

4-45.2. Pure Compression

The ultimate dynamic strength of a short reinforced concrete column subjected to pure axial load (no bending moments) is given by:

$$P_o = 0.85\, f'_{dc}(A_g - A_{st}) + A_{st}\, f_{dy} \qquad 4\text{-}154$$

where:

P_o = maximum axial load

A_g = gross area of section

A_{st} = total area of reinforcing steel

A member subjected to pure axial compression is a hypothetical situation since all columns are subjected to some moment due to actual load conditions. All tied and spiral columns must be designed for a minimum load eccentricity. This minimum design situation is presented in a subsequent section.

4-45.3. Pure Flexure

An interior column of a shear wall type structure cannot be subjected to pure flexure under normal design conditions. For the purpose of plotting a P-M curve, the criteria presented for beams is used.

4-45.4. Balanced Conditions

A balanced strain condition for a column subjected to a dynamic load is achieved when the concrete reaches its limiting strain of 0.003 in/in simultaneously with the tension steel reaching its dynamic yield stress f_{dy}. This condition occurs under the action of the balanced load P_b and the corresponding balanced moment M_b. At balanced conditions, the eccentricity of the load is defined as e_b, and is given by:

$$e_b = M_b/P_b \qquad \text{4-155}$$

The actual values of the balanced load and corresponding balanced moment are generally not required. The balanced eccentricity is the important parameter since a comparison of the actual eccentricity to the balanced eccentricity distinguishes whether the strength of the section is controlled by tension or compression. The comparison of the actual eccentricity to the balanced eccentricity dictates the choice of the appropriate equation for calculating the ultimate axial load capacity, P_u.

Approximate expressions have been derived for the balanced eccentricity for both rectangular and circular members. These expressions are sufficiently accurate for design purposes. For a rectangular tied column with equal reinforcement on opposite faces (Fig. 4-62a). the balanced eccentricity is given by:

$$e_b = 0.20h + (1.54mA_s)/b \qquad \text{4-156}$$

and:

$$m = (f_{dy}) / (0.85 f'_{dc}) \qquad \text{4-157}$$

where:

e_b = balanced eccentricity

h = depth of rectangular section

b = width of rectangular section

A_s = area of reinforcement on one face of the section

For a circular section with spiral reinforcement (Fig. 4-62b), the balanced eccentricity is given by:

$$e_b = (0.24 + 0.39\, p_T m)\, D \qquad \text{4-158}$$

and:

$$p_T = A_{st}/A_g \qquad 4\text{-}159$$

where:

p_T = total percentage of reinforcement

A_{st} = total area of reinforcement

A_g = gross area of circular section

D = overall diameter of circular section

4-45.5. Compression Controls

When the ultimate eccentric load P_u exceeds the balanced value P_b, or when the eccentricity e is less than the balanced value e_b, the member acts more as a column than as a beam. Failure of the section is initiated by crushing of the concrete. When the concrete reaches its ultimate strain, the tension steel has not reached its yield point and may actually be in compression rather than tension. The ultimate eccentric load at a given eccentricity e less than e_b may be obtained by considering the actual strain variation as the unknown and using the principles of statics. However, equations have been developed which approximate the capacity of the column. These approximate procedures are adequate for design purposes.

For a rectangular tied column with equal reinforcement on opposite faces (Fig. 4-62a), the ultimate axial load capacity at a given eccentricity is approximated by:

$$P_u = \frac{A_s f_{dy}}{[e/(2d-h)] + 0.5} + \frac{bh f'_{dc}}{(3he/d^2) + 1.18} \qquad 4\text{-}160$$

where:

P_u = ultimate axial load at actual eccentricity e

e = actual eccentricity of applied load

A_s = area of reinforcement on one face of the section

d = distance from extreme compression fiber to centroid of tension reinforcement

h = depth of rectangular section

b = width of rectangular section

For a circular section with spiral reinforcement, the ultimate axial load capacity at a given eccentricity is approximated by:

$$P_u = \left[\frac{A_{st} \; f_{dy}}{\dfrac{3e}{D_s} + 1.0}\right] + \left[\frac{A_g \; f'_{dc}}{\dfrac{9.6D_e}{(0.8D + 0.67\, D_s)^2} + 1.18}\right] \qquad 4\text{-}161$$

where:

A_{st} = total area of uniformly distributed longitudinal reinforcement

A_g = gross area of circular section

D = overall diameter of circular section

D_s = diameter of the circle through centers of reinforcement arranged in a circular pattern

4-45.6. Tension Controls

When the ultimate eccentric load P_u is less than the balance value P_b or when the eccentricity e is greater than the balanced value e_b, the member acts more as a beam than as a column. Failure of the section is initiated by yielding of the tension steel. The ultimate eccentric load at a given eccentricity e greater than e_b may be obtained by considering the actual strain variation as the unknown and using the principles of statics.

However, again, equations have been developed to approximate the capacity of the column. It should be pointed out that while tension controls are a possible design situation it is not an usual condition for interior columns of a shear wall type structure.

For a rectangular tied column with equal reinforcement on opposite faces (Fig. 4-62a), the ultimate axial load capacity at a given eccentricity is approximated by:

$$P_u = 0.85\, f'_{dc}\, bd \left[1-p - \frac{e'}{d} + \left[\frac{(1-e')^2}{d} + 2p \left[(m-1)\left(2 - \frac{h}{d}\right) + \frac{e'}{d} \right] \right]^{1/2} \right] \tag{4-162}$$

in which:

$$p_s = A\ \ / bd \tag{4-163}$$

$$e' = e + d - (h/2) \tag{4-164}$$

$$m = f_{dy} / (0.85\, f'_{dc}) \tag{4-165}$$

where:

p = percentage of reinforcement on one face of section

e' = eccentricity of axial load at the end of member measured from the centroid of the tension reinforcement

For a circular section with spiral reinforcement (Fig. 4-62b), the ultimate axial load capacity at a given eccentricity is approximated by:

$$P_u = 0.85 f'_{dc}D^2 \left[\left[\left(\frac{0.85e}{D} - 0.38\right)^2 + \frac{p_T m D_s}{2.5\ D} \right]^{1/2} - \left(\frac{0.85e}{D} - 0.38 \right) \right] \qquad 4\text{-}166$$

where:

p_T = total percentage of reinforcement and is defined in Equation 4-159

4-46. Slenderness Effects

4-46.1. General

The preceding section discussed the capacity of short compression members. The strength of these members is based primarily on their cross section. The effects of buckling and lateral deflection on the strength of these short members are small enough to be neglected. Such members are not in danger of buckling prior to achieving their ultimate strength based on the properties of the cross section. Further, the lateral deflections of short compression members subjected to bending moments are small, thus contributing little secondary bending moment (axial load P multiplied by lateral deflection). These buckling and deflection effects reduce the ultimate strength of a compression member below the value given in the preceding section for short columns.

In the design of columns for blast resistant buildings, the use of short columns is preferred. The cross section is selected for the given height and support conditions of the column in accordance with criteria presented below for short columns. If the short column cross section results in a capacity much greater than required, the dimensions may be reduced to achieve an economical design. However, slenderness effects must be evaluated to insure an adequate design. It should be noted that for shear wall type structures, the interior columns are not subjected to sidesway deflections since lateral loads are resisted by the stiff shear walls. Consequently, slenderness effects due to buckling and secondary bending moments (Pü) are the only effects that must be considered.

4-46.2. Slenderness Ratio

The unsupported length L_u of a compression member is taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support for the compression member. Where column capitals or haunches are present, the unsupported length is measured to the lower extremity of capital or haunch in the plane considered.

The effective length of a column kL_u is actually the equivalent length of a pin ended column. For a column with pin ends the effective length is equal to the actual unsupported length (k = 1.0). Where translation of the column at both ends is adequately prevented (braced column), the effective length of the column is the distance between points of inflection (k less than 1.0). It is recommended that for the design of columns in shear wall type structures the effective length factor k may be taken as 0.9 for columns that are definitely restrained by beams and girders at the top and bottom. For all other cases k shall be taken as 1.0 unless analysis shows that a lower value may be used.

For columns braced against sidesway, the effects of slenderness may be neglected when:

$$\frac{kL_u}{r} < 34 - 12\frac{M_1}{M_2} \qquad 4\text{-}167$$

where:

- k = effective length factor
- L_u = unsupported length of column
- r = radius of gyration of cross section of column (r = 0.3h for tied columns and 0.25D for circular columns)
- M_1 = value of smaller end moment on column, positive if member is bent in single curvature and negative in double curvature
- M_2 = value of larger end moment on column

In lieu of a more accurate analysis, the value of M_1/M_2 may conservatively be taken equal to 1.0. Therefore, in the design of columns the effect of slenderness may be neglected when:

$$\frac{kL_u}{r} \le 22 \qquad 4\text{-}168$$

The use of slender columns is not permitted in order to avoid stability problems. Consequently, the slenderness ratio must be limited to a maximum value of 50.

4-46.3. Moment Magnification

Slenderness effects due to buckling and secondary bending moments must be considered in the design of columns whose slenderness ratio is greater than that given by Equation 4-167. The reduction in the ultimate strength of a slender column is accounted for in the design procedure by increasing the design moment. The cross section and/or reinforcement is thereby increased above that required for a short column.

A column braced against sidesway is designed for the applied axial load P and a magnified moment M defined by:

$$M = \delta M_2 \tag{4-169}$$

in which:

$$\delta = \frac{C_m}{1 - \frac{P}{P_c}} \tag{4-170}$$

where:

M = design moment

δ = moment magnifier

M_2 = value of larger end moment on column

C_m = equivalent moment correction factor defined by equation 4-171

M_1 = value of smaller end moment on column

P = design axial load

P_c = critical axial load causing buckling defined by equation 4-172

The value of the moment magnifier δ shall not be taken less than 1.0.

For columns braced against sidesway and not subjected to transverse loads between supports, i.e. interior columns of shear wall type structures, the equivalent moment factor C_m may be taken as:

$$C_m = 0.6 + 0.4\ M_1 / M_2 \tag{4-171}$$

The value of C_m may not under any circumstances be taken less than 0.4. In lieu of a more accurate analysis, the value of M_1/M_2 may conservatively be taken equal to 1.0. Therefore, in the design of interior columns, C_m may be taken as 1.0.

The critical axial load that causes a column to buckle is given by:

$$P_c = \frac{\pi^2 EI}{(kL_u)^2} \tag{4-172}$$

In order to apply Equation 4-172, a realistic value of EI must be obtained for the section at buckling. An approximate expression for EI at the time of buckling is given by:

$$EI = \frac{E_c\ I_a}{1.5} \tag{4-173}$$

in which:

$$I_a = \frac{I_g + I_c}{2} \qquad 4\text{-}174$$

and:

$$I_c = Fbd^3 \qquad 4\text{-}175$$

where:

I_a = average moment of inertia of section

I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement

I_c = moment of inertia of cracked concrete section with equal reinforcement on opposite faces

F = coefficient given in Figure 4-5

4-47. Dynamic Analysis

Columns are not subjected to the blast loading directly. Rather, the load that a column must resist is transmitted through the roof slab, beams and girders. These members "filter" the dynamic effects of the blast load. Thus, in buildings designed to obtain plastic deformations, the dynamic load reaching the columns is typically a fast "static" load, that is, a flat top pressure time load with a relatively long rise time.

The roof members and columns act together to resist the applied blast load. However, a reasonable design can be achieved by considering the column separately from the roof members. The response (resistance-time function) of the roof members to the blast load is taken as the applied dynamic load acting on the columns.

Columns are subjected to an actual axial load (with associated eccentricity) equal to the ultimate resistance of the appropriate roof members acting over the tributary area supported by the column. It is recommended for design of columns the ultimate axial load be equal to 1.2 times the actual axial load. This increase insures that the maximum response of the column will be limited to a ductility ratio (X_m/X_e) of 3.0 or less. If the rise time of the load (time to reach yield for the appropriate roof members) divided by the natural period of the column is small (approximately 0.1), the maximum ductility is limited to 3.0. Whereas, if the time ratio is equal to 1.0 or greater, the column will remain elastic. For the usual design cases, the ratio of the rise time to the natural period will be in the vicinity of 1.0. Therefore, the columns will remain elastic or, at best, sustain slight plastic action.

In some instances, buildings may be designed to remain completely elastic. In these cases, the actual axial load that the column must resist is equal to the maximum actual response of the roof members framing into the column. This response and, therefore, the maximum load on the column, can be no more than two times the blast load.

4-48. Design of Tied Columns

4-48.1. General

Interior columns are not usually subjected to excessive bending moments since sidesway is eliminated by the shear walls. However, significant moments about both axes can result from unsymmetrical loading conditions. These moments may be due to unequal spacing between columns or to time phasing of the applied loads. As a result of the complex load conditions, the columns must be proportioned considering bending about both the x and y axes simultaneously.

One method of analysis is to use the basic principles of equilibrium with the acceptable ultimate strength assumptions. This method essentially involves a trial and error process for obtaining the position of an inclined neutral axis. This method is sufficiently complex so that no formula may be developed for practical use.

An approximate design method has been developed which gives satisfactory results for biaxial bending. The equation is in the form of an interaction formula which for design purposes can be written in the form:

$$\frac{1}{P_u} = \frac{1}{P_x} + \frac{1}{P_y} - \frac{1}{P_o}$$

where:

P_u = ultimate load for biaxial bending with eccentricities e_x and e_y

P_x = ultimate load when eccentricity e_x is present ($e_y = 0$)

P_y = ultimate load when eccentricity e_y is present ($e_x = 0$)

P_o = ultimate load for a concentrically loaded column ($e_x = e_y = 0$)

Equation 4-176 is valid provided P_u is equal to or greater than 0.10 P_o. The usual design cases for interior columns satisfy this limitation. The equation is not reliable where biaxial bending is prevalent and is accompanied by an axial force smaller than 0.10 P_o. In the case of strongly prevalent bending, failure is initiated by yielding of the steel (tension controls region of P-M curve). In this range it is safe and satisfactorily accurate to neglect the axial force entirely and to calculate the section for biaxial bending only.

This procedure is conservative since the addition of axial load in the tension controls region increases the moment capacity. It should be mentioned that the tension controls case would be unusual and, if possible, should be avoided in the design.

Reinforcement must be provided on all four faces of a tied column with the reinforcement on opposite faces of the column equal. In applying Equation 4-176 to the design of tied columns, the values of P_x and P_y are obtained from Equation 4-160 and 4-162 for the regions where compression and tension control the design, respectively. The equations are for rectangular columns with equal reinforcement on the faces of the column parallel to the axis of bending. Consequently, in the calculation of P_x and P_y, the reinforcement perpendicular to the axis of bending is neglected. Conversely, the total quantity of reinforcement provided on all four faces of the column is used to calculate P_o from Equation 4-154. Calculation of P_x, P_y and P_o in the manner described will yield a conservative value of P_u from Equation 4-176.

4-48.2. Minimum Eccentricity

Due to the possible complex load conditions that can result in blast design, all tied columns shall be designed for biaxial bending. If computations show that there are no moments at the ends of the column or that the computed eccentricity of the axial load is less than 0.1h, the column must be designed for a minimum eccentricity equal to 0.1h. The value of h is the depth of the column in the bending direction considered. The minimum eccentricity shall apply to bending in both the x and y directions, simultaneously.

4-48.3. Longitudinal Reinforcement Requirements

To insure proper behavior of a tied column, the longitudinal reinforcement must meet certain restrictions. The area of longitudinal reinforcement shall not be less than 0.01 nor more than 0.04 times the gross area of the section. A minimum of 4 reinforcing bars shall be provided. The size of the longitudinal reinforcing bars shall not be less than #6 nor larger than #11. The use of #14 and #18 bars as well as the use of bundled bars are not recommended due to problems associated with the development and anchorage of such bars. To permit proper placement of the concrete, the minimum clear distance between longitudinal bars shall not be less than 1.5 times the nominal diameter of the longitudinal bars nor 1.5 inches.

4-48.4. Closed Ties Requirements

Lateral ties must enclose all longitudinal bars in compression to insure their full development. These ties must conform to the following:

1. The ties shall be at least #3 bars for longitudinal bars #8 or smaller and at least #4 bars for #9 longitudinal bars or greater.

2. To insure the full development of the ties they shall be closed using 135-degree hooks. The use of 90-degree bends is not recommended.

3. The vertical spacing of the ties shall not exceed 16 longitudinal bar diameters, 48 tie diameters or ½ of the least dimension of the column section.

4. The ties shall be located vertically not more than ½ the tie spacing above the top of footing or slab and not more than ½ the tie spacing below the lowest horizontal reinforcement in a slab or drop panel. Where beams frame into a column, the ties may be terminated not more than 3 inches below the lowest reinforcement in the shallowest of the beams.

5. The ties shall be arranged such that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie with an included angle of not more than 135 degrees and

no bar shall be farther than 6 inches clear on each side along the tie from such a laterally supported bar.

The above requirements for the lateral ties is to insure against buckling of the longitudinal reinforcement in compression. However, if the section is subjected to large shear or torsional stresses, the closed ties must be increased in accordance with the provisions established for beams (see section 4-39).

4-49. Design of Spiral Columns

4-49.1. General

Spiral columns may be subjected to significant bending moments about both axes and should, therefore, be designed for biaxial bending. However, due to the uniform distribution of the longitudinal reinforcement in the form of a circle, the bending moment (or eccentricities) in each direction can be resolved into a resultant bending moment (or eccentricity). The column can then be designed for uniaxial bending using Equations 4-161 and 4-166 for the regions where compression and tension control the design, respectively.

4-49.2. Minimum Eccentricity

Since spiral columns show greater toughness than tied columns, particularly when eccentricities are small, the minimum eccentricity for spiral columns is given as 0.05D in each direction rather than 0.1h in each direction for tied columns. The resultant minimum eccentricity for a spiral column is then equal to 0.0707D. Therefore, if computations show that there are no moments at the ends of a column or that the computed resultant eccentricity of the axial load is less than 0.0707D, the column must be designed for a resultant minimum eccentricity of 0.0707D.

4-49.3. Longitudinal Reinforcement Requirements

To insure proper behavior of a spiral reinforced column, the longitudinal reinforcement must meet the same restrictions given for tied columns concerning minimum and maximum area of reinforcement, smallest and largest reinforcing bars permissible and the minimum clear spacing between bars. The only difference is that for spiral columns the minimum number of longitudinal bars shall not be less than 6 bars.

4-49.4. Spiral Reinforcing Requirements

Continuous spiral reinforcing must enclose all longitudinal bars in compression to insure their full development. The required area of spiral reinforcement A_{sp} is given by:

$$A_{sp} = 0.1125\ s\ D_{sp} \left[\frac{D^2}{D_{sp}^2} - 1 \right] \frac{f'_{dc}}{f_{dy}} \qquad 4\text{-}177$$

where:

A_{sp} = area of spiral reinforcement

s - pitch of spiral

D - overall diameter of circular section

D_{sp} - diameter of the spiral measured through the centerline of the spiral bar

The spiral reinforcement must conform to the following:

1. Spiral column reinforcement shall consist of evenly spaced continuous spirals composed of continuous #3 bars or larger. Circular bars are not permitted.

2. The clear spacing between spiral shall not exceed 3 inches nor be less than 1 inch.

3. Anchorage of spiral reinforcement shall be provided by 1-½ extra turns of spiral bar at each end.

4. Splices in spiral reinforcement shall be lap splices equal to 1-½ turns of spiral bar.

5. Spirals shall extend from top of footing or slab to level of lowest horizontal reinforcement in members supported above.

6. In columns with capitals, spirals shall extend to a level at which the diameter or width of capital is two times that of the column.

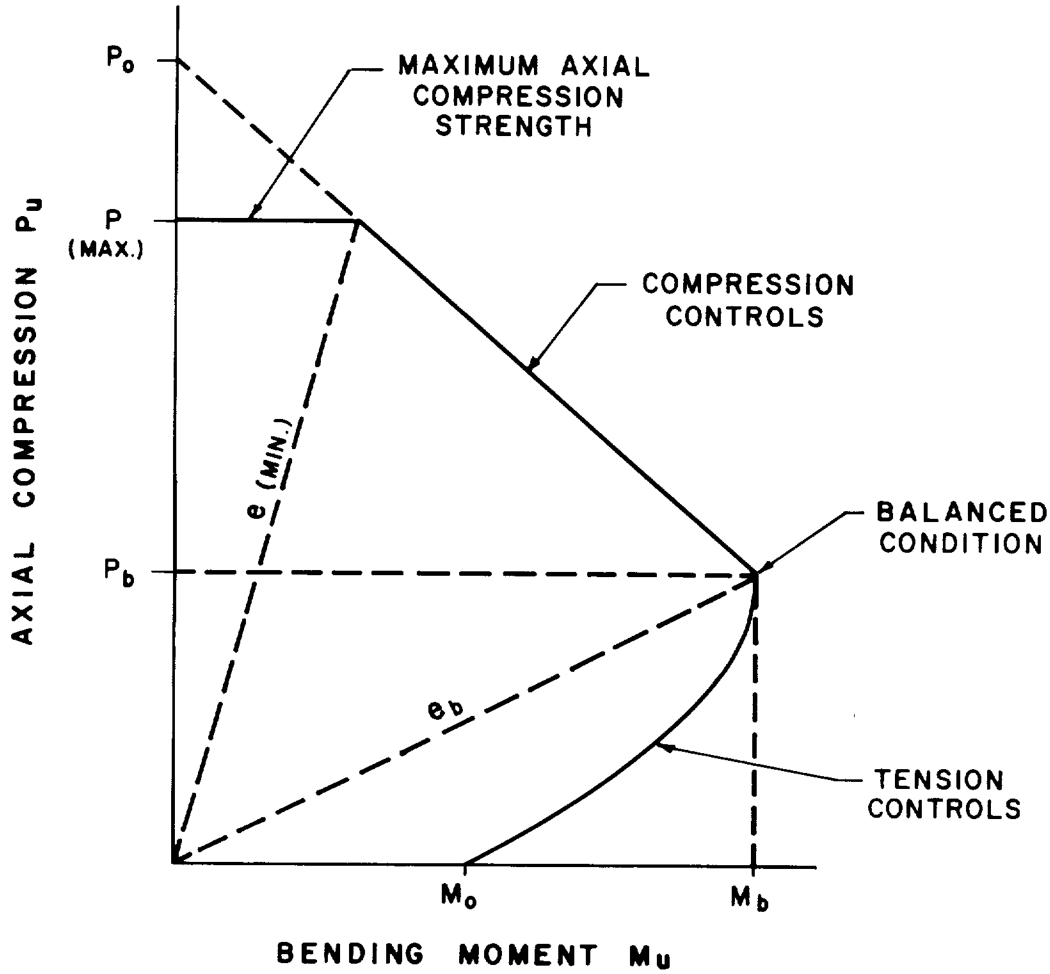
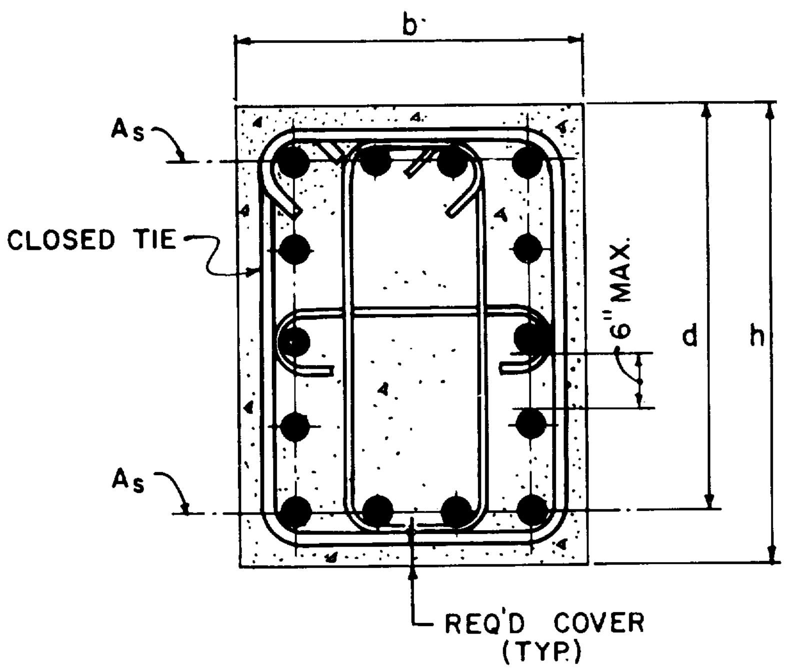
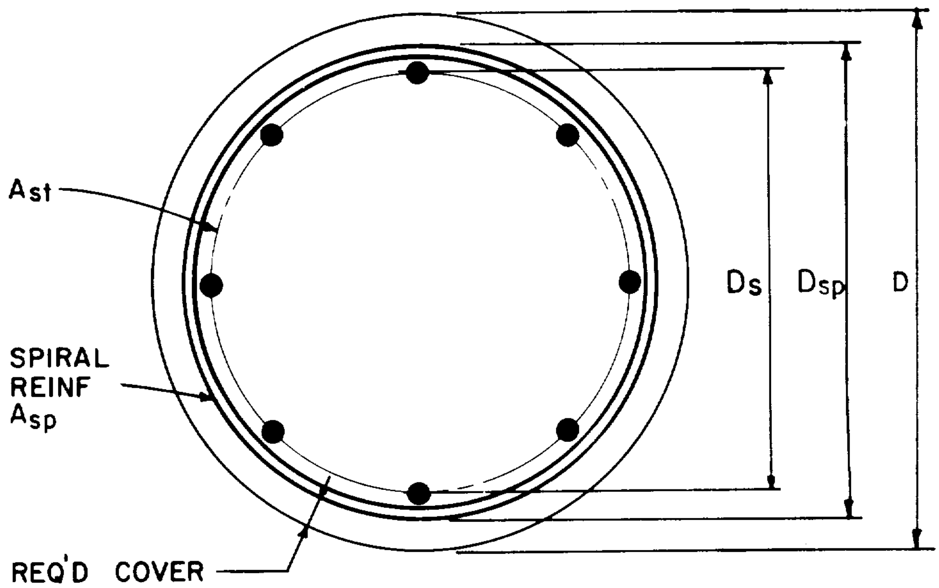


Figure 4-61 Column interaction diagram



a) RECTANGULAR SECTION WITH EQUAL REINFORCEMENT ON OPPOSITE FACES



b) CIRCULAR SECTION WITH UNIFORMLY DISTRIBUTED REINFORCEMENT

Figure 4-62 Typical interior column sections

DYNAMIC DESIGN OF EXTERIOR COLUMNS

4-50. Introduction

Exterior columns may be required for severe loading conditions. These columns could be monolithic with the exterior walls and as such would be subjected to both axial and transverse loading. The axial load results from the direct transfer of floor and roof beam reactions while the transverse load is due to the direct impact of the blast load.

The use of exterior columns would normally be restricted to use in framed structures to transfer roof and floor beam reactions to the foundation. Normally, only tied columns would be used since they are compatible with the placement of wall and beam reinforcement. Exterior columns are not normally required for flat slab structures since roof and floor loads are uniformly transmitted to the exterior walls.

4-51. Design of Exterior Columns

Exterior columns are generally designed as beam elements. The axial load on these columns may be significant, but usually the effect of the transverse load is greater. The column will usually be in the tension controls region (e greater than e_b) of the P-M curve (Fig. 4-61) where the addition of axial load increases the moment capacity of the member.

Consequently, the design of an exterior column as a beam, where the axial load is neglected, is conservative.

Since an exterior column is a primary member which is subjected to an axial load, it is not permitted to attain large plastic deformations. Therefore, the lateral deflection of exterior columns must be limited to a maximum ductility (X_m/X_E) of 3.

STRUCTURAL ANALYSIS AND DESIGN FOR BRITTLE MODE RESPONSE

4-52. Introduction

The response of a structural element in the brittle mode consists of that structural behavior which is associated with either partial or total failure of the element and is characterized by two types of concrete fragmentation: (1) spalling (either direct spalling or scabbing) which is the dynamic disengagement of the surface of the element, and (2) post-failure fragmentation which is associated with structural collapse.

Spalling is usually of concern only for those acceptor systems where personnel, valuable equipment and/or extremely sensitive explosives require protection. Where the acceptor system consists of relatively insensitive explosives so that fragment impact will not result in propagation of explosion or mass detonation, then post-failure fragmentation can be considered in the design. For this latter case, even though the velocity of the spalls can be greater than the velocity of the post-failure fragments, the effects of spalling can be neglected because of the smaller masses involved. Post-failure fragmentation cannot be permitted when personnel are being protected.

4-53. Direct Spalling

Direct spalling of a concrete element (Fig. 4-63) is the result of a tension failure in the concrete normal to its free surface and is caused by the shock pressures of an impinging blast wave being transmitted through the element.

When a shock front strikes the donor surface of a concrete element, compression stresses are transmitted from the air to the element. This stress disturbance propagates through the element in the form of a compression wave, and upon reaching the rear (acceptor) free surface, is reflected as a tension wave identical in shape and magnitude to the compression wave. During the return passage, if the tension stresses in the reflected wave exceed the stresses in the compression wave plus the tensile capacity of the concrete, the material will fracture with that part of the element between the rear free surface and the plane of failure being displaced from the remainder of the element. A portion of the stress wave is trapped in the failed section and contributes to its velocity. The part of the stress wave which remains within the main section continues to propagate with additional reflections and concrete fractures until its magnitude is reduced to that level below which spalling does not occur.

Direct spalling generally results in the formation of small concrete fragments. The size of the fragments is attributed to the nonuniformity of the shock wave (close-in effects) and the further distortions of the wave during its propagation through the element (nonhomogeneous material, nonelastic effects, etc.). Localized failures occur under the action of both flexural and shear stresses resulting in the rupture of the mortar binding the stone aggregate together. The failure zone propagates across the concrete surface forming a large number of comparatively small concrete fragments. The thickness of concrete between the rear (acceptor) surface of the element and the centroid of the rear face reinforcement is the usual depth of concrete dynamically disengaged from the element. Although the concrete between the layers of reinforcement may be cracked to some extent, it is confined by the flexural and lacing reinforcement, thus preventing its disengagement.

The size of the surface area which spalls depends upon the magnitude and duration of the applied blast loads striking and subsequently being transmitted through the element, in addition to the size and shape of the element itself. For long cantilever-type barricades, only a portion of the wall will usually spall, since the magnitudes of the applied blast pressures decrease rapidly along its length, while for cubicle-type structures, the entire wall surface will usually spall because of the amplification of the blast pressures due to their multiple reflections within the structures.

A wide range of velocities exists for spalled fragments. The initial velocity at which spalled fragments leave a structural element has been found to be low (50 feet per second or less). However, concrete elements subjected to the close-in effects of a detonation are generally accelerating before or soon after spalling takes place. This accelerated motion of the element in turn accelerates spalled fragments. The fragment velocities produced by these acceleration effects may be as high as several hundred feet per second. For analytical purposes, an upper limit for the velocities of direct spalled fragments from elements sensitive to impulse may be taken as the initial velocity of the element which is also assumed to be the maximum velocity. However, for elements which respond to the pressure only or pressure-time relationship, an evaluation of the resistance-time and pressure-time curves must be performed to obtain the maximum fragment velocity. The procedures and equations that are necessary to determine the above velocities are contained in Section 4-58.

4-54. Scabbing

Scabbing of reinforced concrete elements (Fig. 4-64) is the end result of a tension failure in the concrete normal to its free surface and is associated with large deflections. In the later stages of the ductile response mode of a reinforced concrete element, extremely large deflections are developed producing large strains in the flexural reinforcement and, consequently, severe cracking and/or crushing of the concrete perpendicular to the free surfaces. Because the tension and compression strains are highest at the surface and since the lacing reinforcement in the later stages of deflection confines the concrete between the layers of flexural reinforcement, damage to the concrete is more severe at the exterior of the reinforcement than between the layers. The applied loads having long since passed, the element is in a stage of deceleration at these large deflections. Therefore, the velocities of scabbed fragments, which are equal to the velocity of the element at $\theta = 5°$ (start of scabbing), are lower than the velocities of accelerated direct spalled fragments. However, the velocities of scabbed fragments also may be in the order of several hundred feet per second. Refer to Section 4-58 to determine the velocity of the element at a support rotation of five degrees.

4-55. Prediction of Concrete Spalling

As previously explained, direct spalling is due to a compression wave traveling through a concrete element, reaching the back face and being reflected as a tension wave. Spalling occurs when the tension is greater than the tensile strength of the concrete. Spalling will occur:

for: $$\frac{Vi_r}{T_c\ P_r} \leq 1.0 \quad \text{when} \quad \frac{P_r}{\sigma_u} \geq 1.0 \qquad \text{4-178}$$

or for:

$$\frac{V i_r}{T_c\ P_r} \leq 1.0 \quad \text{when} \quad \frac{P_r}{\sigma_u}\ \frac{T_c\ P_r}{V i_r} \geq 1.0 \qquad \text{4-179}$$

where:

$$\sigma_u = 0.1\ f'_c \qquad \text{4-180}$$

$$V = (E_c/\rho)^{1/2} \qquad \text{4-181}$$

and:

P_r = peak normal reflected pressure

σ_u = tensile strength of concrete

V = velocity of compression wave through concrete

i_r = normal reflected impulse

T_c = thickness of concrete element

f'_c = static compressive strength of concrete

E_c = modulus of elasticity of concrete

ρ = mass density of concrete

Equations 4-178 and 4-179 are shown graphically in Figure 4-65. Use of this figure predicts the incidence of spalling, that is, whether or not spalling will occur at the point on the element which is subjected to the peak normal reflected pressure P_r and impulse i_r.

If spalling is predicted, the spalled area cannot be calculated from the above data. However, the spalled area may be qualitatively estimated by considering the distance of the plotted point above the line on Figure 4-65. The greater the distance above the line, the larger the spalled area is likely to be. In addition, if the average reflected pressure P_b and impulse i_b are substituted for P_r and i_r respectively, and spalling is still predicted, then the spalled area will likely include the major portion, if not all, of the element's surface.

If a concrete element is subjected to side-on pressures only, Figure 4-65 may still be used to predict the occurrence of spalling. In this case, the peak reflected pressure P_r and impulse i_r are replaced with peak side-on pressure P_{so} and impulse i_s, respectively.

4-56. Minimization of Effects of Spalling and Scabbing

If it is determined that concrete spall due to blast loading will occur (using the procedures outlined in the preceding Section), or that scabbing will occur due to large deflections (>5°), then there are several procedures which can be utilized to minimize its effects.

4-56.1. Design parameters

The occurrence of direct spalling can be eliminated by an adjustment of the charge location, i.e., if an explosive charge is placed at a sufficient distance away from the surface of an element, the magnitude of the blast pressures striking the element will be less than those which will cause tension failure of the concrete. Large adjustments of the donor charge location in a design for the sole purpose of preventing direct spalling is usually not economically feasible.

Although sufficient separation distance between a detonation and an element prevents direct spalling, its effect in reducing scabbing is negligible. A reduction of scabbing is accomplished by limiting the magnitude of the maximum deflection of the element. By reducing this deflection, the strains in the concrete and reinforcement are lowered to a level where tensile failure of the concrete and subsequent scabbed fragment formation is prevented. Scaled tests have indicated that scabbing does not occur when deflections are limited to values less than those corresponding to support rotations of no larger than five degrees. To maintain the same response of the element, the resistance-mass product of the element must be increased proportionally to the decrease in deflection, and this capacity (resistance and/or mass) increase results in an increased construction cost.

4-56.2. Composite Construction

Direct spalling and scabbing can be eliminated through the use of composite elements composed of two concrete panels (donor and acceptor) separated by a sand-filled cavity. Spalling of the donor panel is not generally of concern since resulting fragments enter and are trapped in the sand fill. On the other hand, spalling of the receiver panel will endanger the acceptor system. By maintaining certain design parameters, both direct spalling and scabbing of the receiver panel can be prevented.

To prevent the occurrence of direct spalling, the high peak blast pressures applied to the donor panel of a composite element must be attenuated by the sand fill. This attenuating capability of the sand is attained by providing: (1) a thickness of the sand fill at least equal to twice the thickness of the donor panel where the panel thickness is predicated upon the required strength to resist the applied loads, (2) for one-way elements a ratio of the cavity thickness to span length not less than 0.25 for cantilevers and 0.05 for elements fixed on two opposite sides shall be used, and for two-way elements each direction (span) shall be considered separately as shown above and the larger cavity thickness used, and (3) the sand density shall not be greater than 85 pounds per cubic foot. Since scabbing is eliminated by limiting the element's deflection, scabbing of composite elements is prevented by limiting the deflection of the acceptor panel to the support rotation previously cited.

Figure 4-66 illustrates the use of composite construction to prevent spalling. The magnitude of the donor panel deflection was such that the panel was near incipient failure, the panel experiencing the effects of both direct spalling and scabbing. On the other hand, the deflection of the acceptor panel has been limited and, therefore, had only minor cracking. Direct spalled or scabbed fragments were not formed on the exterior surface of the acceptor panel.

The use of composite barriers for the sole purpose of eliminating spalling is not usually economically feasible. However, if the magnitude of the applied blast loads warrant the use of composite construction, then the elimination of spalling can be achieved at a slight increase in cost by conforming to the previously stated element configurations and response.

4-56.3. Fragment Shields

4-56.3.1. General

Fragment shields are composed of steel plates or other structural material which can be attached to, or placed a short distance from, a protective barrier (fig. 4-67). Unlike the other methods, the use of shields does not reduce or deter the formation of spalled fragments but rather confines and prevents them from striking the acceptor system.

4-56.3.2. Attached Fragment Shield

If the permissible maximum deflection of a barrier is relatively small, then steel plates, blast mats, or other similar material attached rigidly to the barrier may be used to confine the concrete fragments, thus preventing their ejection from the barrier (Fig. 4-67a). It is recommended that for rigidly attached shields the deflection be limited to a five-degree support rotation to prevent failure of the shield and its connections as a result of excessive straining.

The velocity of spalled fragments due to the transmission of air blast is small. However, spalling occurs during the initial response of the element. Since the element is in motion when spalling occurs, the spalled fragments are actually being pushed by the concrete element. After the element reaches maximum velocity, the element and the attached shield decelerate due to the flexural resistance of the concrete element. The attached shield, therefore, decelerates the spalled fragments which are confined between the shield and the unspalled portion of the concrete. The maximum deceleration of the element and, consequently, of the confined spall fragments is given by:

$$a = r_u/m_u \qquad \text{4-182}$$

where:

a = deceleration of the structural element

r_u = ultimate unit resistance of concrete element

m_u = effective unit mass of the concrete element in the plastic range

The force acting on the shield is due to the inertial force of the fragments. Thus, the required resistance of the fragment shield must be equal to this inertial force or:

$$r_{fs} = m_{sp}\, a \qquad \text{4-183}$$

where:

r_{fs} = ultimate unit resistance of fragment shield

m_{sp} = mass of the spalled fragments

When calculating the mass of the spalled fragments the actual mass of the disengaged concrete is used. It is assumed that all the concrete from the rear face of the element to the centroid of the rear face reinforcement disengages. Thus, the spall thickness for a laced section is:

$$d_{sp} = (T_c - d_c) / 2 \qquad \text{4-184}$$

where:

d_{sp} = depth of spalled concrete

T_c = thickness of the concrete element

d_c = average distance between the centroids of the compression and tension reinforcement

An attached fragment shield usually consists of a flat or corrugated steel plate spanning between angles or channels. The angles or channels act as beams spanning between anchor bolts. The anchor bolts connect the plate and beams to the unspalled portion of the concrete. To insure that they do not fail due to concrete pull-out, the anchor bolts are hooked around the flexural reinforcement as shown in Figure 4-68. If the required resistance of the fragment shield is small, the fragment shield may be considered a two-way spanning member without the supporting beams. In such cases, the shield is designed as a flat slab with the anchor bolts supporting it at multiples of the flexural bar intersections. For larger resistances, the plate thickness would become excessive and the design uneconomical.

4-56.3.3. Separated Fragment Shield

If the barrier is permitted to attain large deflections (greater than five-degree support rotation) then the shields must be separated from the barrier. This separation distance should be sufficient to eliminate the possibility of impact between the deflecting barrier and shield. Fragment shields, separated from the main protective elements and affording protection against spalled fragments from the walls and/or roof, are shown in Figure 4-67b. The shield may consist of structural steel, reinforced concrete, wood, etc., and must be designed to resist the impact and penetration of the spalled fragments as well as the overall motion of the main protective structure and any leakage pressure which may occur.

To design a separated shield, the mass and velocity of the spalled fragments must be determined. As an approximation, an average velocity for all spalled fragments can be utilized. The average velocity of the spalled fragments is taken as equal to the maximum velocity of the element (single-degree-of-freedom system) so that:

$$v = i_b/m_u \qquad \text{4-185}$$

where:

v = average velocity of spalled fragments

i_b = blast impulse

m_u = effective unit mass of the concrete element in the plastic range

The impulse imparted to the fragment shield by the spalled fragments is equal to their momentum or:

$$i_{fs} = m_{sp} V \qquad 4\text{-}186$$

where:

i_{fs} = required impulse capacity of the fragment shield

m_{sp} = mass of the spalled fragments (Section 4-56.3.2)

The shield is designed to resist this impulse, i_{fs}.

The cost of separated shields may be somewhat more expensive than shields attached directly to the barrier. However, the cost reduction achieved by permitting the larger barrier deflections may offset the increased cost of the separated shield.

4-57. **Post-Failure Concrete Fragments**

When a reinforced concrete element is substantially overloaded by the blast output, the element fails and concrete fragments (post failure) are formed and displaced at high velocities. The type of failure as well as the size and number of the fragments depends upon whether the element has laced or unlaced reinforcement.

Failure of an unlaced element (Fig. 4-69) is characterized by the dispersal of concrete fragments formed by the cracking and displacement of the concrete between the donor and acceptor layers of the reinforcement. With increased deflections, these compression forces tend to buckle the reinforcement outward thereby initiating the rapid disintegration of the element.

Laced concrete elements exhibit a different type of failure from unlaced elements, the failure being characterized by reinforcement failures occurring at points of maximum flexural stress (plastic hinges) with the sections of the element between the points of failure remaining essentially intact. When fracture due to excessive straining of the tension reinforcement occurs at the positive yield lines, some small concrete fragments will be formed at the acceptor side of the barrier. Quite often, if the overload is not too severe, the compression reinforcement at the hinge points does not fail and thereby prevents total disengagement of the sections between the hinges (Fig. 4-70). In cubicle type structures where continuous laced and flexural reinforcement is used throughout, failure is sometimes initiated at the positive yield lines where flexural and lacing reinforcement fail, while at the supports, only the tension reinforcement fails. The intact sections between the failure points rotate with the compression reinforcement at the supports, acting as the mechanical hinges of an analogous swinging door (Fig. 4-71). The compression reinforcement at the supports, serving as hinges, produces rotational rather than translational motion of the failed sections, and energy which would ordinarily result in translational velocities is transferred to sections of the structure adjacent to the failed element. In other situations, where there is a larger overloading of the element, the failed sections of the laced element are completely disengaged and displaced from the structure. The translational velocities of these sections are usually less than the maximum velocity of the element at incipient failure.

4-58. Post-Failure Impulse Capacity

4-58.1. General

Elements which protect non-sensitive explosives may be designed for controlled post-failure fragments with a substantial cost savings. These elements fail completely, but detonation is prevented by limiting the mass and velocity of the fragments. Barriers and shelters which will protect personnel, equipment and/or sensitive explosives cannot be designed for post-failure criteria. Procedures are presented below for determining the post-failure impulse capacity of laced elements.

4-58.2. Laced Elements

4-58.2.1. General

The idealized curves of Figure 4-72 illustrate the response of an impulse-sensitive two-way element when the applied blast impulse load is larger than it flexural impulse capacity (area under the resistance-time curve, Fig. 4-72a). The assumptions made in these curves are the same as those for impulse-sensitive systems whose response is less than or equal to incipient failure (Vol. III) namely: (1) the element prior to being loaded is at rest, and (2) the duration of the applied blast load and the time to reach to yield are small in comparison to the time to reach the ultimate deflection. In Figure 4-72 the duration of the applied blast load and time to reach yield have been taken as equal to zero so that the element will respond and reach its maximum velocity instantaneously (i.e., at $t_o = 0$, $v_o = i_b/m_u$).

When the element is designed to remain intact (equal to or less than incipient failure conditions), its velocity at time tu (deflection X_u) is equal to zero. However, if the element is overloaded, then the velocity just prior to failure is a finite value, the magnitude of which depends upon: (1) the magnitude of the overload, (2) the magnitude of the flexural capacity, and (3) mass of the element.

When laced elements are overloaded, failure occurs at the hinge lines and the element breaks into a small number of large sections. The magnitude of the velocity of each sector at failure varies from a maximum at the point of maximum deflection to zero at the supports. The variation of the fragment velocity across a section produces tumbling. This tumbling action may result in an acceptor charge which is located close to the barrier being stuck by that portion of a failed sector traveling at the highest velocity. If the acceptor charge is located at a considerable distance from the barrier, the velocity of the fragments should be taken as the translational velocity. The translational velocity is approximately equal to the average velocity of the sector before failure (the average momentum of the element before failure is equal to the average momentum after failure).

The analytical relationship which describes the response of a laced element, both in its flexural and post-failure ranges of action, is obtained through a semigraphical solution of Newton's equation of motion similar to that described for incipient failure design in Chapter 3.

If the areas under the pressure-time and resistance-time curves (Fig. 4-72a) are considered to be positive and negative, respectively, and the velocity of the system before the onset of the load is zero, the summation of the areas at any time divided by the appropriate effective mass for each range is equal to the instantaneous velocity at that time. The velocity vi at the incipient failure deflection X_u (time, t_u) may be expressed as:

$$v_i = \frac{i_b}{m_u} - \frac{r_u t_1}{m_u} - \frac{r_{up} (t_u - t_1)}{m_{up}} \qquad \text{4-187}$$

where the values of i_b, r_u, t_1 and t_u are defined in Figure 4-72 and m_u and m_{up} are the effective masses of the single-degree-of-freedom system in the various flexural ranges (ultimate and post-ultimate) of the two-way element. The acceptor charge is assumed to be close to the barrier, so that the maximum velocity of the fragment after failure v_f is equal to the maximum velocity of the element at incipient failure v_i.

The expression for the deflection at any time may be found by multiplying each differential area (between the time t_o and the time in question), divided by the appropriate effective mass, by the time which is defined by the distance between the centroid of the area and the time in question, and adding these values algebraically. Using this procedure and the expression for the deflection at partial failure (initial failure of a two-way element) the deflection at time t_1 is:

$$X_1 = \frac{i_b t_1}{m_u} - \frac{r_u t_1^2}{2m_u} \qquad \text{4-188}$$

while the equation for the deflection at incipient failure at t_u is given by:

$$X_u = \frac{i_b t_u}{m_u} - \frac{r_u t_1}{m_u}\left[t_u - \frac{t_1}{2}\right] - \frac{r_{up} (t_u - t_1)^2}{2m_{up}} \qquad \text{4-189}$$

4-58.2.2. Post-Failure Impulse Capacity

The expression for the blast overload impulse capacity of a two-way element includes both the flexural capacity and the post-failure fragment momentum portion of the element's response. Solving Equations 4-187 through 4-189 simultaneously gives the expression for the blast overload impulse capacity. For a two-way element the resulting equation is:

$$\frac{i_b^2}{2m_u} = r_u X_1 + \left[\frac{m_u}{m_{up}}\right] r_{up} \; (X_u - X_1) + \frac{m_u v_f^2}{2} \qquad \text{4-190}$$

For a one-way element or a two-way element which does not exhibit a post-ultimate range, the blast overload impulse capacity is:

$$\frac{i_b^2}{2m_u} = r_u X_u + \frac{m_u \ v_f^2}{2} \qquad 4\text{-}191$$

4-58.2.3. **Response Time**

The response time is the time at which the elements reach the ultimate deflection X_u, and failure occurs. The expression for the response time t_u is found by solving Equations 4-187 and 4-188 simultaneously. The response time for a two-way element is:

$$t_u = \frac{i_b}{m_u} + \left[\frac{m_{up}}{m_u \ r_{up}} - \frac{1}{r_u} \right] \left[i_b^2 - 2m_u \ r_u \ X_1 \right]^{1/2} - \frac{m_u}{r_{up}} \ v_f \qquad 4\text{-}192$$

The response time of a one-way element or for a two-way element which does not exhibit a post-ultimate range is:

$$t_u = \frac{i_b}{r_u} - \left[\frac{m_u}{r_u} \right] v_f \qquad 4\text{-}193$$

4-58.2.4. **Design Equations**

The basic equations for the analysis of the blast impulse capacity of an element are given above. However, the form of these equations is not suitable for design purposes. The use of these equations would require a tedious trial-and-error solution. Design equations can be derived in the same manner as for elements designed for incipient failure or less (see Sect. 4-33).

If Equations 4-190 and 4-191 are compared with Equations 4-95 and 4-96, it may be seen that except for the right-hand term in each of the above equations, the corresponding analytical expressions are the same if X_u is substituted for X_m The additional term in the above equations is the kinetic energy of the fragments after failure. Because of this similarity, Equations 4-190 and 4-191 may be expressed in a form which will be a function of the impulse coefficients (section 4-33), the geometry of the element, applied blast impulse and a post-failure fragment coefficient. The resulting equation is:

$$i_b^2 = C_u \left[\frac{p_H \ d_c^3 f_{ds}}{H} \right] + C_f \ d_c^2 \ v_f^2 \qquad 4\text{-}194$$

where

i_b = applied unit blast impulse

p_H = reinforcement ratio in the horizontal direction

d_c = distance between centroids of the compression and tension reinforcement

f_{ds} = dynamic design stress for the reinforcement

H = span height

v_f = maximum velocity of the post-failure fragments

C_u = impulse coefficient for ultimate deflection X_u

C_f = post-failure fragment coefficient

Equation 4-194 is applicable to both one-way and two-way elements which are uniformly loaded. Values of C_u can be found in Section 4-33. The post-failure fragment coefficient takes into account the variation in the effective mass. It is a function of the element's horizontal and vertical reinforcement ratios, aspect ratio and boundary conditions. To facilitate the design procedure, values of the post-failure fragment coefficient C_f have been plotted in Figures 4-73, 4-74 and 4-75 for two-way elements supported on two adjacent edges, three edges and four edges respectively. For one-way elements, the value of coefficient C_f is a constant (C_f = 22,500).

4-58.2.5. Optimum Reinforcement

The optimum arrangement of the flexural reinforcement in two-way elements designed for post-failure fragments will not necessarily be the same as that for similar elements which are designed for incipient failure damage or less. The optimum ratio of the vertical to horizontal reinforcement p_V/p_H will be a function of the amount of the blast impulse absorbed through the flexural action of the element in comparison to that which contributes to the momentum of post-failure fragments. Unlike incipient failure design, the optimum ratio will vary with a variation in the depth of the element. As discussed earlier, the optimum depth of an element and total amount of reinforcement p_T is a function of the relative costs of concrete and reinforcing steel. In a given situation, the designer must establish, through a trial and error design procedure and a cost analysis, the optimum depth, reinforcement ratio p_V/p_H and total amount of reinforcement p_T.

For very large projects. this type of detailed analysis may result significant cost benefits, but for most projects using the values of p_V/p_H and p_T recommended for incipient failure design will yield an economical design.

Figure 4-63   Direct spalled element

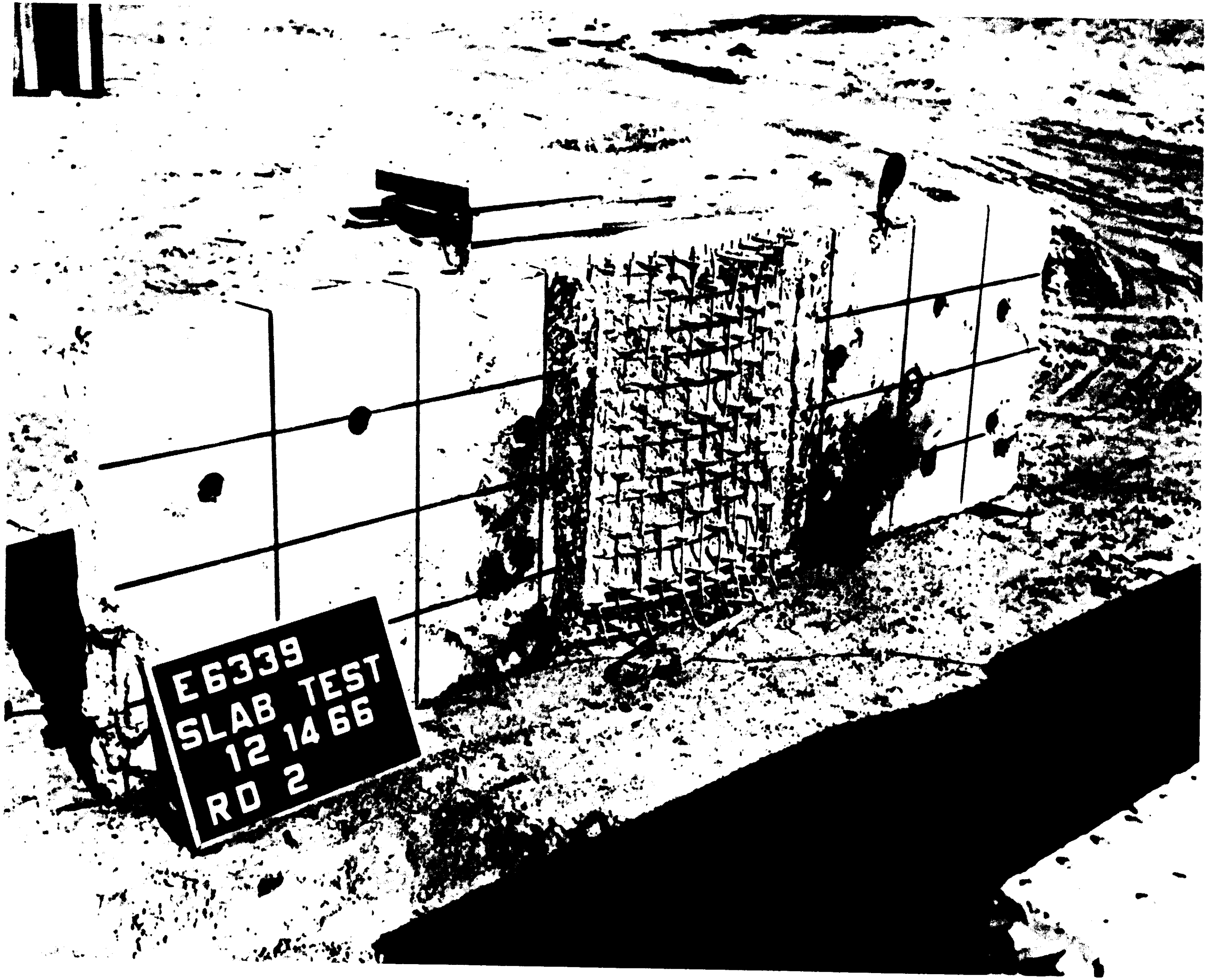


Figure 4-64 Scabbed element

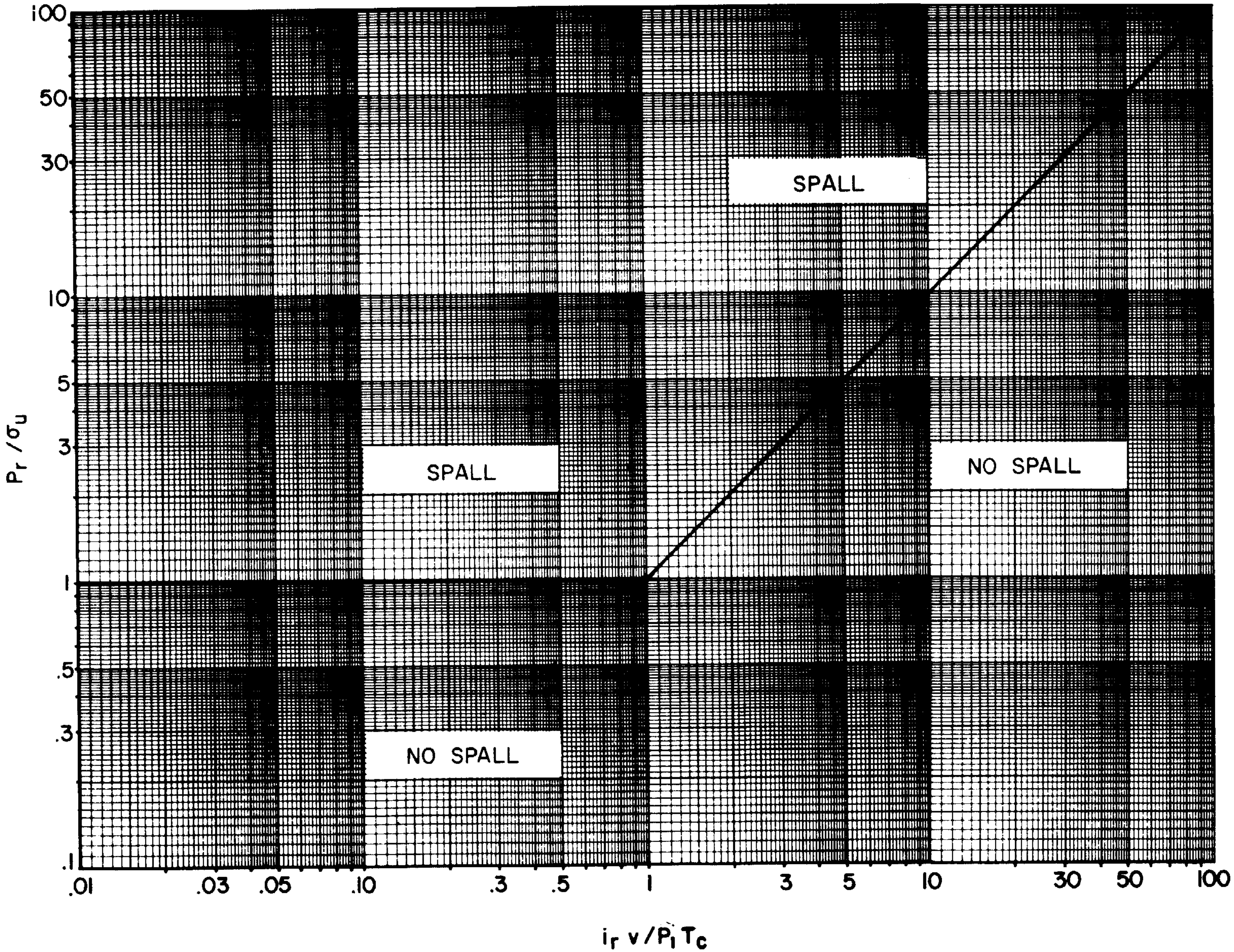


Figure 4-65 Spall threshold for blast waves loading walls

Figure 4-66 Unspalled acceptor panel of composite panel

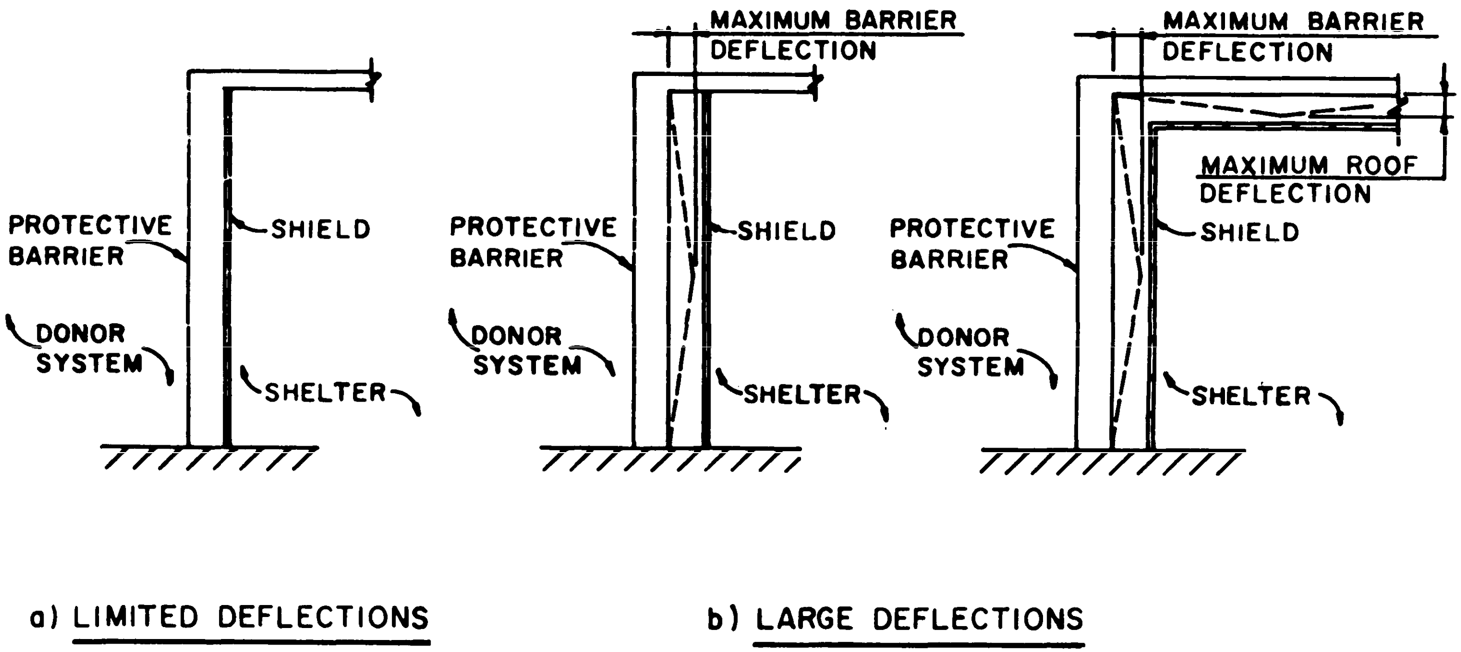


Figure 4-67 Sheilding systems for protection against concrete fragments

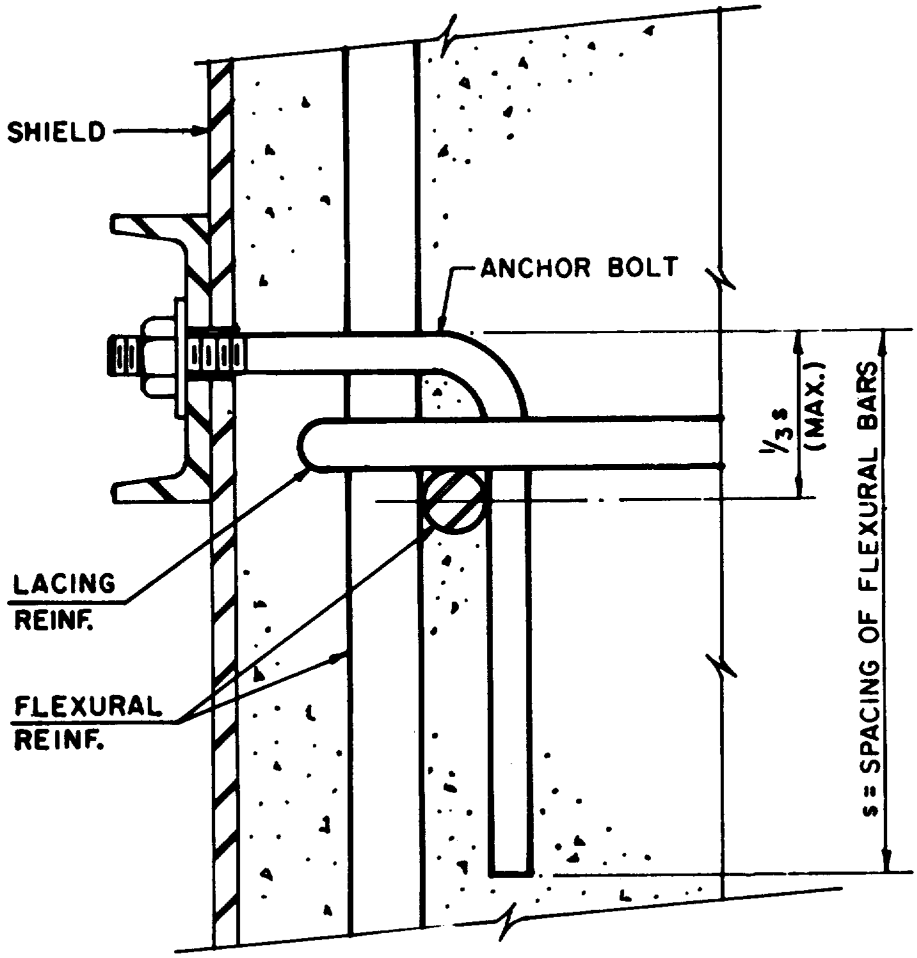


Figure 4-68 Rigid attachment of fragment shield to barrier

Figure 4-69   Failure of an unlaced element

Figure 4-70 Failure at plastic hinges of laced elements

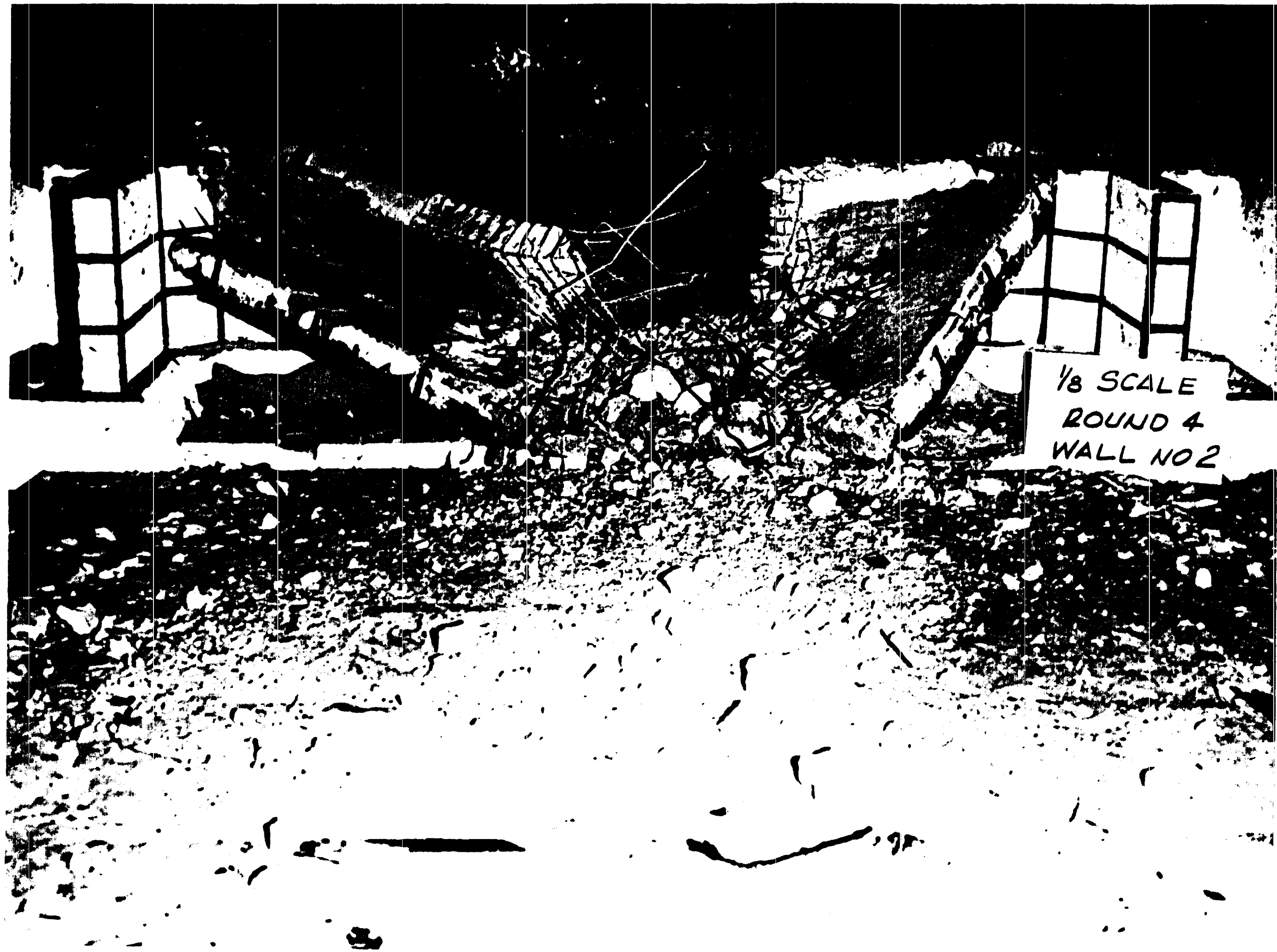
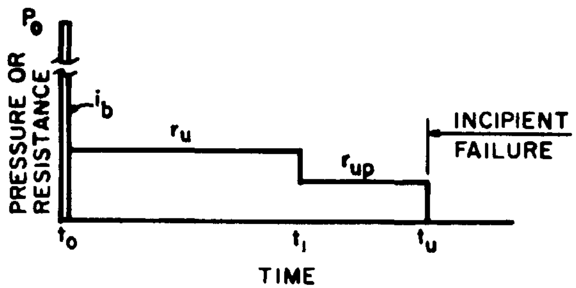
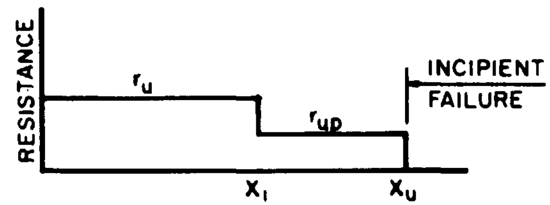


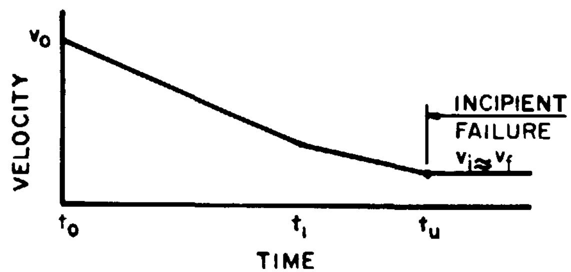
Figure 4-71 Backwall failure of cubicle with laced reinforcement



a) IDEALIZED PRESSURE – TIME AND RESISTANCE – TIME CURVES



C) IDEALIZED RESISTANCE - DEFLECTION CURVE



b) IDEALIZED VELOCITY - TIME CURVE

i_b = unit blast impulse
r_u = ultimate unit resistance
r_{up} = post-ultimate unit resistance
t_o = duration of positive phase of blast pressure
t_1 = time at which partial failure occurs
t_u = time at which ultimate deflection occurs
v_f = post-failure fragment velocity
v_i = velocity of element at incipient failure
v_o = initial velocity of element
X_1 = partial failure deflection
X_u = ultimate deflection

Figure 4-72 Idealized curves for determination of post-failure fragment velocities

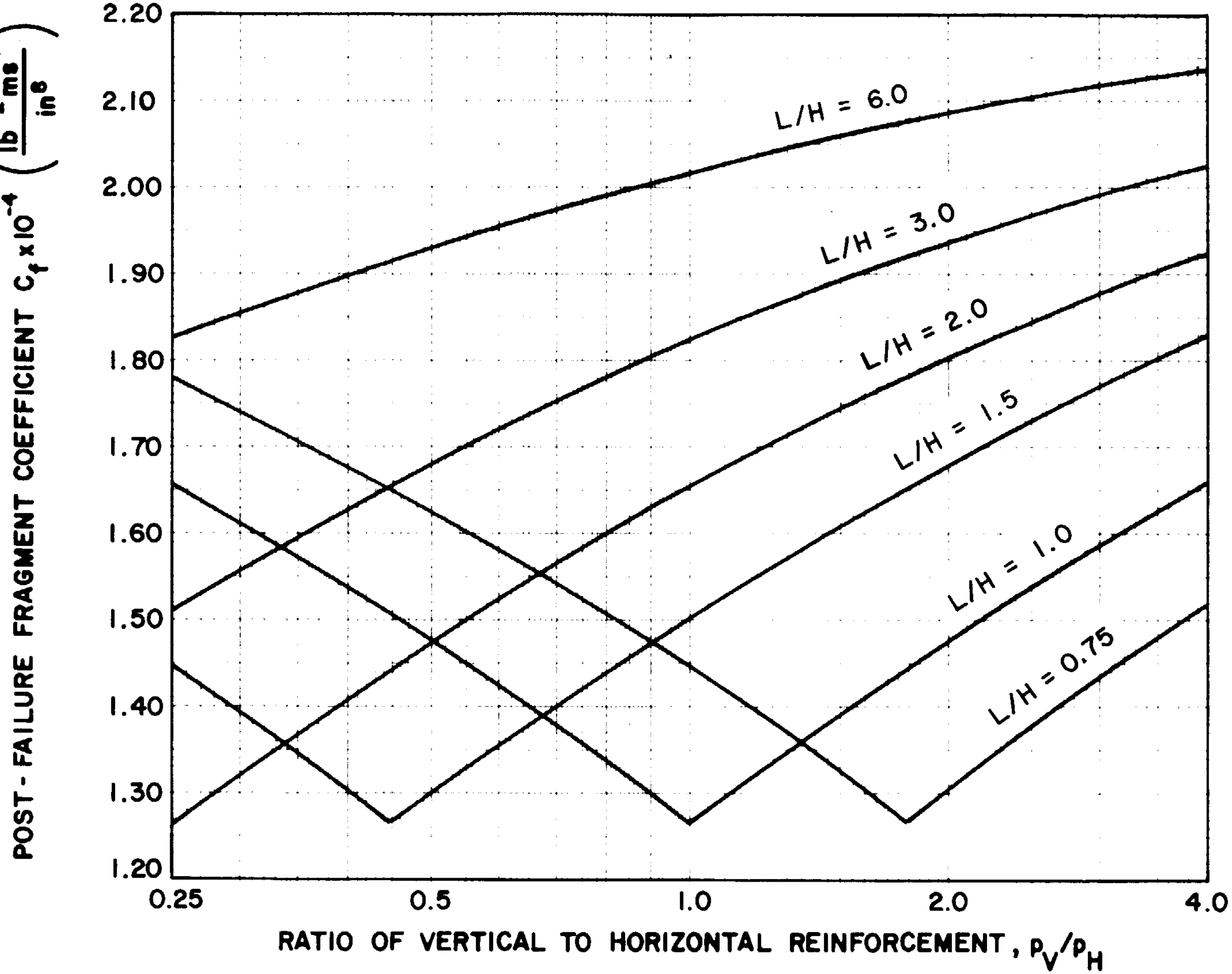


Figure 4-73 Post-failure coefficient C_f for an element fixed on two adjacent edges and two edges free

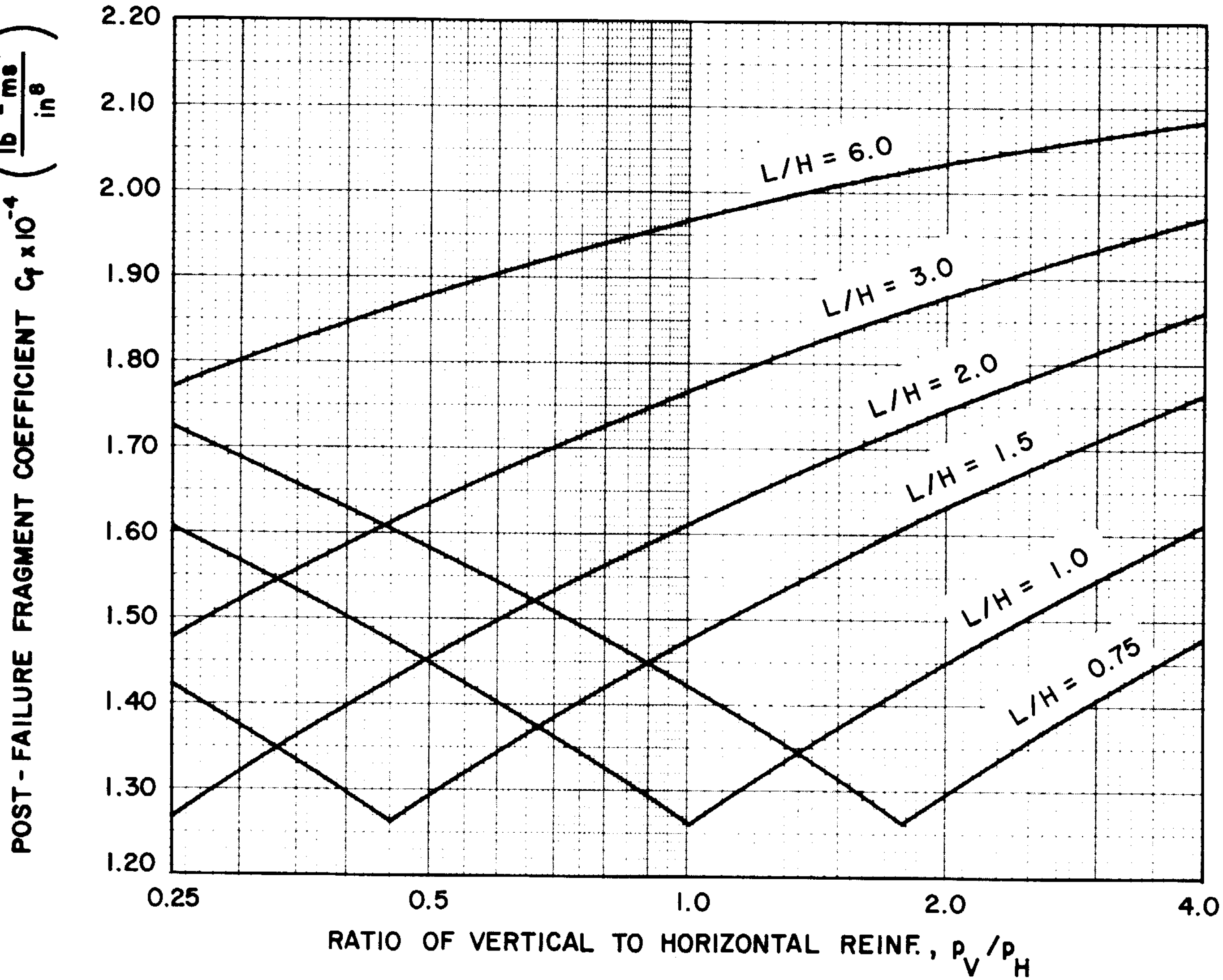


Figure 4-74 Post-failure coefficient C_f for an element fixed on three edges and one edge free

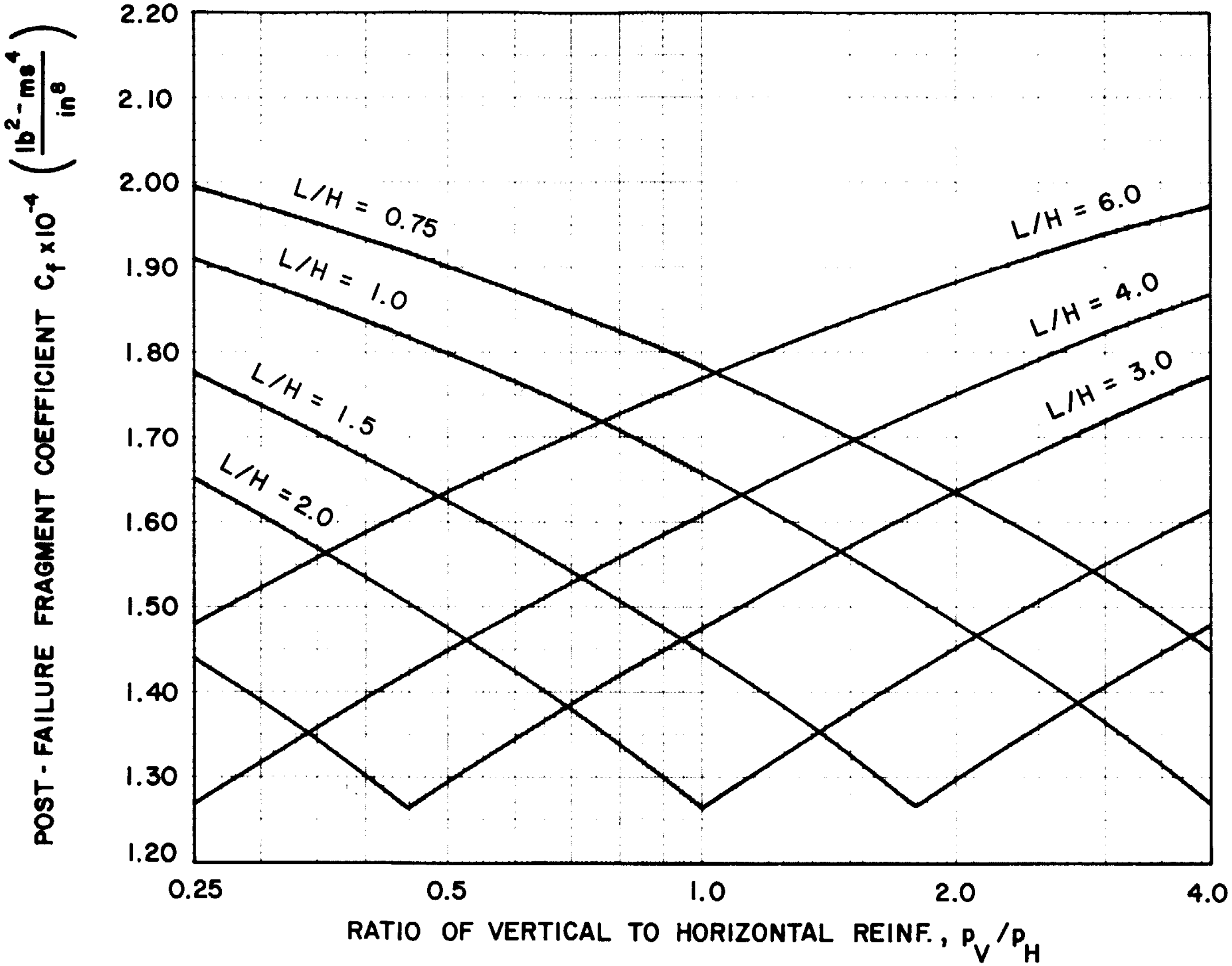


Figure 4-75 Post-failure coefficient C_f for an element fixed on four edges

STRUCTURAL BEHAVIOR TO PRIMARY FRAGMENT IMPACT

4-59. Introduction

4-59.1. Fragment Characteristics

Detonation of cased explosives results in the formation of primary fragments due to the shattering of the casing. These fragments are usually small in size and initially travel at velocities in the order of thousands of feet per second. Upon contact with a barrier, the fragments will either pass through (perforate), be embedded in (penetrate with or without spalling), or be deflected by the barrier. The resulting effect is dependent on the interaction of the following factors: (1) the magnitude of the initial velocity, (2) the distance between the explosion and the barrier, (3) the angle at which the fragment strikes the barrier (angle of obliquity), and (4) the physical properties of the fragment (mass, shape and material strength) and the barrier (concrete strength and thickness).

4-59.2. Velocity and Impact Limitations

This section deals with the situation where a fragment with given properties strikes a barrier element with a known velocity and where the angle of obliquity between the trajectory path of the fragment and a normal to the surface of the barrier is zero degrees (normal impact). The striking velocity of the fragment is assumed equal to its initial velocity for a detonation in close proximity to the element or is determined according to the procedure in Chapter 2 for cases where the fragment travels a distance greater than 20 feet.

4-60. Fragment Impact on Concrete

4-60.1.. General

When a primary fragment strikes a concrete barrier, penetration resisting pressures, in the order of thousands of psi, act on the cross-sectional area of the fragment. For a given mass of the fragment, as the striking velocity is increased, the resisting pressures also increase, while for an increase in the cross-sectional area of the fragment, the resisting pressures decrease. If the fragment can withstand these pressures acting on its frontal surface, then the amount of penetration will be governed by its mass, shape, and striking velocity. On the other hand, if the fragment deforms under the applied loads, then the resisting pressures in the concrete become effective over an increased cross-sectional area, thereby reducing the possible penetration for a given available kinetic energy of the fragment. Generally, larger penetrations may be expected with less ductile metals such as fragments from armor piercing casings.

As a fragment impinges on a wall surface, a section of the wall adjacent to the point of impact spalls, forming a crater around the impact area (Fig. 4-76). This crater is conical in shape but irregular. As the striking velocity of the fragment increases, the size of the crater also increases. At small velocities, the increase in the crater size for a given velocity increment is more rapid than at higher velocities in the order of several thousand feet per second. At a striking velocity of approximately 1,000 feet per second or less, the fragment does not usually penetrate beyond the depth of the crater,

while for larger velocities, the fragment penetrates beyond the front of the crater and either lodges within or passes through the barrier.
A crater similar to that formed on the front face of the barrier is also produced on the back side if the kinetic energy of the fragment upon impact is sufficient to produce excessive tensile stresses in the concrete. As in the case of the front-face crater the size of the crater on the back face increases with an increase in striking velocity of a particular fragment. The back-face crater is generally wider and shallower than the front-face crater, though again the surface of the crater is irregular. Quite often, the kinetic energy afforded by the striking fragment is only sufficient to dislodge the concrete on the exterior side of the rear face reinforcement. In this case, only spalling will occur. As the striking velocity is increased beyond the limit to cause spalling, the penetration of the fragment into the slab increases more and more until perforation is attained.

4-60.2. Penetration by Armor-Piercing Fragments

A certain amount of experimental data, which is analogous to primary fragment penetration, has been accumulated in connection with projects to determine the effects of bombs and projectile impact on concrete structures. This data has been analyzed in order to develop relationships for the amount of fragment penetration into concrete elements in terms of the physical properties of both the metal fragment and the concrete. A general expression for the maximum penetration into a massive concrete slab (i.e. a slab with infinite thickness) by an armor-piercing fragment has been obtained as follows:

$$X_f = 4.0 \times 10^{-3} (KND)^{0.5} d^{1.1} v_s^{0.9} \qquad \text{for } X_f \le 2d, \qquad 4\text{-}195$$

$$X_f = 4.0 \times 10^{-6} KND\, d^{1.2} v_s^{1.8} + d \qquad \text{for } X_f > 2d \qquad 4\text{-}196$$

and

$$K = 12.91/(f'_c)^{1/2} \qquad 4\text{-}197$$

where:

X_f = penetration by armor-piercing steel fragments

K = penetrability constant

N = nose shape factor as defined in Figure 4-77

D = caliber density as defined in Figure 4-77

d = fragment diameter

v_s = striking velocity

For the standard primary fragment and concrete strength f'_c equal to 4,000 psi, Equations 4-195 and 4-196 reduce to the following equations in terms of fragment diameter (in) and the striking velocity v_s (fps):

$$X_f = 2.86 \times 10^{-3} d^{1.1} v_s^{0.9} \qquad \text{for } X_f \le 2d \qquad 4\text{-}198$$

$$X_f = 2.04 \times 10^{-6} d^{1.2} v_s^{1.8} + d \qquad \text{for } X_f > 2d \qquad 4\text{-}199$$

Equations 4-198 and 4-199 can also be expressed in terms of fragment weight (standard shape) as:

$$X_f = 1.92 \times 10^{-3} W_f^{0.37} v_s^{0.9} \qquad \text{for } X_f \leq 2d \qquad 4\text{-}200$$

$$X_f = 1.32 \times 10^{-6} W_f^{0.4} v_s^{1.8} + 0.695 W_f^{0.33} \qquad 4\text{-}201$$

$$\text{for } X_f > 2d$$

Figure 4-78 is a plot of the maximum penetration of a standard fragment through 4,000 psi concrete for various fragment weights and striking velocities.

Maximum penetration of fragments in concrete of strengths other than 4,000 psi, may be calculated using the values of X_f from Equation 4-200, 4-201 or Figure 4-78 and the following equation:

$$X'_f = X_f \left[\frac{4,000}{f'_c} \right]^{1/2} \qquad 4\text{-}202$$

where:

X_f = maximum penetration into 4,000 psi concrete

X'_f = maximum penetration into concrete with compressive strength equal to f'_c

In addition to the weight and striking velocity of a primary fragment, its shape will also affect the resulting penetration. The sharper the leading edge of a fragment, the greater the distance traveled through the concrete. The shape indicated in Figure 4-77 is not necessarily the most critical. When the container of an explosive shatters, it is statistically probable that some of the resulting fragments will have a sharper shape than the standard bullet shape assumed in this manual. However, the number of these fragments is usually very small in comparison to the total number formed, and the probability that these sharper fragments will have normal penetrations though the concrete is low. In most instances, the majority of the primary fragments will have a more blunt shape than that shown. Therefore, for design purposes, the normal penetrations defined for a bullet-shaped fragment can usually be assumed as critical.

4-60.3. Penetration of Fragments Other than Armor-Piercing

To estimate the concrete penetration of metal fragments other than armor-piercing, a procedure has been developed where the concrete penetrating capabilities of armor-piercing fragments have been related to those of other metal fragments. This relationship is expressed in terms of relative metal hardness (the ability of the metal to resist deformation) and density, and is represented by the constant in Equation 4-203.

$$X'_f = k X_f \qquad 4\text{-}203$$

where:

X'_f - maximum penetration in concrete of metal fragments other than armor-piercing fragments

k - constant depending on the casing metal, from Table 4-16

X_f - maximum penetration of armor-piercing fragment

It should be noted that X_f is calculated from Equation 4-200, 4-201 or Figure 4-78 if f'_c = 4,000 psi, and from Equation 4-202 when X_f is modified for concrete strengths other than 4,000 psi.

4-60.4. Perforation of Concrete

Quite often the magnitude of the initial kinetic energy of primary fragments will be large enough to produce perforation of the concrete. The depth of penetration X_f of a fragment into massive concrete is less than into a wall of finite thickness due to the high resisting stresses afforded by the massive concrete. Consequently, the concrete thickness required to prevent perforation is always greater than the depth of penetration X_f into massive concrete. The minimum thickness of concrete required to prevent perforation can be expressed in terms of the equivalent depth of penetration into massive concrete and the fragment size according to the following relationship:

$$T_{pf} = 1.13\ X_f\ d^{0.1} + 1.311\ d \qquad 4\text{-}204$$

where:

T_{pf} - minimum thickness of concrete to prevent perforation by a given fragment

X_f - depth of penetration corrected for concrete strength and fragment material

Fragments which perforate a concrete element will have a residual velocity which may endanger the receiver system. The magnitude of this velocity may be approximated from the expression which defines the velocity of the fragment at any time as it penetrates the concrete.

For cases where X_f is less than 2d:

$$v_r / v_s = \left[1 - (T_c / T_{pf})^2 \right]^{0.555} \qquad 4\text{-}205$$

and for cases where X_f is greater than 2d:

$$v_r / v_s = \left[1 - (T_c / T_{pf}) \right]^{0.555} \qquad 4\text{-}206$$

where:

T_c - thickness of the concrete, less than or equal to T_{pf}

v_r - residual velocity of fragment as it leaves the element

Plots of v_r / v_s against T_c / T_{pf} according to Equations 4-205 and 4-206 are presented in Figures 4-79 and 4-80, respectively.

4-60.5. Spalling due to Fragment Impact

When a primary fragment traveling at a high velocity strikes the donor surface of a concrete barrier, large compression stresses are produced in the vicinity of the point of impact. These stresses form a compression wave which travels from the impact point, expanding spherically until it reaches the back face element. At this free surface, the compression wave is reflected (reversed in direction and changed from a compression wave to a tension wave). When the stresses in the resulting tension wave exceed the tensile capacity of the concrete, spalling of the concrete at the receiver surface occurs. The spalling forms a crater on the receiver surface. This crater does not usually penetrate beyond the reinforcement at the receiver surface.

The occurrence of spalling is a function of the fragment penetration; i.e., the fragment must penetrate a barrier element to such a depth that sufficiently large stresses are formed at the receiver surface to produce spalling. If the thickness of the element is increased above the critical thickness at which spalling occurs for a particular fragment, then the spalling is eliminated. On the other hand, concrete spalling always occurs with fragment perforation. The minimum thickness of concrete barrier required to prevent spalling due to penetration of a given fragment can be expressed in terms of the fragment penetration into massive concrete and the fragment diameter:

$$T_{sp} = 1.215\ X_f\ d^{0.1} + 2.12d \qquad \text{4-207}$$

where:

T_{sp} = minimum concrete thickness to prevent spalling

X_f = depth of penetration corrected for concrete strength and fragment material

The secondary fragment velocities associated with spalling resulting from fragment impact are usually small (less than 5 fps). However, when blast pressures are also involved, the magnitude of the resulting velocities can be quite large. The secondary concrete fragments are accelerated by the motion of the barrier resulting in possible fragment velocities up to several hundred feet per second.

Because of the potentially large secondary fragment velocities associated with primary fragment impact, full protection is usually required for personnel, valuable equipment, and sensitive explosives. This protection may be accomplished either by providing sufficient concrete thickness to eliminate spalling or by other mechanical means used to minimize the effects of spalling resulting from blast pressures as described in section 4-56. The required concrete thickness to eliminate spalling caused by primary fragment impact may be obtained from Equation 4-207.

4-61. Fragment Impact on Composite Construction

4-61.1. General To evaluate the effect of primary fragments on composite (concrete-sand-concrete) barriers, the penetration of the fragment through both the concrete and sand must be considered. For damage to be sustained at the rear of a composite barrier a fragment must first perforate both the donor concrete panel and the sand, and then penetrate or perforate the receiver panel. If the fragment penetrates only part of the way through the receiver panel, then spalling may or may not occur, depending on the panel thickness. Obviously, fragment perforation of the receiver panel indicates perforation of the entire barrier.

4-61.2. Penetration of Composite Barriers

To determine the degree of damage at the receiver side, the penetration of the fragment in each section of the barrier must be investigated in sequence. Starting with the striking velocity and the weight of the primary fragment, the donor concrete section is first analyzed to determine whether or not perforation of that section occurs. If the calculations indicate that the fragment will stop within this section, then no damage will be sustained by the remainder of the barrier. On the other hand, if perforation does occur in the forward (donor) section, then the fragment penetration through the sand must be investigated.

The amount of the penetration through the sand depends upon the magnitude of the residual velocity as the fragment leaves the rear of the donor panel. This residual velocity is determined from Figures 4-79 and 4-80 utilizing the striking velocity, the thickness of the donor panel, and the theoretical maximum fragment penetration obtained from Figure 4-78.

The maximum penetration through massive sand is obtained from Equation 4-208, using the residual velocity calculated above as the striking velocity of the fragment on the surface of the sand.

$$X_s = 1.188\ Dd\ \ln\ (1 + 2.16 \times 10^{-3}\ v_s^2\ ) \qquad 4\text{-}208$$

where:

X_s = penetration of the fragment into the sand

Substituting the caliber density D for a standard shape (Fig. 4-77), Equation 4-208 becomes:

$$X_s = 3.53d\ \ln\ (1 + 2.16 \times 10^{-3}\ v_s^2) \qquad 4\text{-}209$$

A plot of Equation 4-209 for a range of fragment weights and striking properties is shown in Figure 4-81. If the penetration in the sand is found to be less than the thickness of the sand layer, no damage is sustained at the rear surface of the barrier. In case of perforation, the penetration of the fragment into the rear section (receiver) of the barrier is governed by the residual velocity as the fragment leaves the sand. This residual velocity is calculated in a manner similar to that used for computing the residual velocity for the donor panel, except that the fragment penetration and striking velocity are those associated with movement of the fragment through the sand, that is:

$$v_r\ /\ v_s = [1 - T_s\ /\ X_s\ ]^{0.555} \qquad 4\text{-}210$$

where:

T_s = actual thickness of the sand layer

Figure 4-80 can be used to calculate the residual velocity of a fragment perforating the sand layer.

Similar to the fragment penetration through the donor panel and sand, penetration of the fragment through the receiver panel is a function of the magnitude of the fragment velocity as the fragment strikes the forward surface of the panel. This velocity is equal to the residual velocity as the fragment leaves the sand. Once the penetration in the receiver panel is known, then the damage sustained at the rear of the composite barrier can be defined in terms of either spalling or fragment perforation.

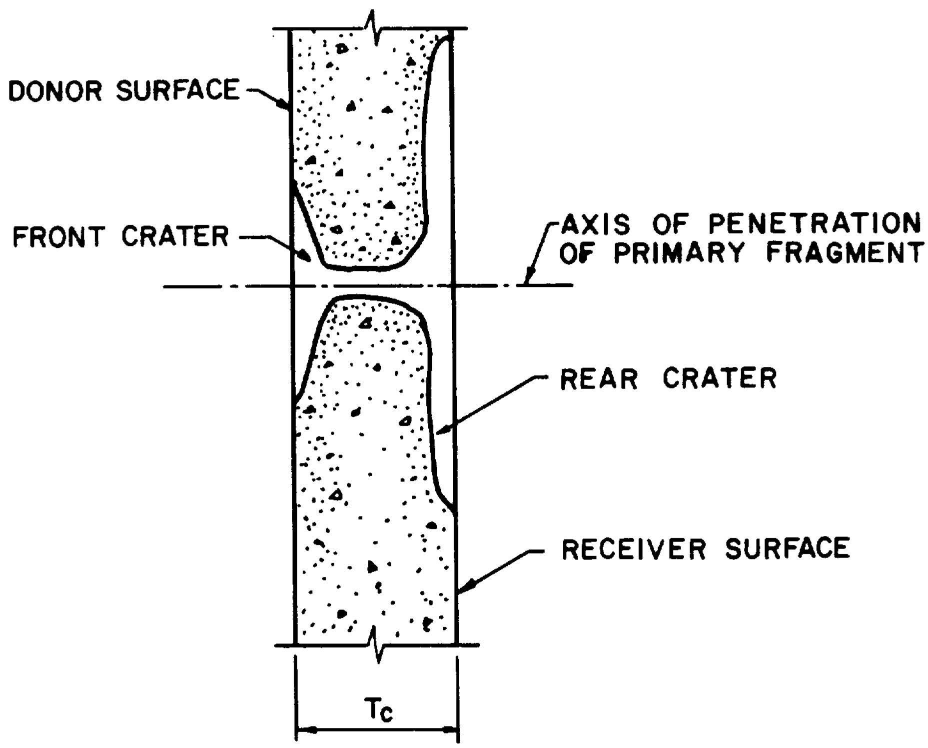
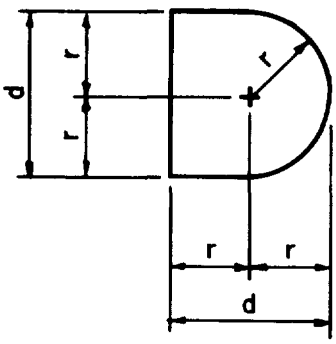


Figure 4-76 Perforation and spalling of concrete due to primary fragments



d = DIAMETER OF CYLINDRICAL PORTION OF FRAGMENT

r = RADIUS OF HEMISPHERICAL PORTION OF FRAGMENT

$r = \frac{d}{2}$

D = CALIBER DENSITY, W_f/d^3 = 2.976 oz./in.^{3}

W_f = FRAGMENT WEIGHT

N = NOSE SHAPE FACTOR = $0.72 + 0.25\sqrt{n - 0.25} = 0.845$

n = CALIBER RADIUS OF THE TANGENT OGIVE OF THE FRAGMENT NOSE = r/d

Figure 4-77 Shape of standard primary fragments

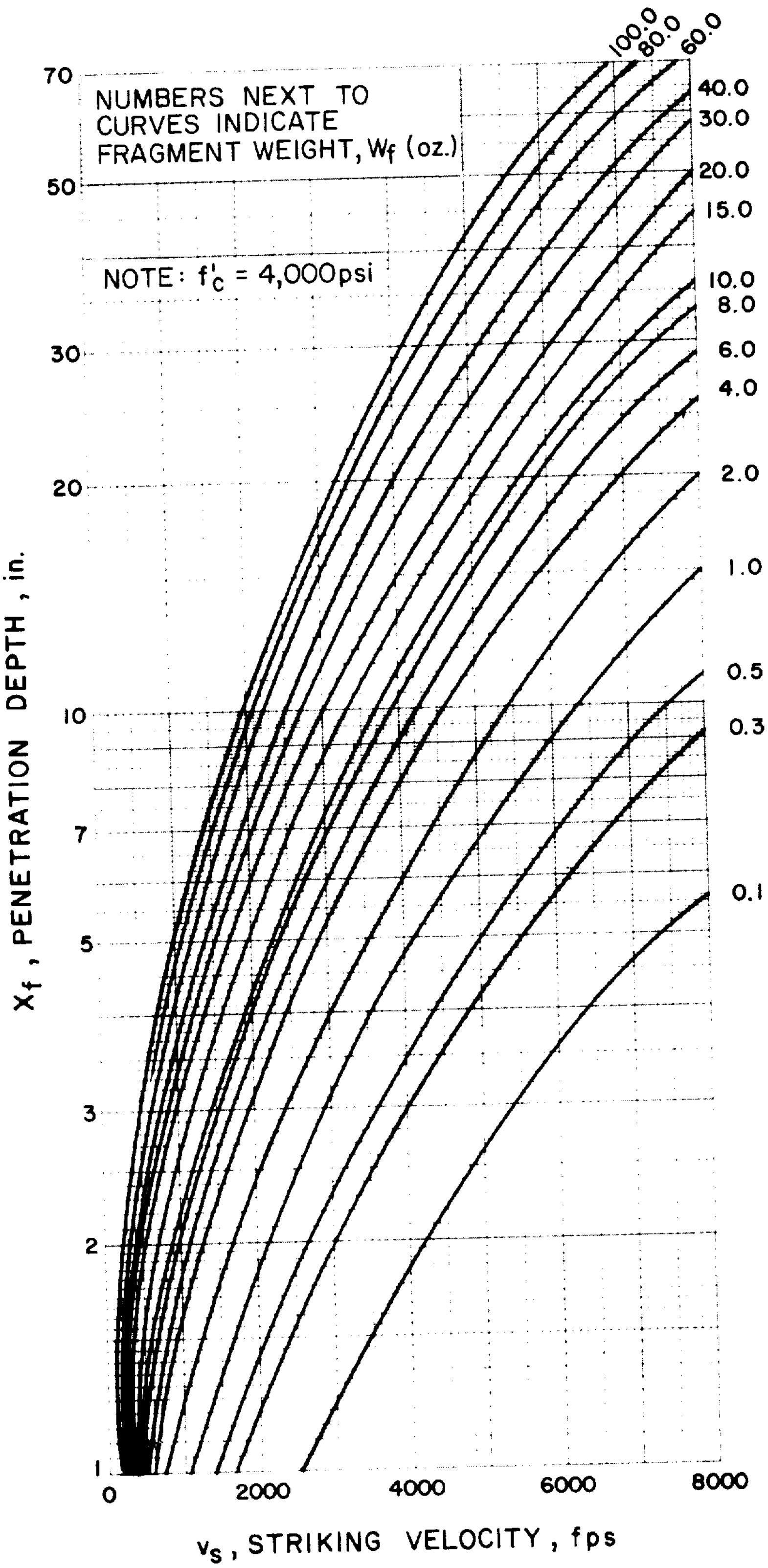


Figure 4-78 Concrete penetration chart for armor-piercing steel fragments

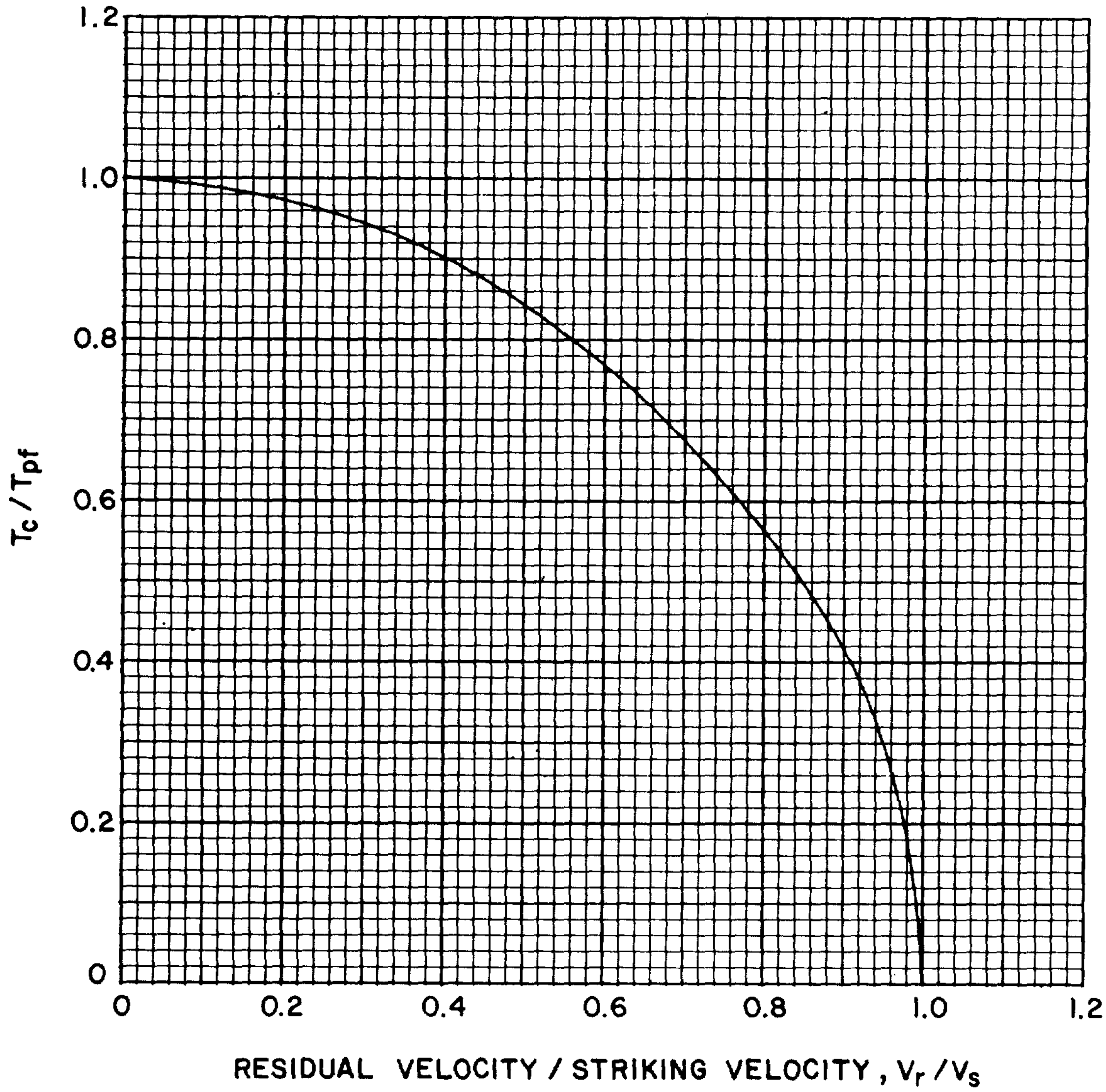


Figure 4-79 Residual fragment velocity upon perforation of concrete barriers (for cases where x < 2d)

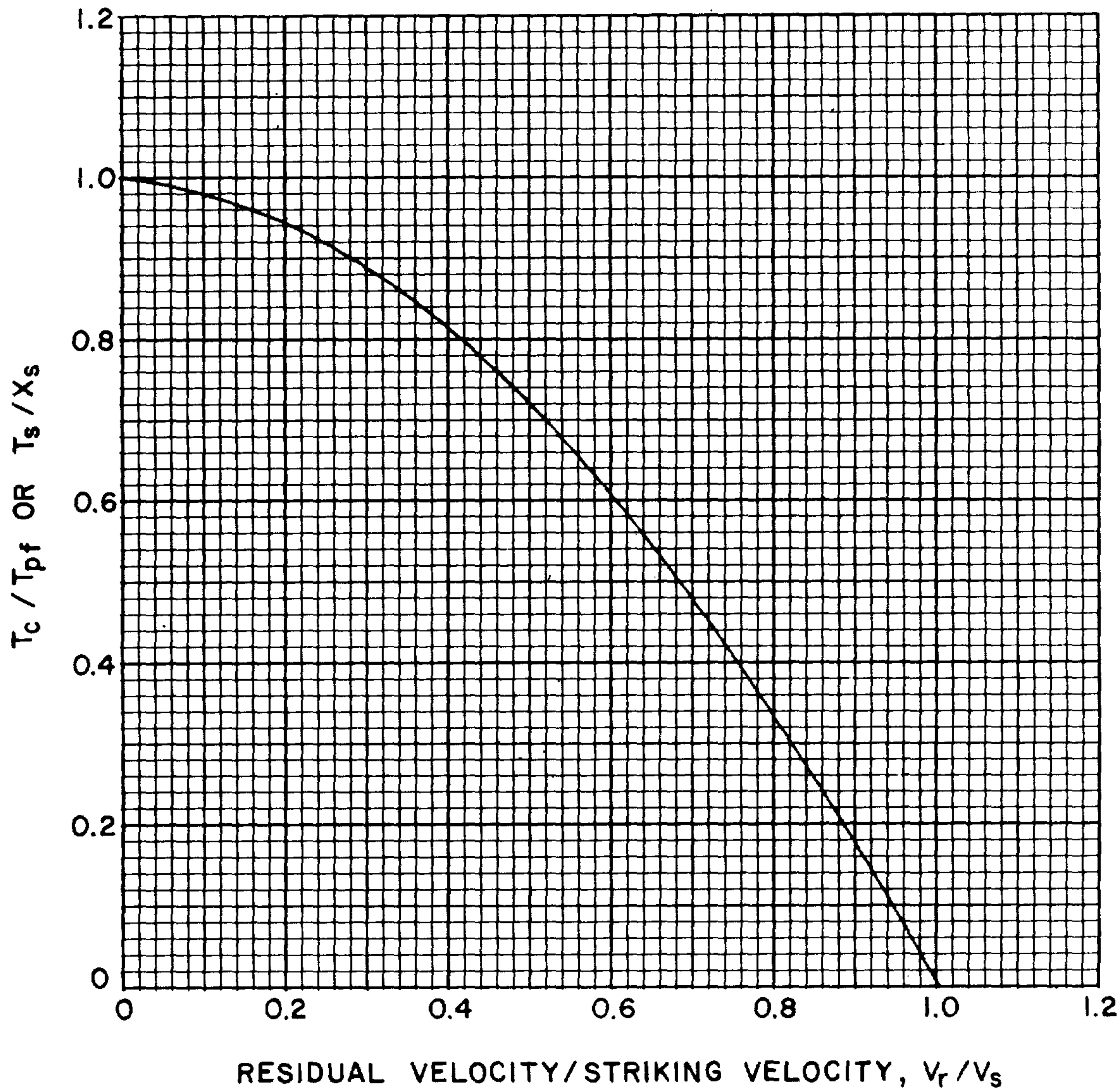
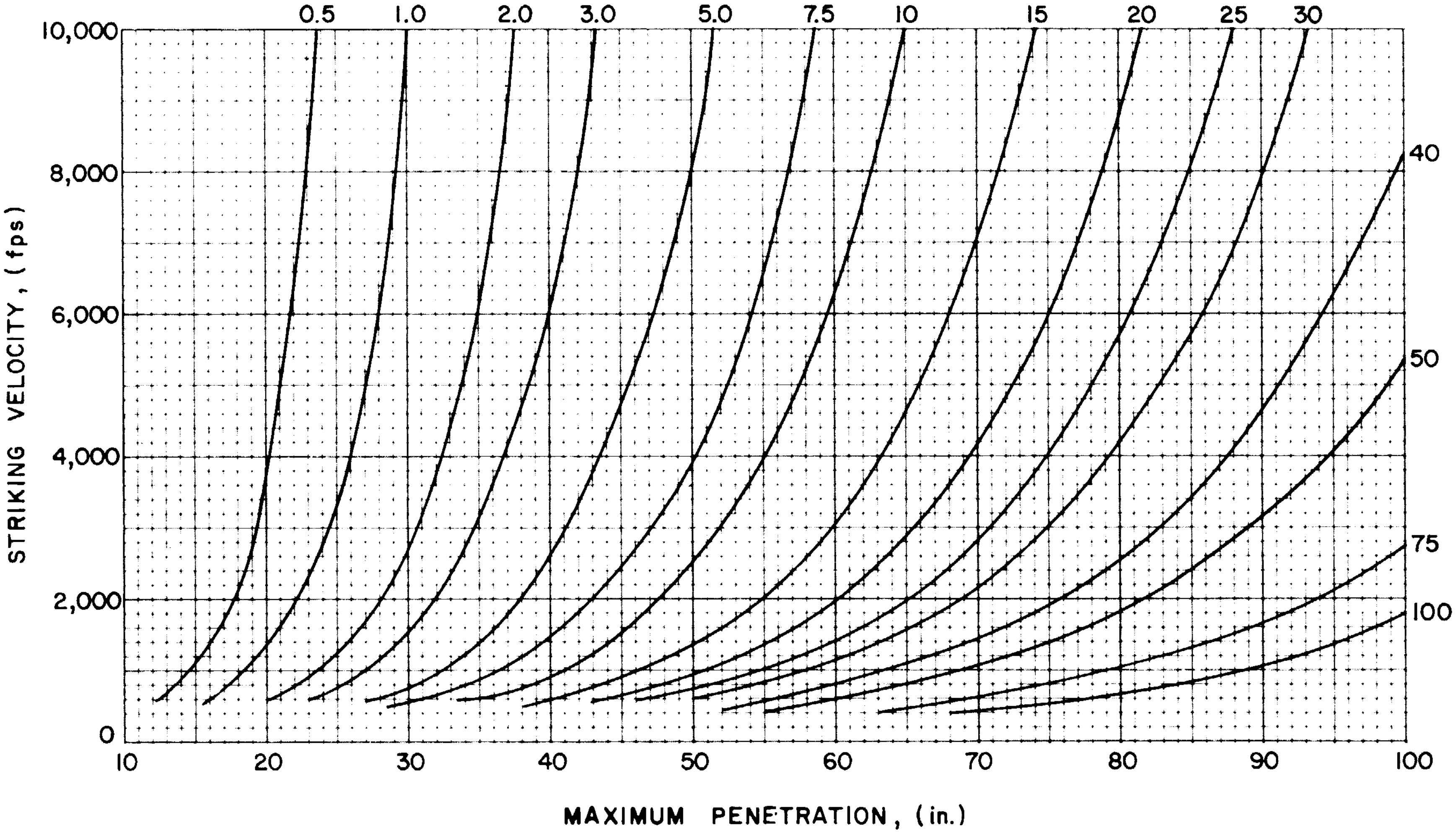


Figure 4-80 Residual fragment velocity upon perforation of :
(1) Concrete barriers (for cases where x > 2d)
(2) sand layers



NOTE: NUMBERS NEXT TO CURVES INDICATE FRAGMENT WEIGHT, (oz.)

Figure 4-81 Depth of penetration into sand by standard primary fragments

Table 4-16 **Relative Penetrability Coefficients for Various Missile Materials**

Type of Metal	Constant k
Armor-piercing steel	1.00
Mild steel	0.70
Lead	0.50
Aluminum	0.15

CONSTRUCTION DETAILS AND PROCEDURES

4-62. Introduction

A major portion of the detailing and construction procedures required for structures designed to resist blast pressures is the same as required for structures designed conventionally. However, some differences do exist and neglecting them would result in an unsafe situation, since the structure would not act as assumed in the design. These sections describe the differences in construction. Particular attention is directed towards the construction of structures subjected to close-in blast effects (elements reinforced with lacing or single leg stirrups) but the construction of conventionally reinforced elements, flat slabs and composite elements is also discussed.

Although the construction of blast resistant structures is similar to conventional structures, some changes in the fabrication and construction procedures are required to insure full development of both the concrete and the reinforcement well into the range of plastic action of the various elements. Since these changes primarily affect the reinforcement rather than the concrete, the major portion of the following discussion pertains to reinforcing steel details. Typical details are presented to illustrate detailing procedures and design considerations. These details may not be applicable to all design situations, and may have to be modified by the engineers within the guidelines given below.

4-63. Concrete

The dynamic characteristics and high magnitude of the applied blast load require the strength of the concrete used in blast resistant construction to be higher than that required for conventional construction. Because of the flexural action of blast resistant elements while large deflections are required and the high pressures associated with blast loadings, it is recommended that a minimum concrete strength of 4,000 psi be used.

The properties and testing of the concrete materials (cement, aggregate, water) used in blast resistant concrete construction are the same as those normally used and should conform to the standards specified in the ACI Building Code. High early-strength Portland cement (type III) may be used. To minimize the effects of spalling, it is recommended that the size of the aggregate used be not greater than 1 inch. This limitation of the aggregate size also facilitates the placement of the concrete, particularly where the cover over the reinforcement is held to a minimum. In all cases, the minimum concrete cover should conform to that specified in the ACI Building Code and, whenever possible, should also be the maximum thickness of the concrete cover.

Because of the large amounts of flexural reinforcement and, in laced elements, the presence of lacing reinforcement, the concrete slump used is usually larger than that permitted for conventional construction. A concrete slump of 4 to 6 inches is recommended for laced elements to insure that concrete voids do not occur.

Wherever possible, both horizontal and vertical construction joints should be avoided. A wall whose height is equal to or less than 10 feet usually can be poured without a horizontal joint. On the other hand, good construction techniques and economy may require the use of horizontal joints for higher walls. These joints should be located at points of low stress intensity. A more detailed discussion of joint locations is given later in sections.

4-64. Flexural Reinforcement

The flexural reinforcement used in blast resistant construction should be designated as ASTM A 615, Grade 60. Slabs must be reinforced in two mutually perpendicular directions. In all elements, the reinforcement should be continuous in any direction. All flexural reinforcement should consist of straight bars, and bends in the reinforcing within the span of an element should be avoided. However, the reinforcement may be bent well within the element's supports when additional anchorage is required.

The spacing of flexural bars is governed by the required area of the reinforcing steel, the selected bar size and, as discussed later, the spacing required to achieve a working and economical arrangement of lacing reinforcement and single leg stirrups. In general, flexural bars should be spaced fairly close together to insure that the cracked concrete between the layers of reinforcement will not be dislodged from the element. Tests have indicated that a maximum spacing of approximately 15 inches will insure confinement of the concrete.

Because of their reduced ductility, reinforcing steel larger than No. 14 bars (No. 18) should not be used as flexural reinforcement in blast resistant elements. Also, the size of the flexural steel should be at least equal to a No. 4 bar. Where necessary, the area of steel normally furnished by the special large bars should be provided by bundling smaller bars. However, the use of these bundled bars should be limited to one direction only, for any laced element. If bundled bars are used in an element whose main span is between two opposite supports, then all bars of each bundle should be continuous across the full span. On the other hand, if the main span is between a support and a free edge, then bundled bars may be cut off at points of reduced stress. At least one bar in each bundle should be continuous across the full span. For two-way elements these cutoffs must be located beyond the positive yield line with sufficient anchorage to develop the bars.

Continuous reinforcement should be used in blast resistant elements, but in many cases, this is a physical impossibility. Therefore, splicing of the reinforcement is necessary. Splices should be located in regions of reduced stress and their number held to a minimum by using the longest reinforcing bars practical (bars 60 feet in length are generally available throughout the country). Tests have indicated that the preferable method of splicing flexural reinforcement in laced concrete elements is by lapping the reinforcing bars. The length of each lap should be at least equal to 40 diameters of the larger of the two bars spliced, but not less than 2 feet (usually the same size bars will be spliced). In addition, splices of adjacent parallel bars should be staggered to prevent the formation of a plane of weakness. Figure 4-82 illustrates typical splicing patterns for both single and bundled bars used in the close-in design range.

In conventionally reinforced (non-laced) concrete elements used in the far design range, a two bar splice pattern is used. The length of the lap splices must be calculated to ensure full development of the reinforcement. If the splices occur in regions of low stress, the length of the lap is taken as 40 bar diameters. Lap splices of No. 14 bars are not permitted in either laced or non-laced concrete elements.

Mechanical splices may also be used, but they must be capable of developing the ultimate strength of the reinforcement without reducing its ductility. If the bar deformations have to be removed in the preparation of these splices, grinding rather than heat should be employed since heat can alter the chemical properties thereby changing the physical properties of the steel and possibly reducing the capacity of the element. Welding of the reinforcement should be prohibited unless it can be determined that the combination of weld and reinforcement steel will not result in a reduction of the ultimate strength and ductility. In those cases where welding is absolutely essential, it may be necessary to obtain special reinforcement manufactured with controlled chemical properties.

4-65. Construction Details for "Far Range" Design

4-65.1. General

Unlaced reinforced concrete elements are generally used in those facilities designed to resist explosive output at the far design range. The facilities generally consist of shelter-type structures.

Blast resistant structures utilizing unlaced reinforced concrete construction will only differ from conventional construction insofar as the increased magnitude and reversibility of applied loads are concerned. These differences are reflected in the details of the blast resistant structure. An increase in the amount of reinforcement is required to resist the large dynamic loads. Also, the reinforcement at both surfaces of an element must be detailed to prevent failure due to rebound stresses.

In general, conventionally reinforced elements are limited to deflections corresponding to support rotations of 2 degrees or less. However, elements designed for far range effects are capable of attaining deflections up to 4 degrees through the use of single leg stirrups and up to 8 degrees by developing tension membrane action. Elements subject to close-in effects require stirrups and are limited to 2 degrees support rotations. Details are presented below for elements designed for limited deflections, elements acting under tension membrane action and columns. For elements subjected to low and intermediate pressure and reinforced with stirrups, the details given below may be used without modification. However, details for elements with stirrups and subject to high pressures are discussed in the following section.

4-65.2. Elements Designed for Limited Deflections

Construction and detailing of unlaced, blast resistant structures is very similar to conventional structures. The major difference is the method of anchoring the reinforcement. A typical section through a non-laced wall is shown in Figure 4-83. At the roof-wall intersection, the exterior wall reinforcement is continued through the regions of high stress and lap spliced with the roof reinforcement in the vicinity of the point of inflection. Thus, the reinforcement is not actually anchored but rather developed. The interior reinforcement of both wall and roof is terminated with a standard hook in order to be effective in resisting rebound tension forces. However, this reinforcement will be in compression during the initial phase of loading and therefore, the straight portion of the bar must be sufficient to develop the reinforcement in compression.

The bottom floor slab reinforcement is extended through the floor-wall intersection into the wall in the same manner as the roof-wall intersection. Again the reinforcement is developed into the wall. The vertical wall reinforcement is supported on the floor slab rather than supported above the floor slab on the reinforcement. Figure 4-83 illustrates a building with a slab-on-grade foundation. Figure 4-84 illustrates several alternative arrangements.

A horizontal section through the intersection through two discontinuous walls (Fig. 4-85) would reveal details very similar to those shown for the roof-wall intersection. The number of splices used would depend on the length of each wall.

Wherever possible, continuous reinforcement should be used. Lap slices may be used when necessary but their number should be held to a minimum and they must be located in regions of reduced stress. To prevent the formation of a plane of weakness, splices on adjacent parallel bars must be staggered. In addition, the splice pattern for the reinforcement on opposite faces of an element should not be in the same location. Figures 4-86 and 4-87 illustrate preferred splice locations for a two-way slab and a multi-span slab respectively.

4-65.3. Elements Designed for Large Deflections

Unlaced concrete slabs are capable of attaining deflections up to 8 degrees support rotation through the development of tension membrane action. Construction details are basically the same whether the slab attains large or small deflections, however, there are two important exceptions. At large deflections, tension membrane action produces large tensile strains in both tension and "compression" reinforcement. Therefore, all anchorage lengths and lap splice lengths must be calculated for the design stress f_{ds} and all the reinforcement is in tension. The second difference in detailing concerns the location of splices. Although splices should still be located in regions of low flexural stress, they will be located in regions of high tensile stress when the element attains its full tension membrane capacity. The length of the lap splice must, therefore, be increased to 1.3 times the modified development length given in Section 4-21.4 of a reinforcing bar in tension.

4-65.4. Column Details

Columns are generally required in blast resistant, shear wall structures. Their details differ little from those used in conventional structures. Both round, spiral reinforced columns and rectangular, tied columns can be used, but one or the other is preferred for a given design situation. Details of the reinforcement and formwork of rectangular columns are compatible with beam details and, therefore, are recommended for beam-slab roof systems. Round columns should be used for flat slab roofs.

Figure 4-88 illustrates a typical section through a circular column with a capital supporting a flat slab. A round column prevents stress concentrations that may cause local failures. The column capital, although not required, is preferred since it simplifies the placement of the diagonal bars as well as decreasing shear stresses. All diagonal bars should extend into the column.

If, however, the number and/or size of the diagonal bars do not permit all the bars to extend into the column, up to half the bars may be cut off in the capital as shown. Lateral reinforcement of a circular column consists of spiral reinforcement beginning at the top of the floor slab and extending to the underside of the drop panel or the underside of the roof slab if no drop panel is used. Within the column capital, No. 4 hoop reinforcement at 6 inches on center must enclose the diagonal bars.

In a beam-slab system, beam reinforcement does not permit the addition of diagonal bars at the top of the column. Therefore, the beams themselves must be able to provide adequate shear strength. Closed ties provide lateral support of the longitudinal reinforcement in rectangular columns. The ties must start not more than ½ tie spacing above the top of the floor slab and end not more than 3 inches below the lowest reinforcement in the shallowest beam. The ties must be arranged so that every corner and alternate longitudinal bar has the lateral support provided by the corner of a tie having an included angle no more than 135 degrees. In addition, no longitudinal bar shall be farther than 6 inches clear on either side from such a laterally supported bar.

The column footing illustrated in Figure 4-88 is the same for both rectangular and circular columns. Dowels anchor the column into its footing. Since there is little or no moment at the bottom of the column, splices need not be staggered as they are in a wall. The splice should be able to fully develop the reinforcement in tension in order to resist rebound tension forces. Within the column, no splices of the longitudinal reinforcement are permitted. Continuous reinforcement should not be a problem as blast resistant structures are limited to one and two-story buildings.

4-66. Construction Details for "Close-In Design"

4-66.1. General

Laced reinforced concrete elements are usually used in those facilities which are designed to resist the explosive output of close-in detonations (high-intensity pressures with short durations). The functional requirements of these facilities (storage and/or manufacture of explosives) normally dictate the use of one-story concrete buildings with austere architecture. Basically, these structures consist of a series of interconnecting structural elements (walls, floor slabs, and/or roofs) forming several compartments or cubicles. Because of this cubicle arrangement, the walls separating the individual areas are the predominant element used in laced construction and are the most critical component in the design. However, in some cases, the roof and/or floor slab can be of equal importance.

Single leg stirrups may replace lacing bars when the scaled normal distance between the charge and the element is equal to or greater than 1.0 $ft/lbs^{1/3}$. However, the maximum allowable support rotation for elements with single leg stirrups is less than for laced elements. Single leg stirrups are more economical to fabricate and slightly more economical to place during construction. Unlike laced reinforcement which requires that the position of the flexural reinforcement be changed to suit the horizontal and vertical lacing, the position of the flexural reinforcement remains constant for single leg stirrups. The stirrups are tied around the outer bars whether they are horizontal or vertical. Details for flexural reinforcement, splice location and length etc., are the same for single leg stirrups and laced reinforcement.

4-66.2. Laced Elements

4-66.2.1. Lacing Reinforcement

All flexural reinforcing bars must be tied with continuous diagonal lacing bars (Fig. 4-89). At any particular section of an element, the longitudinal or main tension and compression reinforcement is placed to the interior of the transverse or secondary flexural reinforcing steel around which the diagonal lacing bars are bent.

Lacing reinforcement must be fabricated without the formation of excessive stress concentrations at the bends. The bending should be performed without the use of heat, and in no case should the radius of the pin used to bend the lacing be less than four times the diameter of the bar. Figure 4-90 illustrates the typical lacing bends used with both single and bundled flexural bars.

The amount of lacing reinforcement required in an element depends upon the element's capacity (quantity and distribution of flexural reinforcement, thickness of the element, and the type and number of supports) while the size of the lacing bars is a function of the required area and spacing. The maximum and minimum size of the lacing bars should be No. 8 and No. 3 bars, respectively. However, the preferred maximum size of lacing bar is No. 6. Several lacing schemes have been developed (Fig. 4-91) which avoid excessive stress concentrations as a result of large angular bends , provide adequate restraint against buckling of the compression reinforcement, and make use of the most efficient arrangement of the lacing reinforcement (lacing making an angle of 45 degrees with the longitudinal reinforcement is most efficient). In these schemes, the transverse flexural reinforcement may or may not be tied at every intersection with a longitudinal bar. However, a grid system is established whereby bar intersections are tied within a distance s_1 or 2 feet, whichever is less. The choice of the scheme to be used depends upon the flexural bar spacing and the thickness of the element so that the angle α is approximately equal to, but never less than, 45 degrees. Although not tying every transverse flexural bar results in a large lacing bar size, the total cost of the lacing may be reduced since the size of the lacing bend associated with an increase in the spacing s_1 reduces the overall length of the bar and the number of bends required to cover a given longitudinal distance.

An additional cost saving in the fabrication of the lacing may be realized by utilizing the equipment that is used to bend the steel bars for open web steel joists. However, when detailing the flexural and lacing reinforcement and the concrete wall thickness, consideration must be given to the physical capabilities (size of bend and bar, depth of lacing, etc.) of the equipment. Alteration of bar joist equipment to meet the requirements of a design is not usually practical with respect to both time and cost.

The placement of the lacing depends upon the distribution of the flexural reinforcement and the number and type of supports. Lacing is always placed perpendicular to the element's supports to resist diagonal tension stresses. Because of the nonuniformity of the blast loads associated with close-in detonations, continuous lacing across the span length should be used to distribute the loads. Except for cantilever elements, lacing in one-way elements is placed in the direction of and continuous across the span. Cantilever elements require lacing in two directions. Discontinuous lacing is located perpendicular and adjacent to the support while continuous lacing is placed across the full width of the element in direction parallel and adjacent to the free edge located opposite the element's support. For two-way elements, diagonal tension stresses must be resisted in two directions. Because of interference, lacing can only be continuous in one direction, which in general is in the direction of the longest span. Figure 4-92 illustrates the location of the lacing used in a cantilever wall as well as in several two-way elements. For two-way elements, the location of the lacing is not affected by the type of supports. Therefore, the supports indicated in Figure 4-92 can be simple, restrained, or fully fixed.

Similar to the flexural reinforcement, lacing will usually require splices. Tests have indicated that the preferable way of splicing lacing is by lapping the bars. The lap length which is measured along the bars should be at least equal to that required for a full tension slice (40-bar diameters). However, the lacing should also be bent around a minimum of three flexural bars. The splices of adjacent lacing bars should be staggered to avoid forming a plane of weakness in the element, and the slices should be located in regions of low stress (away from the supports and positive yield lines). Typical details for the splicing of lacing bars are presented in Figure 4-93. Wherever possible, welding of lacing bars should be avoided and is only permitted while the full development of the ultimate strength can be assured without any reduction in strength or ductility.

The location of the splices is determined from the distance along the length of the element which can be covered by a given length of lacing bar (usually 60 feet). The expression for the actual length of lacing bar L_1 required to cover the length s_1 is a function of the flexural bar spacing and the geometry of the lacing and is given in Figure 4-94.

4-66.2.2. Corner Details

Because of the magnitude of the blast loads associated with close-in detonations and their amplification at corners, the use of concrete haunches has been found to be a satisfactory method of maintaining the integrity of these sections of a structure. All corners should be reinforced with diagonal bars to transfer the high shear forces from the element to its support and to assist in maintaining the integrity of the intersection. Diagonal bars should be used in elements both with and without haunches. Reinforcement details for corners are shown in conjunction with wall intersections in the following section.

4-66.2.3. Walls

Figure 4-95 illustrates the detailing procedure for a typical laced wall. The wall is free at the top, supported at its sides by other walls (not shown), and at its base by the floor slab. It has vertical lacing which is continuous from the bottom of the wall to approximately midheight. In the upper half of the wall, the horizontal lacing reinforcement is continuous over the full length of the wall and anchored in the side supports. It should be noted that the horizontal flexural steel in the lower half of the wall is located at the exterior of the vertical reinforcement, while in the upper half of the wall it is at the interior. This arrangement is necessary for the placement of the lacing so that the lacing can provide full confinement of both the flexural steel and the concrete separating the two layers of reinforcement. If spalling is critical, its effects may be minimized by reducing the concrete cover over the reinforcement in the upper half of the wall. The addition of U-bars at the top of the free edge also minimizes the formation of concrete fragments from this area. Reducing the concrete cover is not cost effective and, therefore, if spalling is not critical, the wall should have a constant thickness.

A portion of the wall should extend below the floor slab a distance no less than that required to anchor the flexural reinforcement (40-bar diameters) and in no case less than 1 foot - 6 inches. In addition to providing anchorage for the vertical reinforcement, the portion of the wall below the floor slab assists in resisting sliding of the structure by developing the passive pressure of the soil adjacent to the wall.

The working pad at the base of the wall provides the support required for the erection of the wall reinforcement and also affords protection for the reinforcement after construction is completed. It should be noted that in the example illustrated in Figure 4-95 the cover over the reinforcement in the portion of the wall above the floor slab is specified as 3/4 inch (minimum reinforcement cover required by the ACI Building Code for concrete not exposed to the weather). while below the floor slab it is specified as 2 inches (minimum cover required by ACI Building Code for concrete in contact with the ground after removal of the forms). The increased cover below the floor slab is achieved by increasing the wall thickness rather than bending the reinforcement.

Diagonal bars are provided at the intersection of the floor slab and wall. These bars transfer the high shear forces from the base of the wall through the haunch and into the floor slab. Section C-C, Figure 4-95 indicates the location of the diagonal bars relative to the lacing reinforcement.

Details of the reinforcement at wall intersections are similar to those at the intersection of the wall and floor slab. Figures 4-96 through 4-98 illustrate these details for the intersection of two continuous walls, one continuous and one discontinuous wall, and two discontinuous walls, respectively. In all cases, both the flexural and lacing reinforcement are fully anchored by being made continuous through the intersection (a distance of at least 40 bar diameters). In discontinuous walls, the wall must be extended a distance sufficient to anchor all reinforcement but not less than 1 foot-6 inches. This extension (or extensions) aids in resisting both the overturning of the structure and the tension force produced in the walls (discussed further in subsequent sections). Diagonal bars have been provided at all intersections to transfer the support shears and to maintain the integrity of the section. Where discontinuous walls are encountered, only one diagonal bar is effective for the continuous wall of Figure 4-97 and for both discontinuous walls of Figure 4-98. For these cases, bundled diagonal bars (2 bars maximum) may have to be used.

If wall extensions are not permitted due to architectural or other criteria, the reinforcement is anchored by bending it within the corner (figs. 4-99 and 4-100). However, the distance from the face of the support to the end of the hook must be at least 20 bar diameters. A standard hook may be used if; (1)

the total distance from the face of the support to the end of the bar (including the hook) is 40 bar diameters and (2) the length of the hook is at least 12 inches. The use of hooks may cause problems in the placement of the lacing reinforcement and is discussed in conjunction with the sequence of construction below.

In addition to flexural reinforcement, the walls of containment structures may require the addition of tension reinforcement. This reinforcement is placed at the mid-depth of the wall and has the same spacing as the flexural reinforcement. It may be required in the vertical and/or horizontal directions. The reinforcement is anchored at wall intersections in the same manner as the flexural reinforcement. Vertical tension reinforcement usually does not require splices. However, if the horizontal tension reinforcement requires splicing, the splice (lap, mechanical, weld) and pattern should be same as the flexural reinforcement. Of course, the tension steel should never be spliced at the same location as the flexural reinforcement.

4-66.2.4. Floors Slabs

Floor slabs on grade must provide sufficient capacity to fully develop the wall reinforcement. If sufficient resistance is provided by the soil, slabs poured on grade usually do not require lacing nor other shear reinforcement; although lacing reinforcement is always required in slabs exposed to multiple detonations. Soil strata having enough bearing capacity to support the dead load of the structure can be considered to provide the support required by the slab. Before placement of the slab, the top 6 inches of the subgrade should be compacted to 95 percent of maximum density in accordance with ASTM Standard D1557.

Piles are used to support a structure where the bearing capacity of the soil is inadequate. The piles are placed under the walls and the floor must span between them. Lacing or single leg stirrups must be provided. The reactions of the slab are transferred to the portion of the wall below the floor slab which acts as a beam spanning between piles.

In addition to flexural reinforcement, floor slabs may require tension reinforcement located at mid-depth of the slab. Tension forces are discussed in conjunction with single and multi-cubicle structures.

4-66.2.5. Roof Slabs

Roof slabs are similar to walls since they are usually supported only at their periphery and require the addition of lacing to distribute and resist the applied blast loads. In those facilities where the explosion occurs within a structure, the blast pressures acting on the interior surface of a nonfrangible roof causes tension stresses in the walls which require the addition of tension reinforcement above that needed for bending. Tension forces are discussed in conjunction with single and multi-cubicle structures.

4-66.3. Elements Reinforced with Single Leg Stirrups

4-66.3.1. Single Leg Stirrups

A single leg stirrup consists of a straight bar with a hook of at least 135 degrees at each end. Hooks shall conform to the ACI Building Code. At any particular section of an element, the longitudinal flexural reinforcement is placed to the interior of the transverse reinforcement and the stirrups are bent around the transverse reinforcement (Fig. 4-101).

The required quantity of single leg stirrups is calculated in the same manner as lacing. They are a function of the element's flexural capacity while size rebar used is a function of the required area and spacing of the stirrups. The maximum and minimum size of stirrup bars are No. 8 and No. 3, respectively, while the spacing between stirrups is limited to a maximum of d/2 or $d_c/2$ for type I and type II or III, respectively.

The preferable placement of single-leg stirrups is at every flexural bar intersection. However, the transverse flexural reinforcement does not have to be tied at every intersection with a longitudinal bar. A grid system may be established whereby alternate bar intersections in one or both directions are tied within a distance not greater than 2 feet. The choice of the three possible schemes depends upon the quantity of flexural reinforcement, the spacing of the flexural bars and the thickness of the concrete element. For thick, lightly reinforced elements, stirrups may be furnished at alternate bar intersections, whereas for thin and/or heavily reinforced elements, stirrups will be required at every bar intersection. For those cites where large stirrups are required at every flexural bar intersection, the bar size used may be reduced by furnishing two stirrups at each flexural bar intersection. In this situation, a stirrup is provided at each side of longitudinal bar.

Single leg stirrups must be distributed throughout an element. Unlike shear reinforcement in conventionally loaded elements, the stirrups cannot be reduced in regions of low shear stress. The size of the stirrups is determined for the high stress areas and, because of the non-uniformity of the blast loads associated with close-in detonations, this size stirrup is placed across the span length to distribute the loads. For two-way elements, diagonal tension stresses must be resisted in two directions. The size of stirrup determined for each direction is placed to the same extent as the lacing shown in Figure 4-92. However, the distribution does not apply for cantilever elements since they are one-way elements requiring only one stirrup size which is uniformly distributed throughout.

4-66.3.2. Corner Details

Corner details for elements with single leg stirrups are the same as for laced elements. Concrete haunches, reinforced with diagonal bars, should be used at all corners. For those cases where compelling operational requirements prohibit the use of haunches, diagonal bars must still be placed at these corners. In addition to diagonal bars, closed ties must be placed at all corners (fig. 4-102) to assist in maintaining the integrity of the intersection. The tie should be the same size as the stirrups but not less than a No. 4 bar. The spacing of the ties should be the same as the flexural reinforcement.

4-66.3.3. Walls

The detailing procedure for a wall with single leg stirrups is similar to a laced wall. Figure 4-103 illustrates the detailing procedure for a typical wall with single leg stirrups. This wall is the same as the wall shown in Figure 4-95 except that stirrups are used rather than lacing. These are only two differences between the two walls. First, there is no need to alter the position of the horizontal flexural reinforcement for the placement of stirrups. The horizontal reinforcement is in the main direction (assumed for wall illustrated) and, therefore, this steel is placed exterior of the vertical reinforcement for the entire height of the wall. Second, closed ties are placed at the wall and floor slab intersection to assist in maintaining the integrity of the section. The common requirements for both walls include the addition of U-bars, diagonal bars, concrete haunches, increased cover over the reinforcement below the floor slab by increasing the wall thickness, shear reinforcement (stirrups or lacing) in the wall extension below the floor slab, and the preferred use of a working pad.

Details of the reinforcement at wall intersections are similar to those at the intersection of the wall and floor slab. The requirements for anchorage of the flexural reinforcement and diagonal bars at wall intersections are exactly the same as laced walls (fig. 4-96 through 4-100). The placement of the single leg stirrups and the required closed ties are shown in Figure 4-102. Similar to laced walls, the use of wall extensions is the preferred method of reinforcement anchorage at discontinuous walls.

4-66.3.4. Floor Slabs

The floor slab must provide sufficient capacity to fully develop the wall reinforcement. The requirements are the same as a floor slab for laced walls.

4-66.3.5. Roof Slabs

Roof slabs are similar to walls since they are usually supported only at their periphery and require the addition of single leg stirrups to distribute and resist the applied blast loads. For interior explosions, the roof causes tension forces in the walls. Tension reinforcement is discussed in conjunction with single and multi-cubicle structures.

4-67. Composite Construction

Composite construction is primarily used for barricades and consists of two concrete panels separated by sand fill (Fig. 4-104). Details of each panel are similar to those described for single laced walls (or walls reinforced with single leg stirrups).

The concrete panels may be supported at the base either by the floor slabs or a concrete pedestal. When the pedestal is used, reinforcement across the base of both panels terminates in the floor slab and provides a monolithic connection between the two panels. The floor reinforcement serves as the monolithic connection when pedestals are not used.

The upper portion of the wall may either be open or solid. Open sections are usually used when the upper edge of the wall is unsupported; the solid section is used when an external tie system is used to restrain the motion and provide support for the top of the wall. The solid section must be reinforced to resist torsion and bending induced by the ties and the panels.

The impulse capacity of composite walls is a function of the density of the sand fill. The sand will be compacted after construction due to its own weight and/or by water drainage when the wall is exposed to the weather. The sand fill must be continuously maintained at the level stipulated in the design by mechanical means which will allow periodic rearrangement of the sand fill. Clay pipe or other similar material may be placed in the wall cavity with the sand so that when the wall is loaded the clay pipe will be crushed by the impact of the donor panel, thereby providing space within the wall cavity into which the compacted sand may flow, hence reducing its density. If possible, the sand should be protected from the elements by sealing the top of the cavity.

4-68. Single and Multicubicle Structures

In single-cell structures (Fig. 4-105) unbalanced force (support reactions) exist at all element intersections (walls, and floor and wall intersections) and must be resisted by tension force produced in the support elements. In addition to the reinforcement required to resist flexural and shear stresses, tension reinforcement, distributed along the centerline of the elements, is required. Horizontal tension reinforcement in the side wall and floor slab (parallel to the side walls) is required to resist the vertical and horizontal reactions of the back wall, while horizontal steel in the back wall and floor slab (parallel to the back wall) resists the tension force produced by the side wall reactions.

These unbalanced forces are transmitted to the structure's foundation and, depending upon their magnitude, the size and configuration of the structure and the subgrade conditions, the structure may be subject to both translational and vertical rotational motions. Translation of the structure is resisted by the extension of the walls below the floor slab (shear key) and the friction developed between the floor slab and subgrade, whereas rotation is resisted by the mass of the structure with assistance from the blast load acting on the floor slab of the donor cell. The stability of the structure can be substantially increased by the extension of the walls and floor slab as illustrated in Figure 4-105b. This extension of the walls and floor slab (1) increases the resistance of the structure to overturning (rotation), (2) increases the rigidity of the structure, (3) reduces the effects of the unbalanced wall moments which cause twisting of the corners, (4) reduces the required thickness and/or reinforcement in the floor slab (moment capacity of the floor slab extension must be developed by bearing on the subgrade). and (5) eliminates the need to anchor the reinforcement by bending at the corners which would ordinarily hinder the placement of the concrete.

End cells of multicubicle arrangements also require the addition of tension reinforcement to resist unbalanced blast loads acting on the end walls. The interior cells do not require this additional reinforcement since the mass and base friction of adjoining cells provide the restraint to resist the lateral forces. Two possible multicubicle arrangements are shown in Figure 4-106. In both arrangements the back walls of the cells are continuous, whereas the side walls between the adjoining cells are either continuous or discontinuous. The type of cell arrangement (either one of those shown in Figure 4-106 or a combination of both arrangements) used in a particular design depends primarily upon the functional requirements of the facility and the economy involved. However, there are certain structural features which should be considered in the final selection of either structural arrangement. The horizontal reinforcement (flexural and lacing) in the side walls may be placed continuous across the back wall of scheme a, whereas with scheme b, the side wall horizontal reinforcement must be bent and anchored in the back wall. This latter arrangement can result in congestion of the horizontal reinforcement at the wall intersections. On the other hand, by offsetting the side walls at each side of the back wall, the span length of the back wall between adjacent side walls is reduced thereby reducing the required strength (concrete thickness and/or reinforcement) of the back wall.

In general, continuous walls usually require constant concrete thickness and horizontal reinforcement. However, where economy can be achieved, it may be desirable to reduce the thickness and reinforcement of the continuous wall of one cell in comparison to those of adjoining cells. This reduction should only be made between the supports in order that a constant moment capacity can be maintained across the length of the reduced element. This capacity reduction requires the horizontal reinforcement to be sliced at the supports, and extreme caution should be exercised in the detailing of the splices.

4-69. Sequence of Construction

Although the construction procedure for all blast resistant concrete elements is similar, each structure must be evaluated to determine the specific sequence of construction which is most appropriate for the particular situation. This evaluation should consider: (1) type and location of shear reinforcement (single leg stirrups, horizontal and vertical lacing). (2) location of reinforcement splices, (3) erection sequence of the reinforcement (flexural and shear), (4) location of horizontal construction joints and (5) pouring sequence of concrete.

To illustrate the construction of a laced concrete structure, consider the recommended construction procedure for the cubicle structure shown in Figure 4-105 b. A vertical section through any wall is similar to the wall described in Figure 4-95. Figure 4-107 illustrates the pouring sequence for the following procedure.

1. Fabricate the reinforcement as indicated on the drawings.

2. Pour a working pad.

3. Erect vertical flexural reinforcement, vertical lacing and vertical diagonal bars in all walls. Thread horizontal flexural bars between vertical lacing and vertical flexural bars up to the top of the floor slab. Also place reinforcement for the floor slab.

4. Adjust reinforcement to required positions and complete second pour to the top of the floor slab. As an alternative, place sufficient horizontal lacing (as described in step 7) to insure proper positioning of the vertical flexural reinforcement and then complete second pour. Additional horizontal flexural bars may be placed beyond the limit of the pour to help stabilize the reinforcement.

5. Thread horizontal flexural bars between the vertical lacing and vertical flexural bars beyond the limit of the third pour. Adjust reinforcement and complete the third pour.

6. Thread the remainder of the horizontal flexural bars up to the top of the vertical lacing.

7. Place horizontal flexural and lacing reinforcement and diagonal bars between the top of the vertical lacing and the top of the wall. Placement of the horizontal flexural and horizontal lacing reinforcement is accomplished by lowering this reinforcement over the vertical reinforcement. At wall intersections the proper sequence is to first lower diagonal bars, type a. Then in the north-south walls lower horizontal lacing bar type b, place horizontal flexural reinforcement and lower opposing lacing bar type a. Repeat this sequence with the reinforcement in the east-west walls and complete this individual layer of reinforcement by placing the horizontal diagonal bars type b. The entire sequence is repeated for the remaining reinforcement.

8. Add U bars at the top of the wall and adjust reinforcement to required positions. Pour remainder of the wall.

The above procedure is for the cubicle structure of Figure 4-105 b, where wall extensions are provided at the corners. For the case where wall extensions are not used (Fig. 4-105 a), the horizontal reinforcement must utilize a 90 degree hook for anchorage (Fig. 4-100). The horizontal flexural reinforcement for the side walls requires a 90 degree hook at one end. Therefore, the reinforcement must be threaded between the vertical lacing and vertical flexural bars from behind the back wall. If the back wall was close to an existing structure, the horizontal reinforcement could not be threaded. The horizontal flexural reinforcement in the back wall requires a 90 degree hook at each end which would prohibit threading the bars. To place this steel, the bars would have to be spliced so that they could be threaded through the back wall from each side wall. The use of splices is not desirable and should be avoided, making the use of wall extensions preferable.

The construction procedure for an element reinforced with single leg stirrups is similar, but not quite as complex as laced elements. The single leg stirrups should be lowered into position if the vertical flexural reinforcing bars are exterior of the horizontal bars. However, if the horizontal bars are exterior of the vertical bars (Fig. 4-103). the horizontal bars should be threaded between the stirrups and vertical bars. Again, as for laced elements, the reinforcement of intersecting walls and the diagonal bars must be placed in sequence.

The use of construction joints (both vertical and horizontal) should be avoided wherever possible since all joints are a potential plane of weakness. However, joints in large structures cannot be avoided because good practice for placement of concrete and/or economy requires their use. All joints should be located in regions of low stress intensity, and, if possible, for laced elements, vertical joints should be situated in areas where horizontal lacing is located, and horizontal joints should be situated in areas where vertical lacing is located. However, vertical joints are difficult to form in laced construction. In most cases, vertical joints are not used, and a certain height of all walls is poured simultaneously. In addition, concrete surfaces should be roughened at all joints.

The above construction procedure required the use of two horizontal construction joints. The joint located at the floor slab is generally used in all blast resistant structures while the second joint in the upper section of the wall should only be used if the height of the wall warrants it. The use of vertical construction joints is generally required for multicubicle arrangements. Walls (intersecting walls must be poured simultaneously) and corresponding floor slabs should be poured in checkerboard fashion to guard against joint separation due to shrinkage and temperature variations. To maintain a minimum rate of pour, multiple pouring crews may have to be used, and pumping of concrete, rather than the use of tremies, may be required for high walls.

Expansion joints are generally not required for laced concrete elements due to the presence of relatively large amounts of reinforcement. However, their use should be considered for long buildings and/or structures subjected to extreme temperature changes.

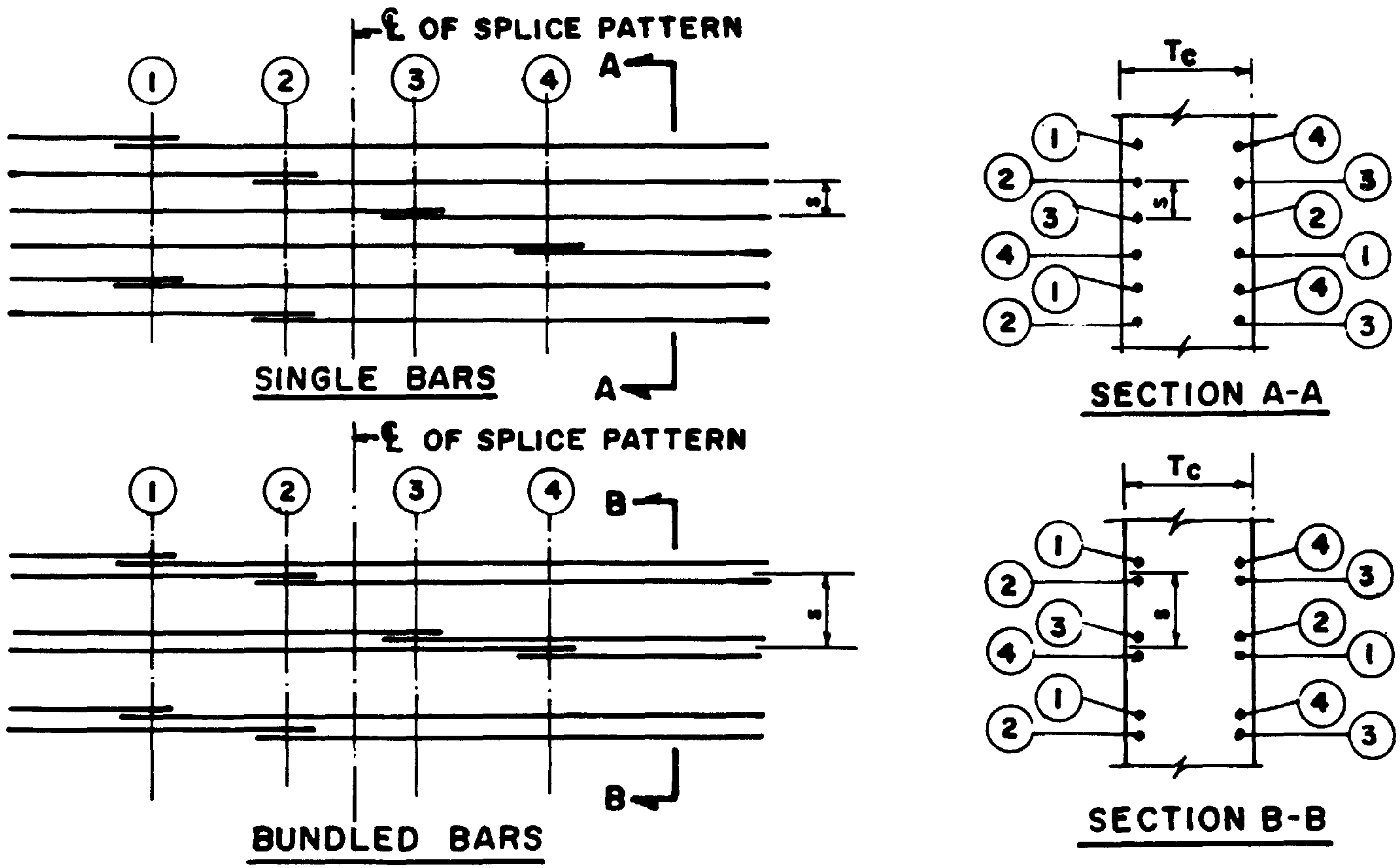


Figure 4-82 Typical flexural reinforcement splice pattern for close-in effects

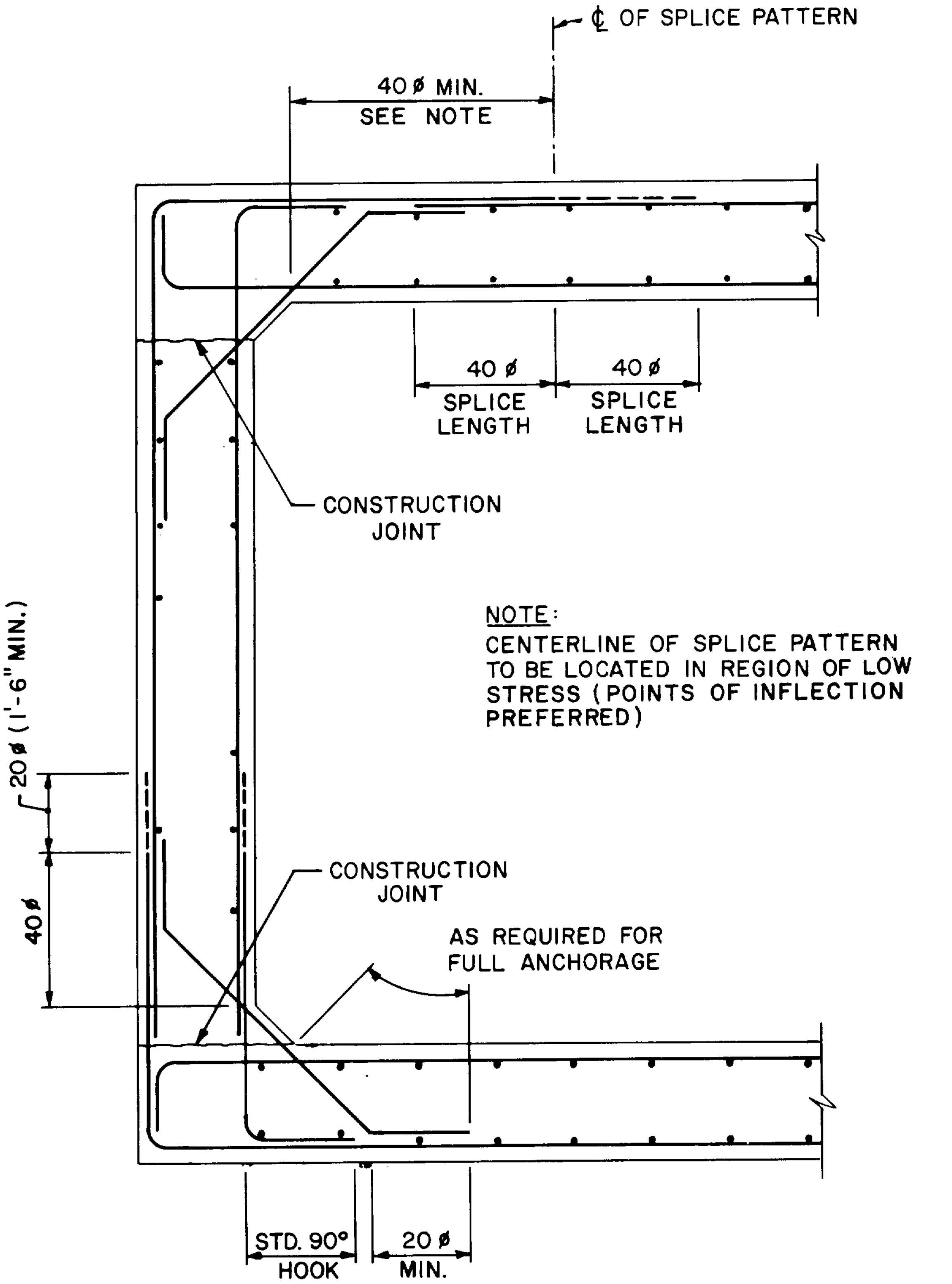


Figure 4-83   Typical section through conventionally reinforced concrete wall

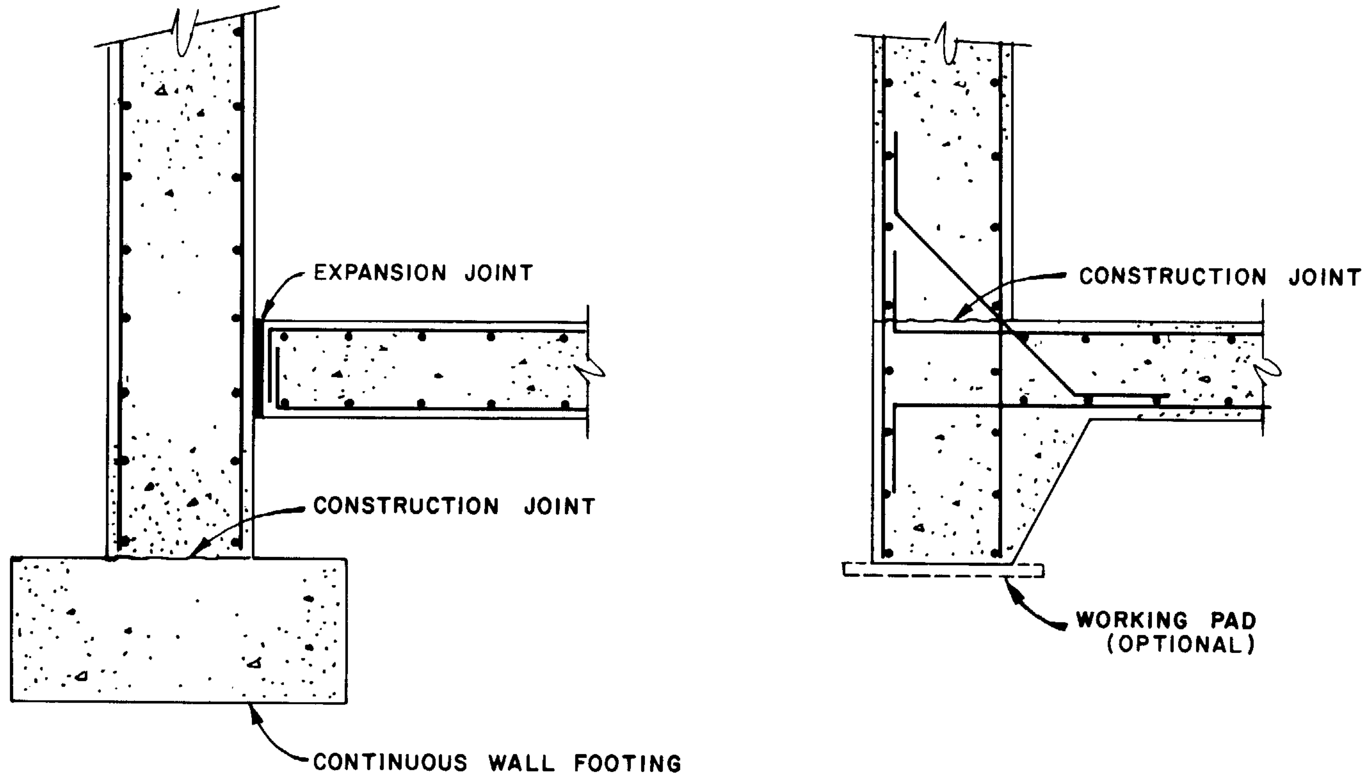


Figure 4-84 Floor slab-wall intersections

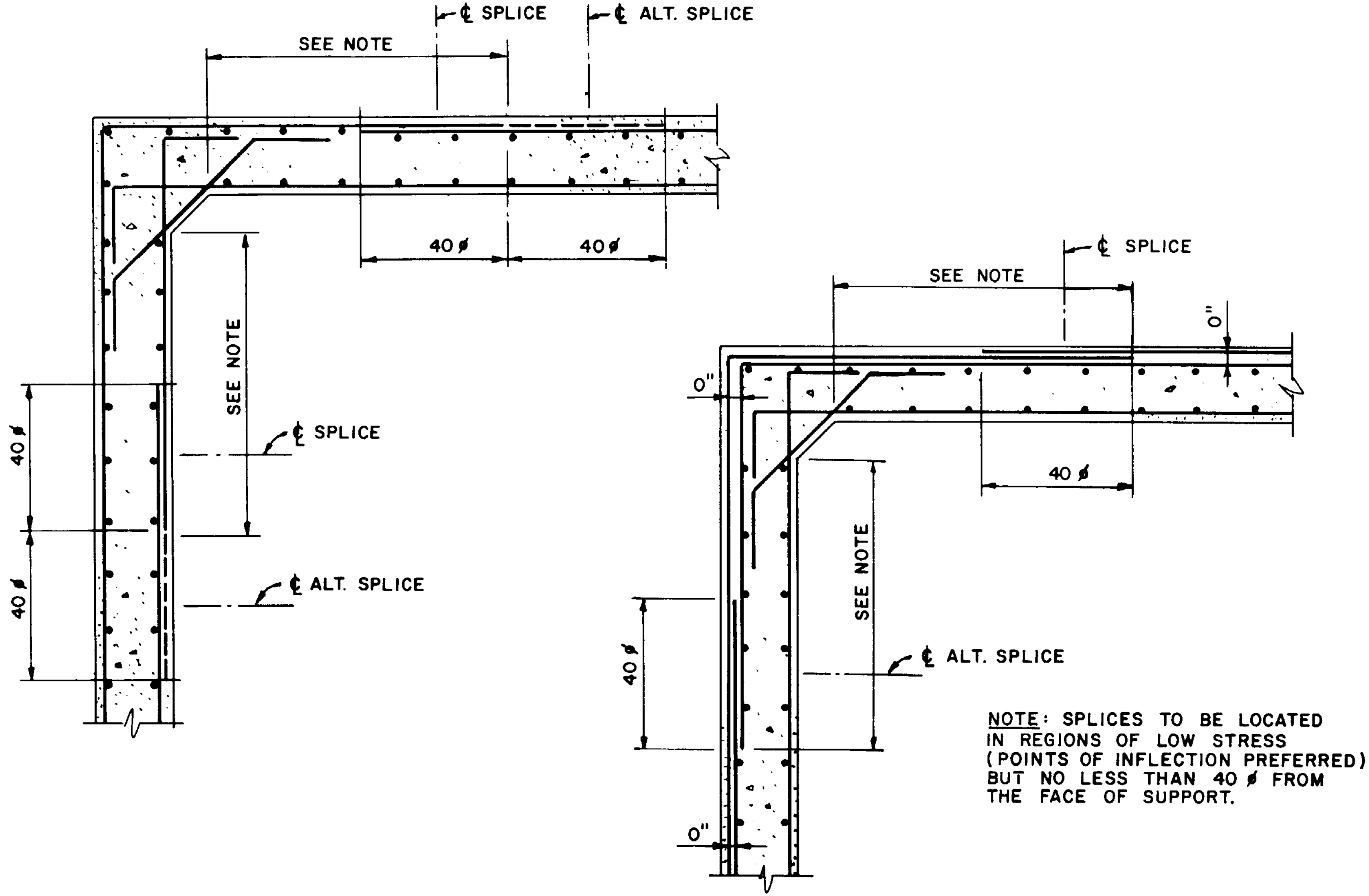


Figure 4-85 Typical horizontal corner details of conventionally reinforced concrete walls

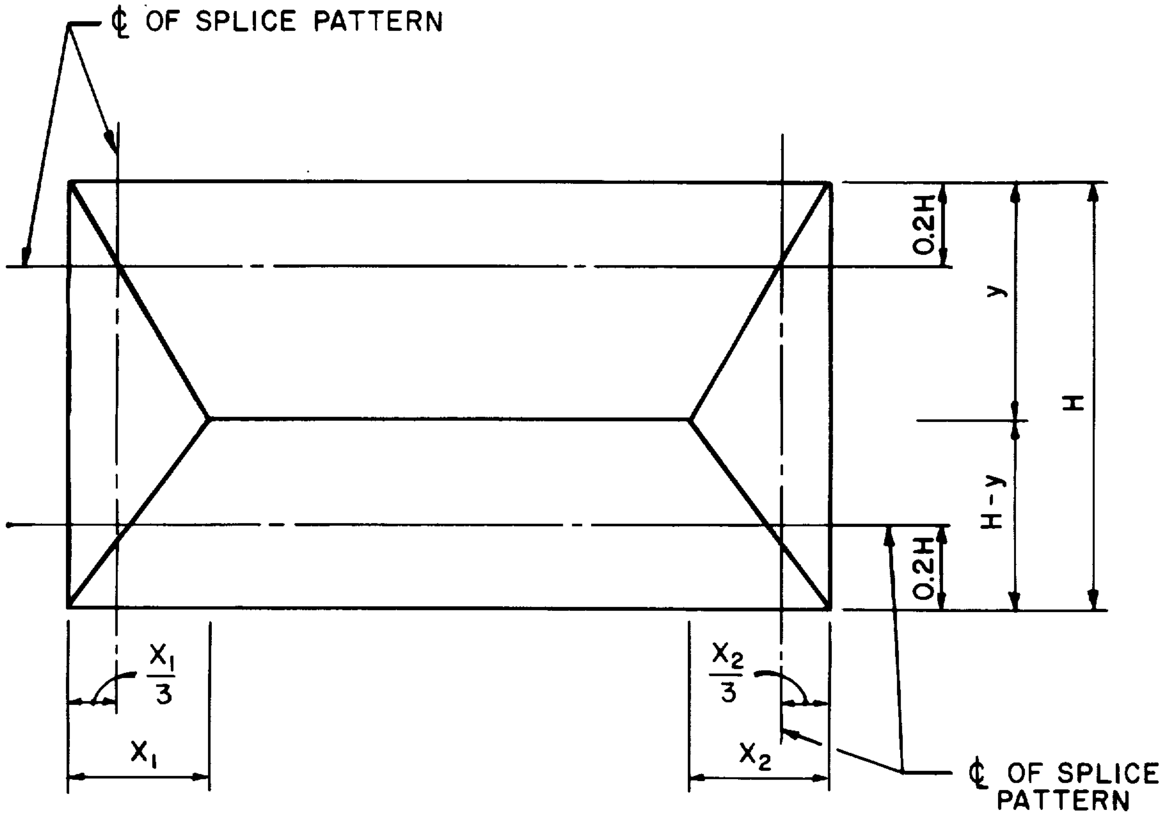
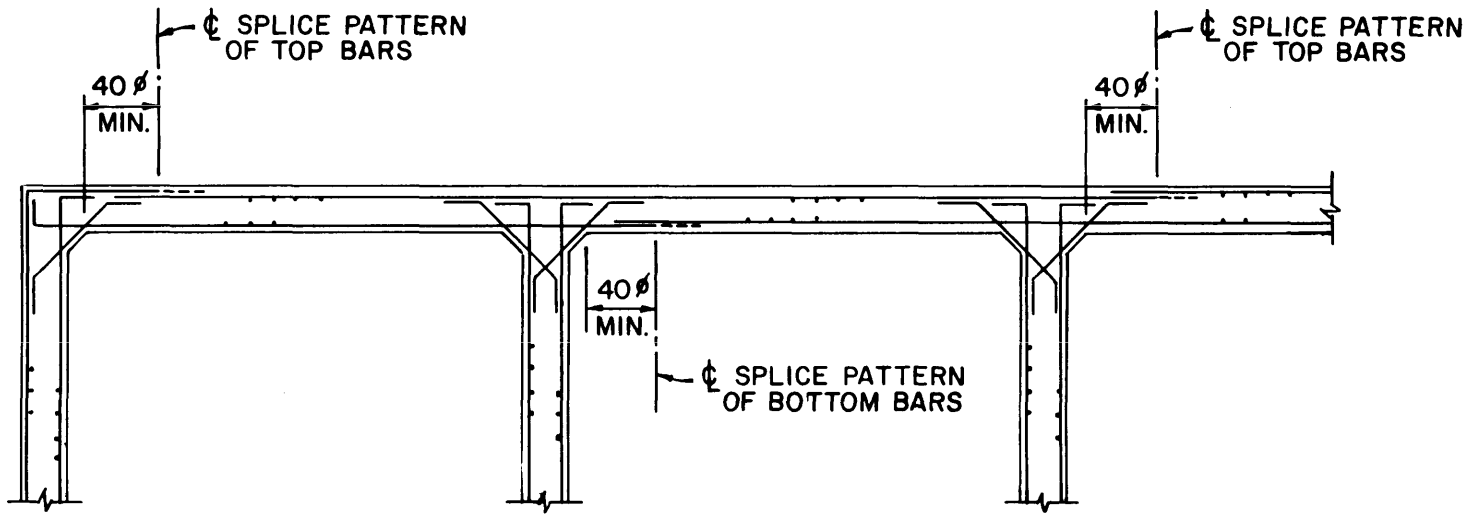


Figure 4-86 Preferred location of lap splices for a two-way element fixed on four edges



NOTE: ℄'s OF SPLICE PATTERNS TO BE LOCATED IN REGION OF LOW STRESS (POINTS OF INFLECTION PREFERRED)

Figure 4-87 Splice locations for multi-span slab

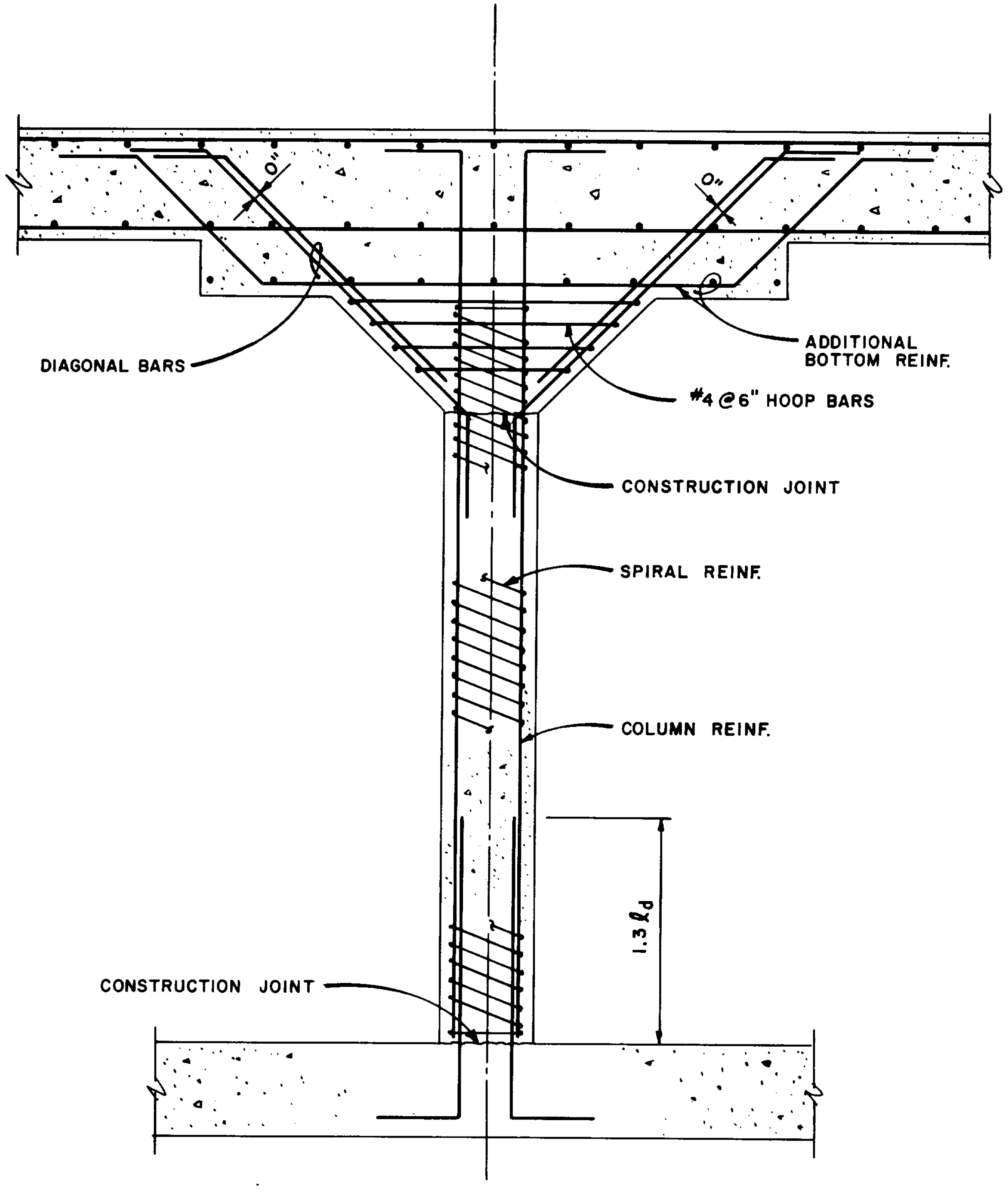


Figure 4-88 Section through column of flat slab

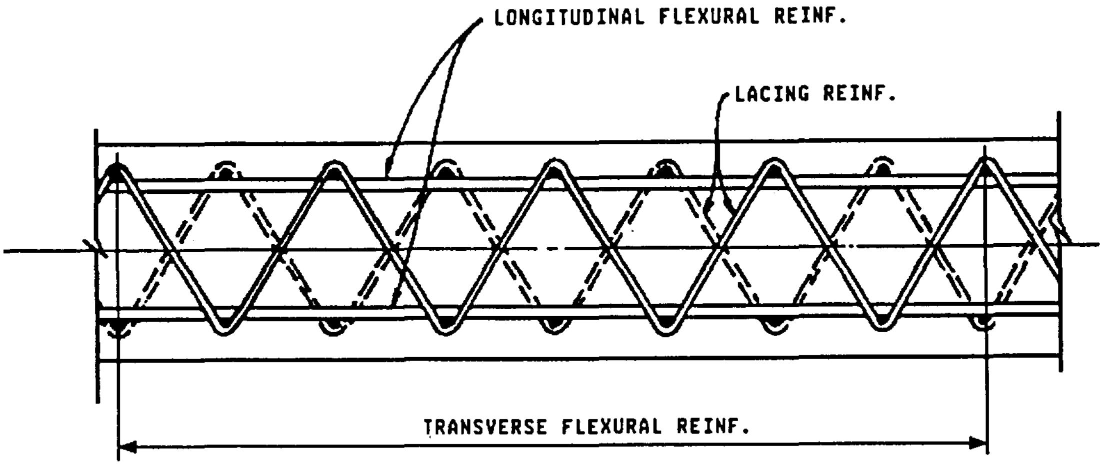


Figure 4-89 Laced reinforced element

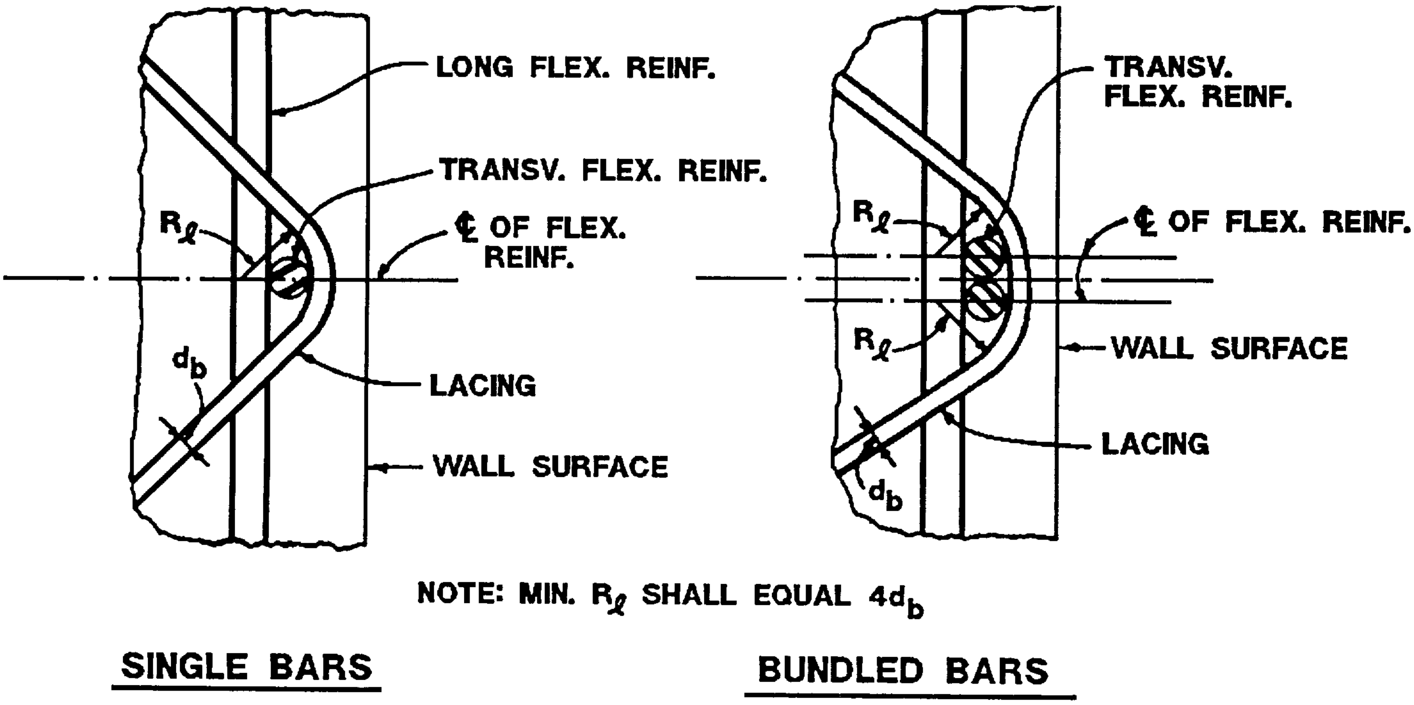


Figure 4-90 Typical lacing bend details

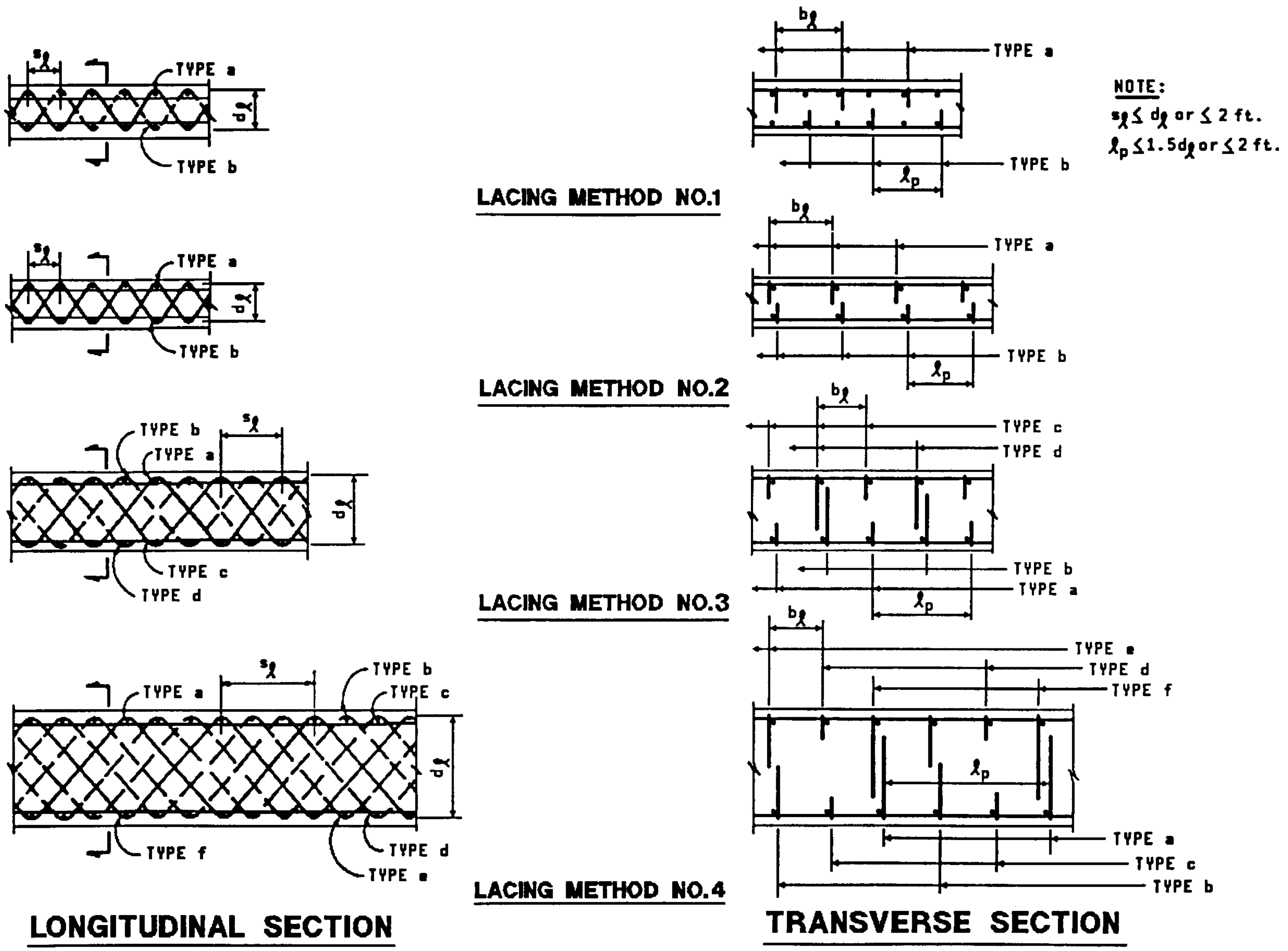


Figure 4-91   Typical methods of lacing

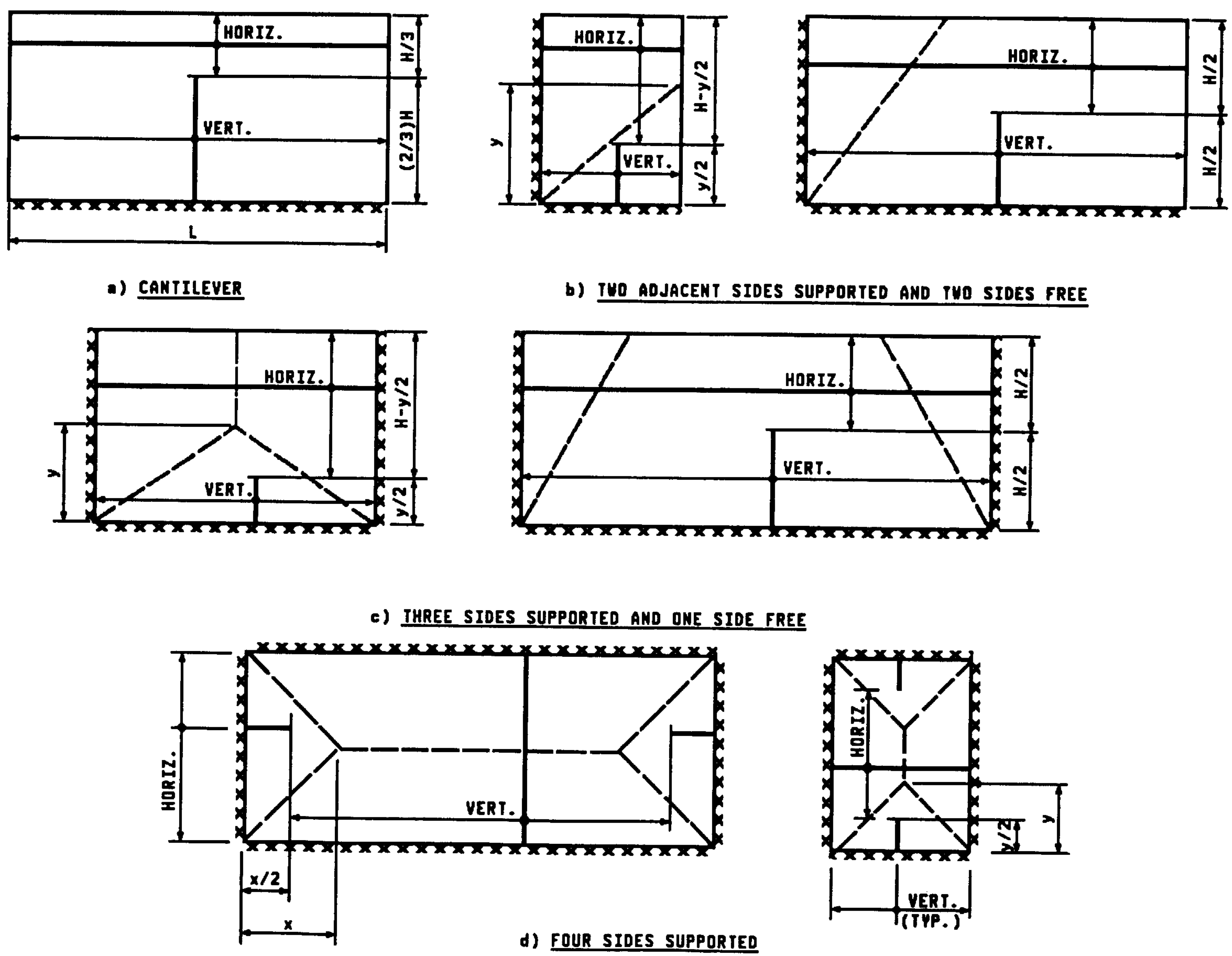


Figure 4-92 Typical location of continuous and discontinuous lacing

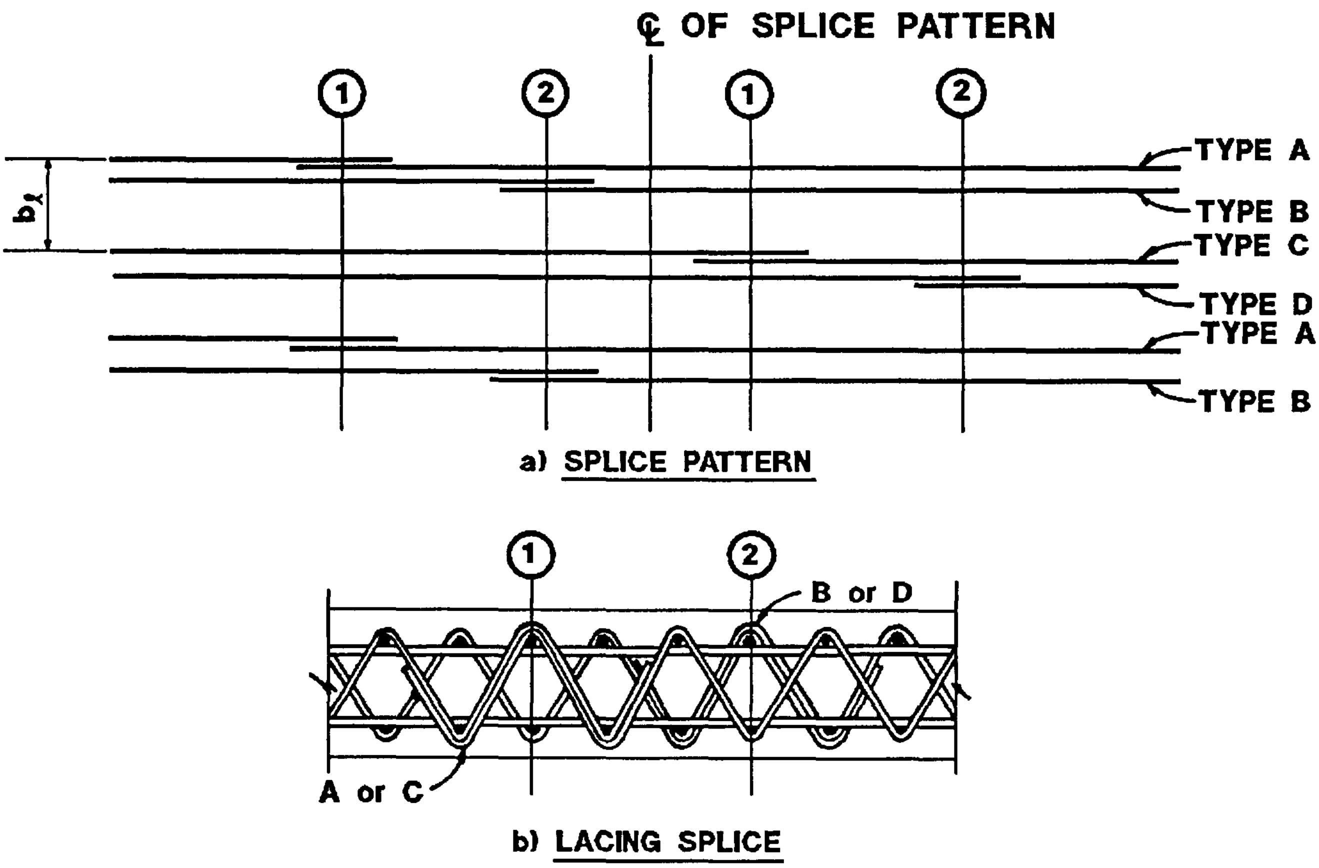
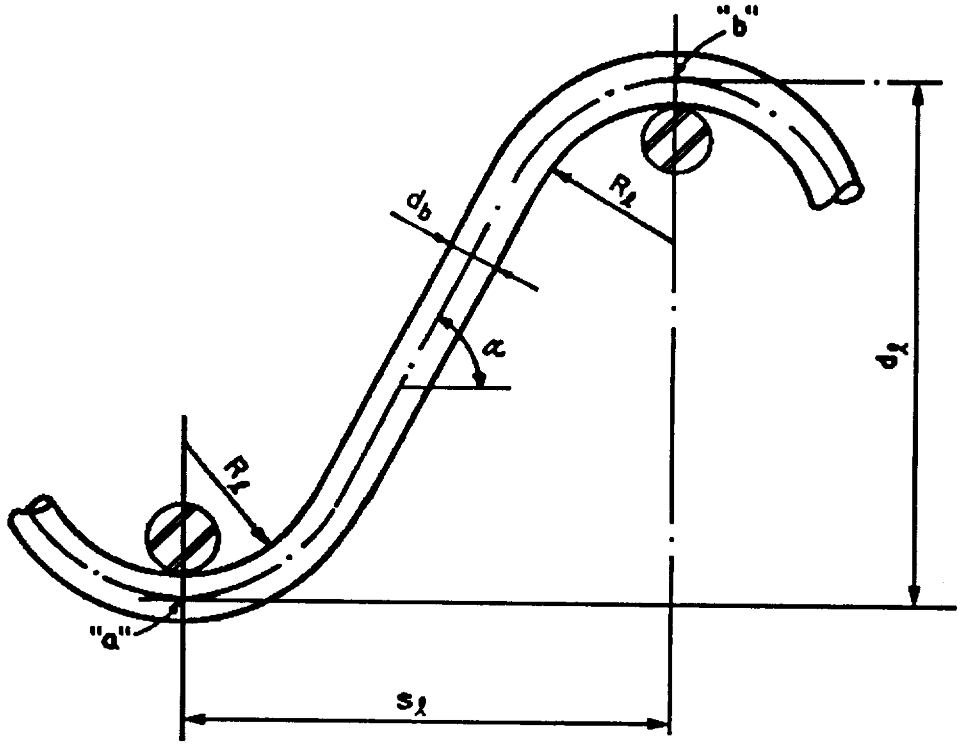


Figure 4-93 Typical details for splicing of lacing bars



NOTE: L_ℓ IS MEASURED ALONG CENTER LINE OF LACING BAR BETWEEN POINTS "a" AND "b"

$$L_\ell = \frac{s_\ell - (2R_\ell + d_b)\text{SIN}\,\alpha}{\text{COS}\,\alpha} + \pi(2R_\ell + d_b)\left(\frac{\alpha}{180}\right)$$

$$\text{COS}\,\alpha = \frac{-2B(1-B) \pm \sqrt{[2B(1-B)]^2 - 4[(1-B)^2 + A^2][B^2 - A^2]}}{2[(1-B)^2 + A^2]}$$

$$A = s_\ell / d_\ell \quad \text{AND} \quad B = \frac{2R_\ell + d_b}{d_\ell}$$

Figure 4-94 Length of lacing bars

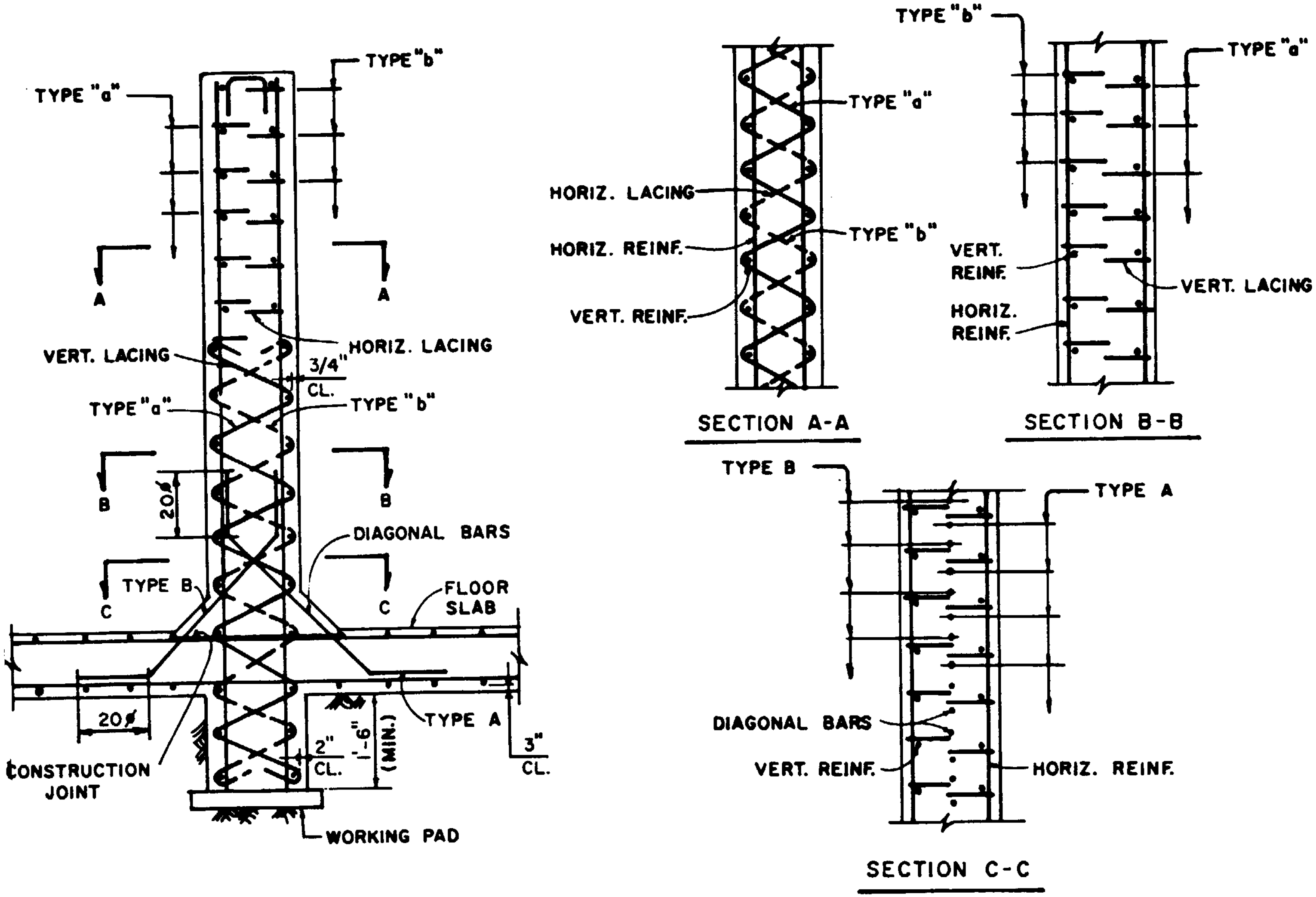


Figure 4-95 Reinforcement details of laced concrete walls

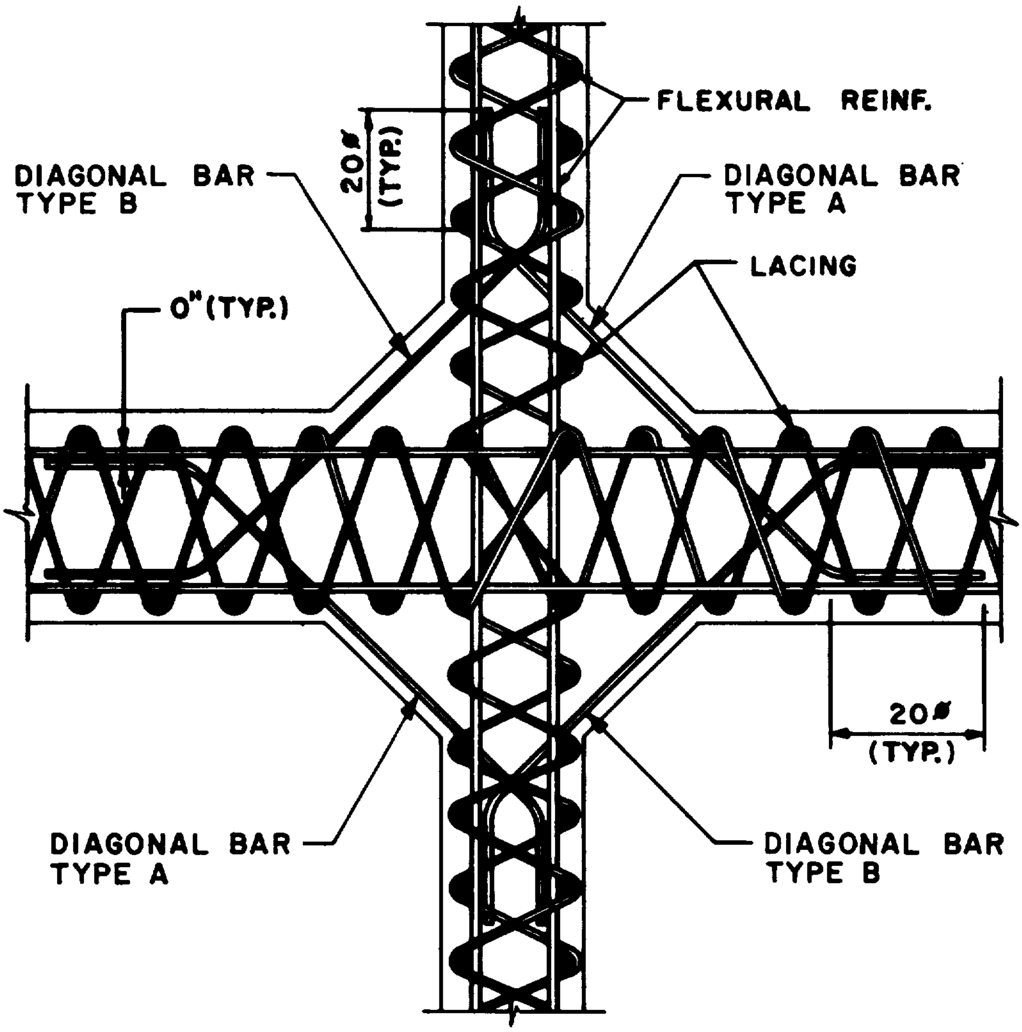


Figure 4-96 Typical detail at intersection of two continuous laced walls

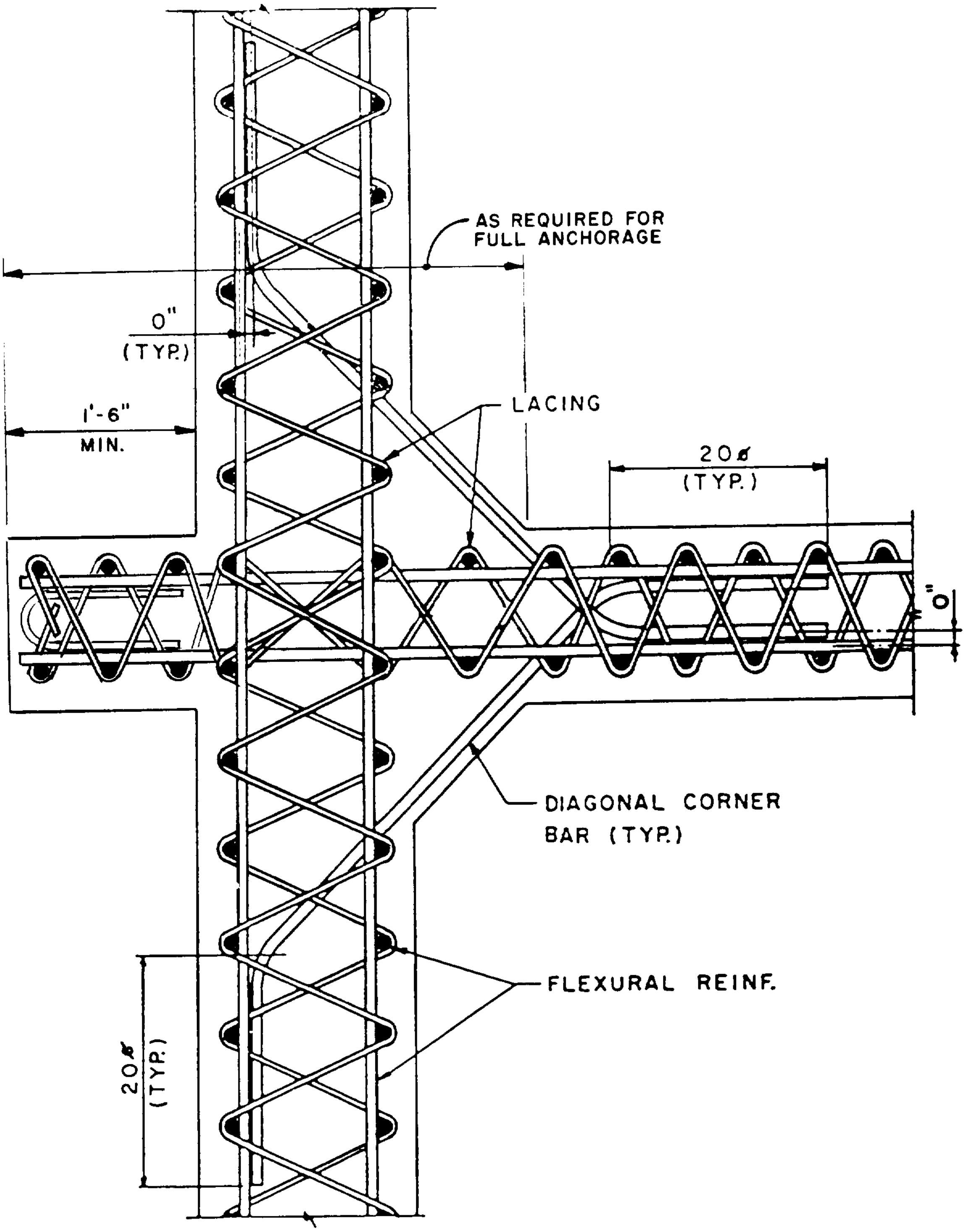


Figure 4-97 Typical detail at intersection of continuous and discontinuous laced walls

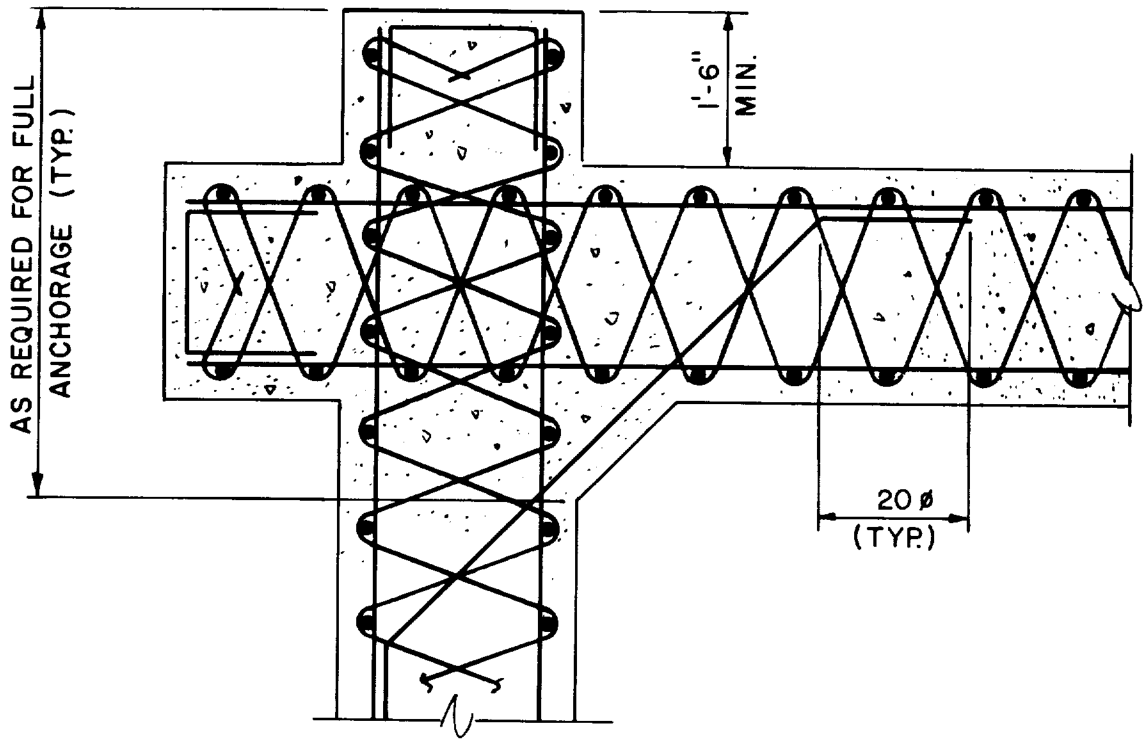


Figure 4-98 Typical detail at corner of laced walls

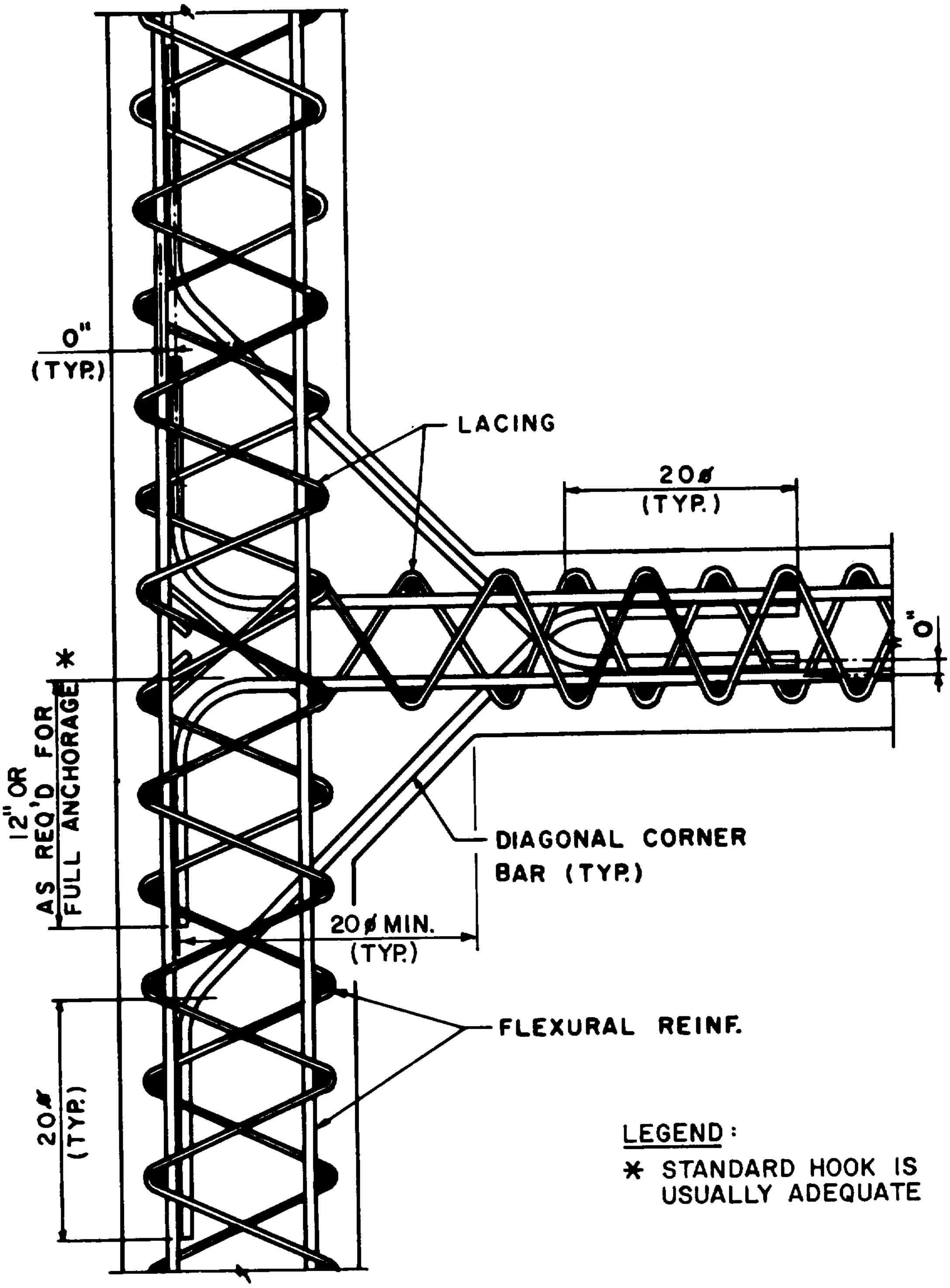


Figure 4-99 Intersection of continuous and discontinuous laced walls without wall extensions

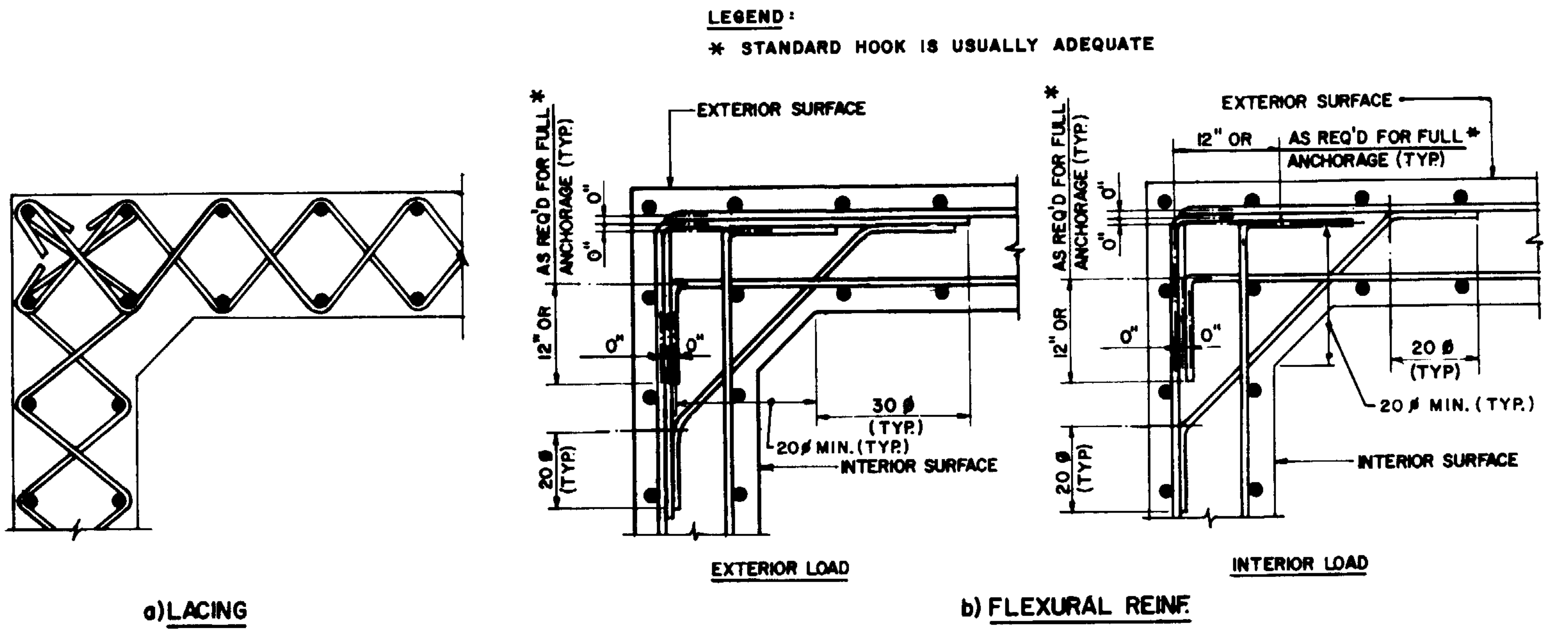


Figure 4-100 Corner details for laced walls without wall extensions

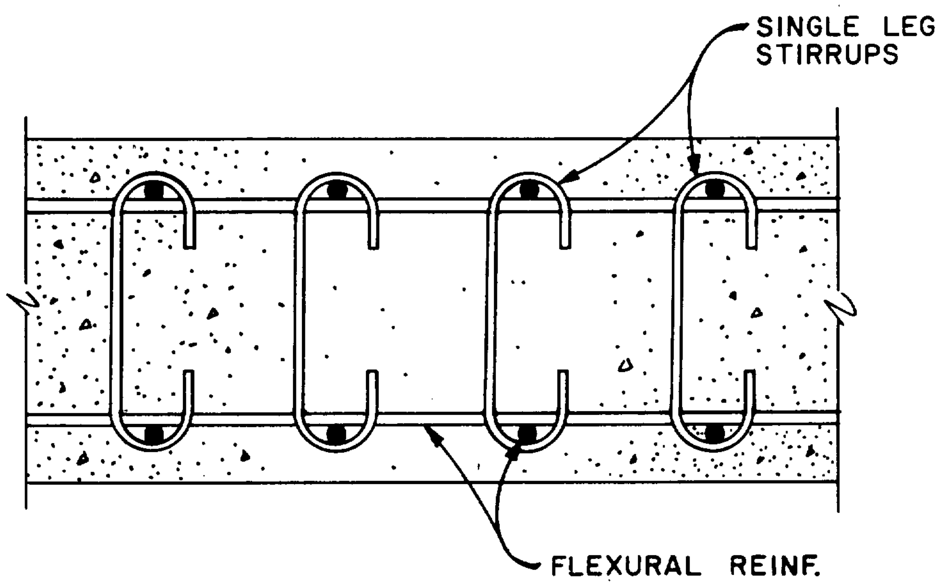


Figure 4-101 Element reinforced with single leg stirrups

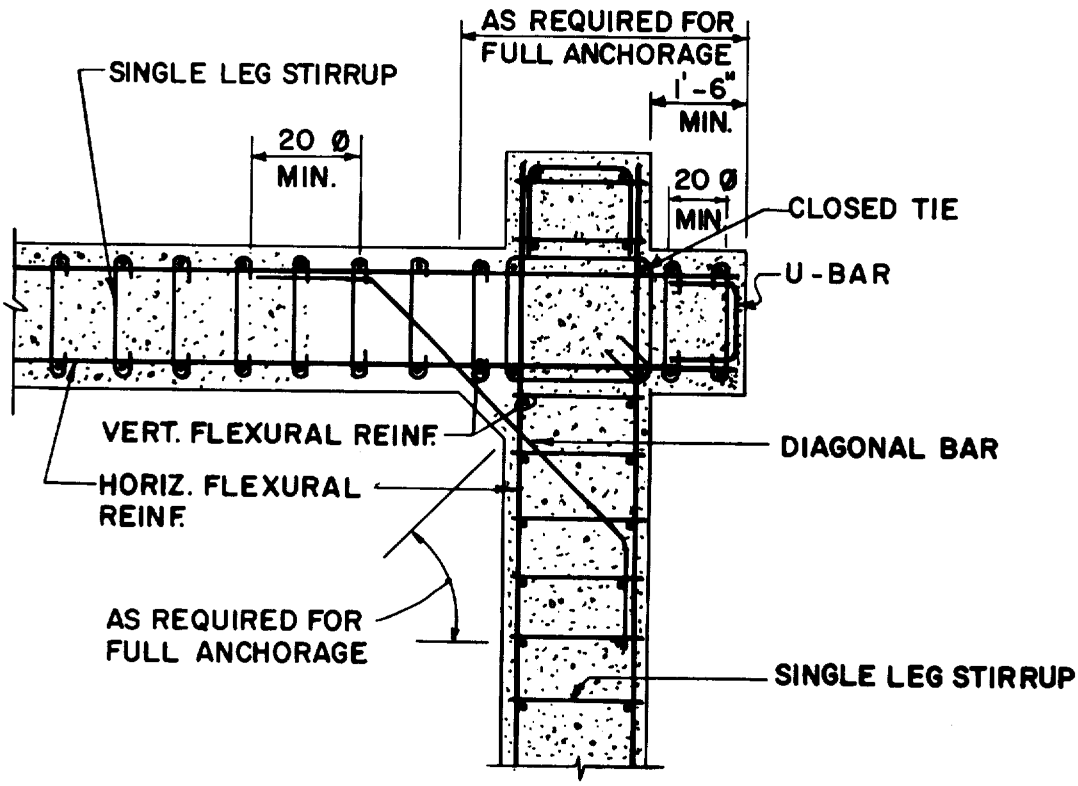


Figure 4-102 Typical detail at corner for walls reinforced with single leg stirrups

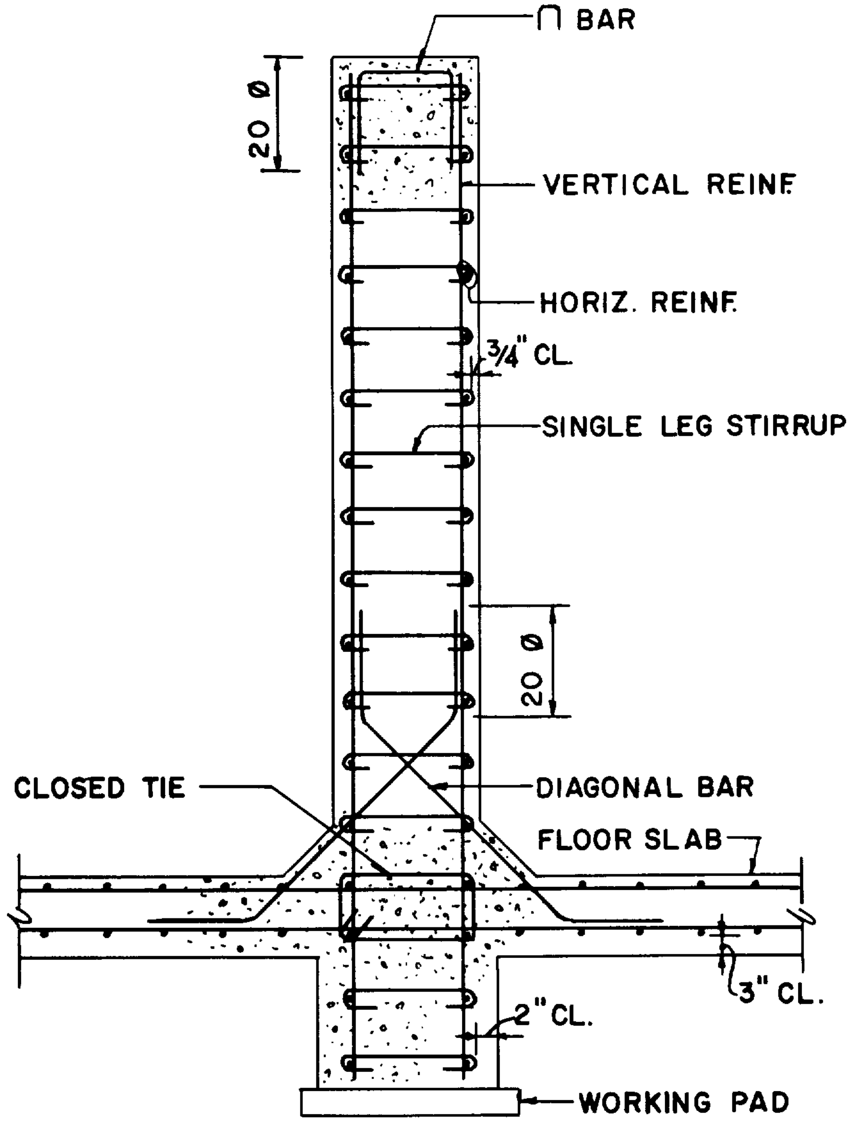


Figure 4-103 Reinforcement detail of wall with single leg stirrups

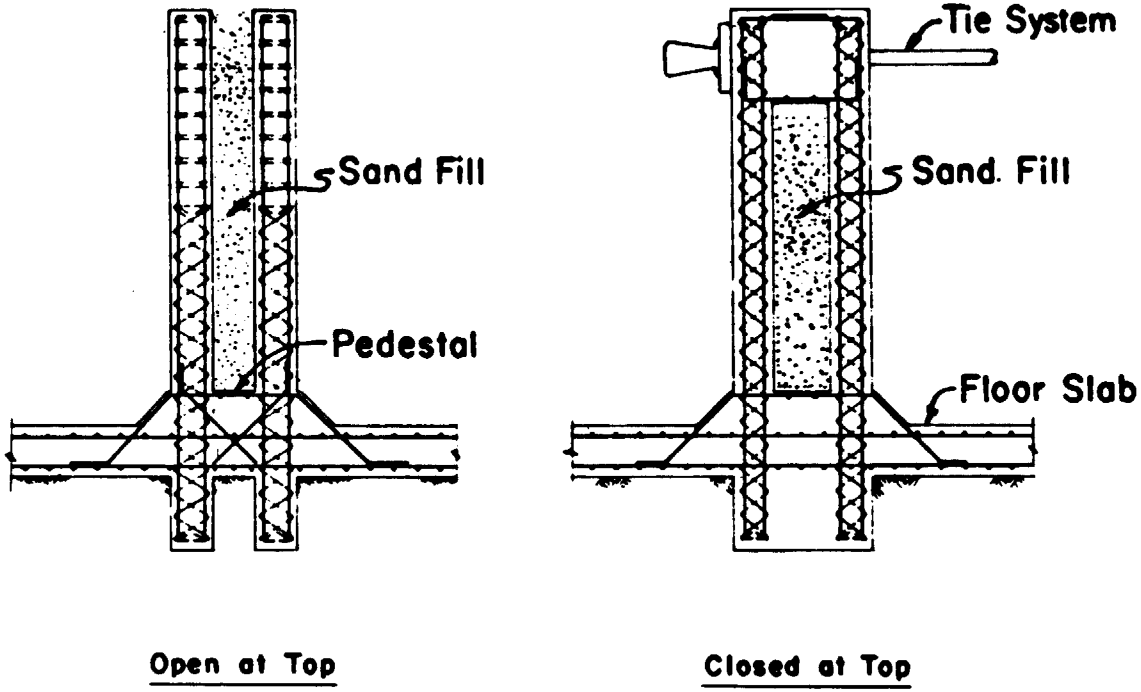


Figure 4-104 Typical composite wall details

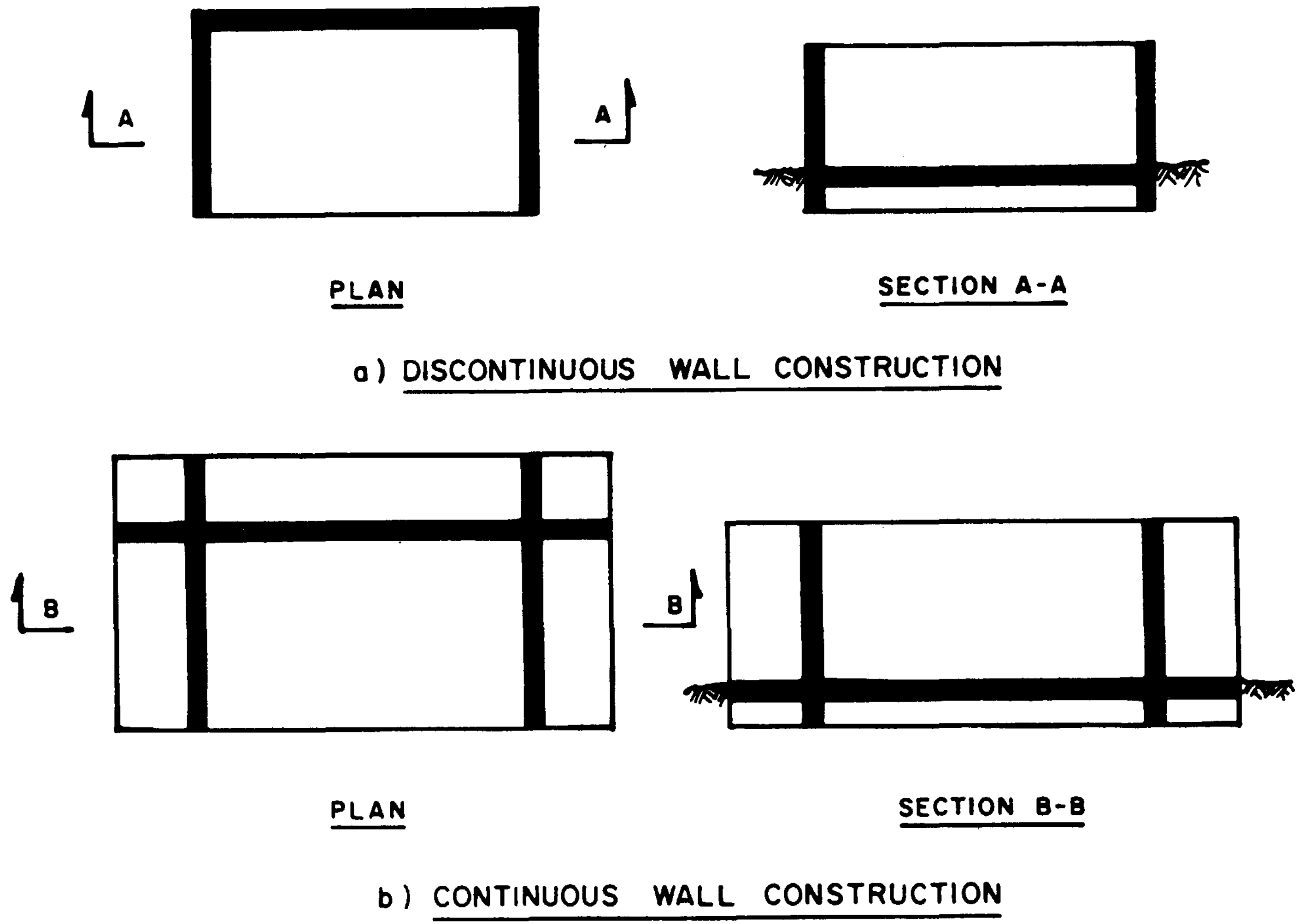
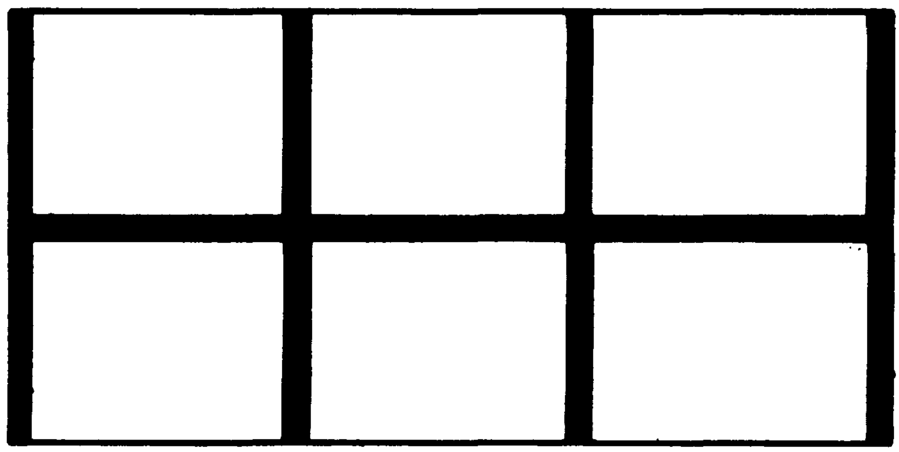
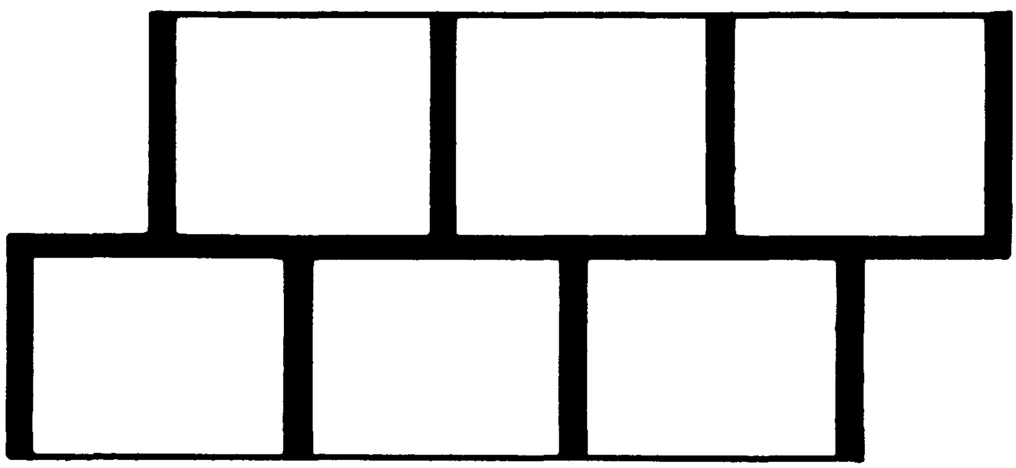


Figure 4-105 Typical single-cell structures



SCHEME a) CONTINUOUS SIDE WALL CONSTRUCTION



SCHEME b) DISCONTINUOUS SIDE WALL CONSTRUCTION

Figure 4-106 Typical multicubicle structures

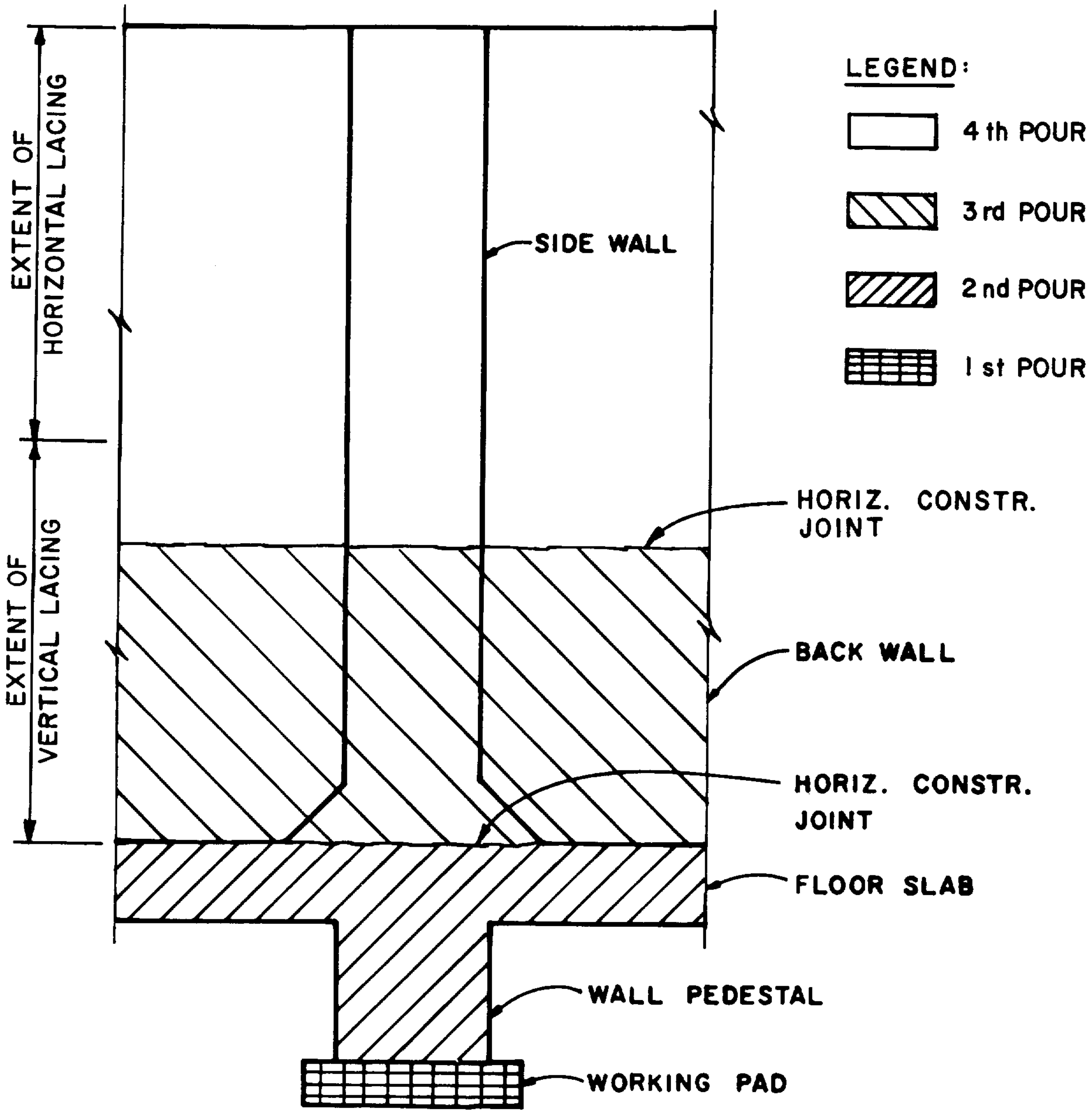


Figure 4-107 Pouring sequence

APPENDIX 4A

ILLUSTRATIVE EXAMPLES

Problem 4A-1. Elements Designed for the Pressure-Time Relationship

Problem: Design an element which responds to the pressure-time relationship.

Note:
Steps 5, 8, 10 through 18, 22 and 23 are specific for two-way elements, however, references are given defining similar procedures for one-way elements.

Procedure:

Step 1. Establish design parameters:

a. Blast loads including pressure-time relationship (Chapter 2).
b. Deflection criteria.
c. Structural configuration including geometry and support conditions.
d. Type of section available to resist blast, Type I, II or III depending upon the occurrence of spalling and/or crushing of the concrete cover.

Step 2. Select cross section of element including thickness and concrete cover over reinforcement. Also determine static stresses of concrete and reinforcing steel (Section 4-12).

Step 3. Determine dynamic increase factors for both concrete and reinforcement from table 4-1. Using the above DIF and the static stresses of step 2, calculate the dynamic strength of materials.

Step 4. Determine the dynamic design stresses using table 4-2 and the results from step 3.

Step 5. Assume vertical and horizontal reinforcement bars to yield the optimum steel distribution. The steel distribution is optimum when the resulting yield lines make an angle of 45 degrees with supports.

Step 6. Calculate d_e (d or d_c, depending upon type of cross section available to resist blast) for both the positive and negative moments in both vertical and horizontal directions. Determine reinforcing ratios. Also check for minimum steel ratios from table 4-3.

Step 7. Using the area of reinforcement, the value of d_e from step 6, and the dynamic design stresses of step 4, calculate the moment capacity (Section 4-17) of both the positive and negative reinforcement.

Note: Steps 8, 10 through 18 are required to determine the actual and equivalent resistance-deflection curves for two-way elements. To obtain these curves for one-way elements, see problem 4A-6.

Step 8. Using the equations of table 3-2 or table 3-3 and the moment capacities of step 7. calculate the ultimate resistance in the plastic range.

Step 9. Using equation 4-4, the static concrete stress of step 2, and unit weight for concrete equal to 150 psf, calculate the modulus of elasticity for concrete. With the above modulus for concrete and that for steel (eq. 4-5) and equation 4-6, calculate the modular ratio.

Step 10. With the use of equation 4-9a and the assumed concrete thickness of step 2, calculate the gross moment of inertia of the concrete. Using the value of d_e for the negative and positive reinforcement of step 6, calculate an average value of d_e. Also calculate an average percent of positive and negative reinforcement using the above d_e and the area of reinforcement of step 6 in both vertical and horizontal directions. With the values of p (average) and figure 4-11 or 4-12, determine the values of the constants F and calculate the moment of inertia of a cracked section with equation 49b in both directions. Calculate the average cracked moment of inertia for the element using equation 4-10, and also, the average moment of inertia of the element from equation 4-7.

Step 11. Using equation 3-33, and the modulus of elasticity of step 9 and the moment of inertia of step 10, calculate the unit flexural rigidity.

Step 12. Establish points of interest and their ultimate moment capacities (fig. 3-23).

Step 13. Compute properties of first yield.

a. Location of first yield.
b. Resistance at first yield r_e.
c. Moments at remaining points consistent with r_e.
d. Maximum deflection at first yield.

Step 14. Compute properties at second yield.

a. Remaining moment capacity at other points.
b. Location of second yield.
c. Change in unit resistance Δr, between first and second yield.
d. Unit resistance at second yield r_{ep}.
e. Moment at remaining points consistent with r_{ep}.
f. Change in maximum deflection.
g. Total maximum deflection.

Note:

An element with unsymmetrical support conditions may exhibit three or four support yields. Therefore, repeat step 14 as many times as necessary to obtain properties at various yield points.

Step 15. Compute properties at final yield (ultimate unit resistance).

a. Ultimate unit resistance.

b. Change in resistance between ultimate unit resistance and resistance at prior yield.

c. Change in maximum deflection (for elements supported on two, three or four edges, use stiffness obtained from figure 3-26, 3-30 and 3-36, respectively).

d. Total maximum deflection.

Step 16. Draw the actual resistance-deflection curve (fig. 3-39).

Step 17. Calculate equivalent maximum elastic deflection of the element.

Step 18. Calculate the equivalent elastic unit stiffness K_E from equation 3-36.

Step 19. Determine the load-mass factor K_{LM} for the elastic, elasto-plastic and plastic ranges from table 3-13 and figure 3-44. The average load mass factor is obtained by taking the average K_{LM} for the elastic and elasto-plastic ranges and averaging this value with the K_{LM} of the plastic range. In addition, calculate the unit mass of the element (account for reduced concrete thickness if spalling is anticipated) and multiply this unit mass by K_{LM} for the element to obtain the effective unit mass of the element.

Note:

For one-way elements, use table 3-12 to determine the average load mass factor.

Step 20. Using the effective mass of step 19 and the equivalent stiffness, calculate the natural period of vibration T_N from equation 3-60.

Step 21. Determine the response chart parameters:

a. Peak pressure P (step 1).

b. Peak resistance r_u (step 8).

c. Duration of load T (step 1).

d. Natural period of vibration T_N (step 20).

Also calculate the ratios of peak pressure P to peak resistance r_u and duration T to period T_N. Using these ratios and the response charts of Chapter 3, determine the value of X_m/X_E and t_m/T_N . Compute the value of X_m and compare it to the maximum permissible deflection of step 1, and if found satisfactory, proceed to step 22. If comparison is unsatisfactory, repeat steps 2 to step 21. In addition, compute the value of t_m/t_o from t_m/T_N and T/T_N and assuming that $T = t_o$, determine whether or not correct procedure has been used; for elements to respond to the pressure-time relationship, $0.1 < t_m/t_o < 3$.

Step 22. Using the ultimate resistance of step 8, the value of d_e of step 6 and equations of table 4-7, calculate the ultimate diagonal tension shear stresses at distance d_e from each support. Also calculate the shear capacity of the element from equation 4-23. If the capacity is greater than that produced by the load, shear reinforcement is not required. However, if the shear produced by the load is greater than the capacity, then shear reinforcement must be added to resist the excess.

Note:

For one-way elements, use table 4-6 to establish diagonal tension shear stress.

Step 23. Using the equations of table 3-10 or 3-11 and the ultimate resistance of step 8, calculate the shear at the supports. Determine required area of diagonal bars using equation 4-30. However, if section type I is used, then the minimum diagonal bars must be provided (eq. 4-31).

Note:

For one-way elements, use table 3-9 to calculate the shear at the supports.

Example 4A-1, Elements Designed for the Pressure-Time Relationship

Required: Design a wall which spans in two directions and is fully restrained at all supports for a given blast load.

Solution:

Step 1. Given:

a. Pressure-time loading (fig. 4A-1).

b. Maximum deflection equal to 3 times elastic deflection.

c. L = 180 in., H = 144 in. and fixed on four sides (fig. 4A-1).

d. Type I cross section.

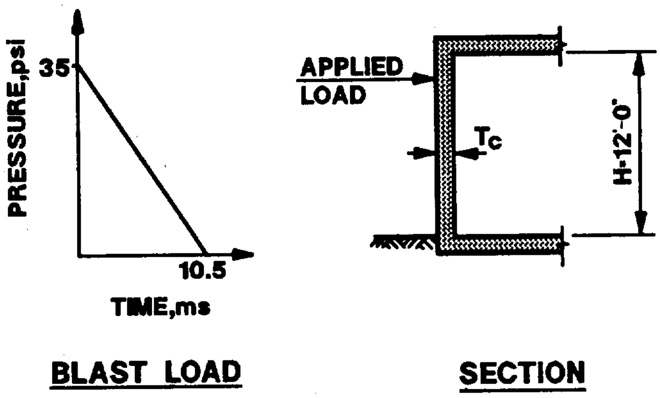
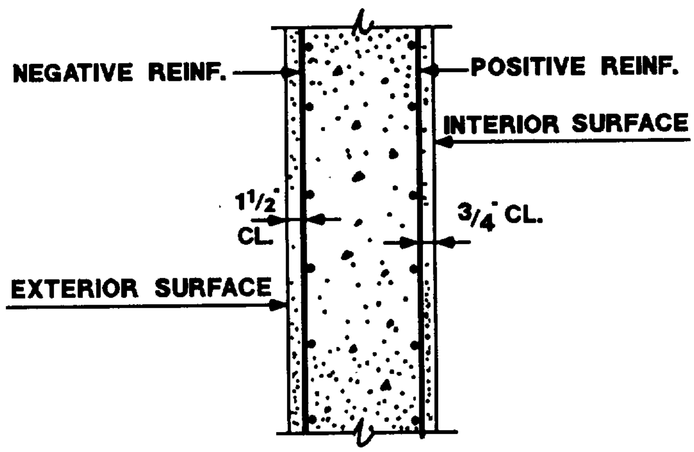


Figure 4A-1

Step 2. Select cross section of element and static stress of reinforcement and concrete (fig. 4A-2).



f'_c = 4,000 psi

f_y = 66,000 psi

Assume T_c = 12 in. and concrete cover as shown.

Figure 4A-2

Step 3. Determine dynamic stresses.

a. Dynamic increase factors - DIF (table 4-1).

Concrete:

Bending	- 1.19
Diagonal tension	- 1.00

Reinforcement:

Bending	- 1.17
Diagonal tension	- 1.00
Direct shear	- 1.10

b. Dynamic strength of materials

Concrete (f'_{dc}):

Compression	- 1.19 (4,000) = 4,760 psi
Diagonal tension	- 1.00 (4,000) = 4,000 psi

Reinforcement (f_{dy}):

Bending	- 1.17 (66,000) = 77,200 psi
Diagonal tension	- 1.00 (66,000) = 66,000 psi
Direct shear	- 1.10 (66,000) = 72,600 psi

Step 4. Dynamic design stresses from table 4-2.

Concrete (f'_{dc}):

Compression	- 4,760 psi
Diagonal tension	- 4,000 psi

Reinforcement ($f_{ds} = f_{dy}$):

Bending	- 77,200 psi
Diagonal tension	- 66,000 psi
Direct shear	- 72,600 psi

Step 5. In order to obtain optimum steel ratio p_V/p_H, set x = H/2 to have 45 degrees yield lines.

$$\frac{x}{L} = \frac{H}{2L} = \frac{144}{2 \times 180} = 0.40$$

From figure 3-17,

$$\frac{L}{H}\left[\frac{M_{VN} \quad M_{VP}}{M_{HN} + M_{HP}}\right]^{1/2} = 1.43$$

Therefore,

$$\frac{M_{VN} + M_{VP}}{M_{HN} + M_{HP}} = \left[\frac{1.43 \times 144}{180}\right]^2 = 1.31$$

Try No. 4 bars at 10 in. o.c. in vertical direction, and No. 4 bars at 12 in. o.c. in horizontal direction.

Step 6. Calculate d_c and steel ratios for each direction, minimum reinforcement ratio is equal to 0.15 percent.

Vertical:

Vertical reinforcement bars are No. 4 at 10 in. o.c. from step 5.

$$A_{sV} = .20 \text{ X } 12/10 = 0.24 \text{ in }/\text{ft}^2$$

Negative moment $d_V = 12 - 1.5 - 0.25 = 10.25$ in

Positive moment $d_V = 12 - 0.75 - 0.25 = 11.0$ in

$$p_V = \frac{A_{sV}}{bd_V} = \frac{0.24}{12 \text{ X } 11.0} = 0.00182 > 0.0015 \text{ O.K.}$$

Horizontal:

Horizontal reinforcement bars are No. 4 at 12 in. o.c. from step 5.

$$A_{sH} = .20 \text{ X } 12/12 = 0.20 \text{ in }/\text{ft}_2$$

Negative moment $d_H = 10.25 - 0.25 - 0.25 = 9.75$ in

Positive moment $d_H = 11.0 - 0.25 - 0.25 = 10.5$ in

$$p_H = \frac{A_{sH}}{bd_H} = \frac{.20}{12 \text{ X } 10.5} = 0.00158 > 0.0015 \text{ O.K.}$$

Step 7. Calculate moment capacity of both positive and negative reinforcement in both directions.

a. Depth of equivalent rectangular stress blocks.

$$a = \frac{A_s f_{ds}}{.85\, bf'_{dc}} \qquad \text{(eq. 4-12)}$$

$$a_V = \frac{.24 \text{ X } 77{,}200}{.85 \text{ X } 12 \text{ X } 4760} = .382 \text{ in}$$

$$a_H = \frac{.20 \text{ X } 77{,}200}{.85 \text{ X } 12 \text{ X } 4760} = .318 \text{ in}$$

b. Moment capacity (eq. 4-11).

$$M_u = \frac{A_s f_{ds}}{b} (d - a/2)$$

$$M_{VN} = \frac{0.24\ (77,200)(10.25 - .382/2)}{12} = 15531 \text{ in-lbs/in}$$

$$M_{VP} = \frac{0.24\ (77,200)\ (11.0 - .382/2)}{12} = 16689 \text{ in-lbs/in}$$

$$M_{HN} = \frac{0.20\ (77,200)\ (9.75 - .318/2)}{12} = 12340 \text{ in-lbs/in}$$

$$M_{HP} = \frac{0.20\ (77,200)\ (10.5 - .318/2)}{12} = 13305 \text{ in-lbs/in}$$

Step 8. Determine ultimate resistance of the element.

$$\frac{L}{H}\left[\frac{M_{VN} + M_{VP}}{M_{HN} + M_{HP}}\right]^{1/2} = \frac{180}{144}\left[\frac{15531 + 16689}{12340 + 13305}\right]^{1/2} = 1.40 = 1.43 \text{ (step 5)}$$

From figure 3-17,

$$\frac{x}{L} = .405$$

$$x = .405 \text{ X } 180 = 72.9 \text{ in}$$

Ultimate resistance (table 3-2).

$$r_u = \frac{5\ (M_{HN} + M_{HP})}{x^2} = \frac{5(12,340 + 13,305)}{(72.9)^2} = 24.13 \text{ psi}$$

Step 9. Determine modulus of elasticity and modular ratio.

a. Concrete (eq. 4-4)

$$E_c = w^{1.5}\ 33\ (f'_c)^{1/2} = (150)^{1.5}\ (33)(4000)^{1/2}$$

$$= 3.83 \times 10^6 \text{ psi}$$

b. Steel (eq. 4-5)

$$E_s = 29 \times 10^6 \text{ psi}$$

c. $$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.83 \times 10^6} = 7.56 \qquad \text{(eq. 4-6)}$$

Step 10. Determine average moment of inertia for an inch strip.

a. Gross moment of inertia (eq. 4-9a)

$$I_g = \frac{T_c^{\,3}}{12} = \frac{12^3}{12} = 144 \text{ in}^4/\text{in}$$

b. Moment of inertia of cracked section (eq. 4-9b).

Vertical direction:

$$d_{(avg)} = \frac{10.25 + 11.0}{2} = 10.625 \text{ in}$$

$$p_{(avg)} = \frac{A_s}{bd_{(avg)}} = \frac{0.24}{12(10.625)} = 0.00188$$

$\therefore F = 0.0102$ (fig. 4-12)

$$I_{cV} = Fd_{(avg)}^{\ 3} = .0102 \times (10.625)^3 = 12.2 \text{ in}^4/\text{in}$$

Horizontal direction:

$$d_{(avg)} = \frac{9.75 + 10.50}{2} = 10.125 \text{ in}$$

$$P_{(avg)} = \frac{.20}{12\ (10.125)} = .00165$$

$$\therefore F = 0.0092$$

$$I_{cH} = .0092 \times (10.125)^3 = 9.5 \text{ in}^4/\text{in}$$

c. Average moment of inertia of cracked section.

$$I_c = \frac{LI_{cV} + HI_{cH}}{L + H} \qquad \text{(eq. 4-10)}$$

$$I_c = \frac{(180 \times 12.2) + (144 \times 9.5)}{180 + 144} = 11.0 \text{ in}_4/\text{in}$$

d. Average moment of inertia (eq. 4-7).

$$I_a = \frac{I_g + I_c}{2} = \frac{144.0 + 11.0}{2} = 77.5 \text{ in}^4/\text{in}$$

Step 11. Calculate unit flexural rigidity.

$$D = \frac{E_c I_a}{1 - \nu^2} \qquad \text{(eq. 3-33)}$$

Use $\nu = .167$ for concrete

Therefore,

$$D = \frac{(3.83 \times 10^6)\ 77.5}{1 - (.167)^2} = 305.34 \times 10^6 \text{ in-lbs}$$

Step 12. For points of interest, see figure 4A-3.

Step 13. Properties at first yield.

From figure 3-33 of Chapter 3 for $H/L = 0.80$

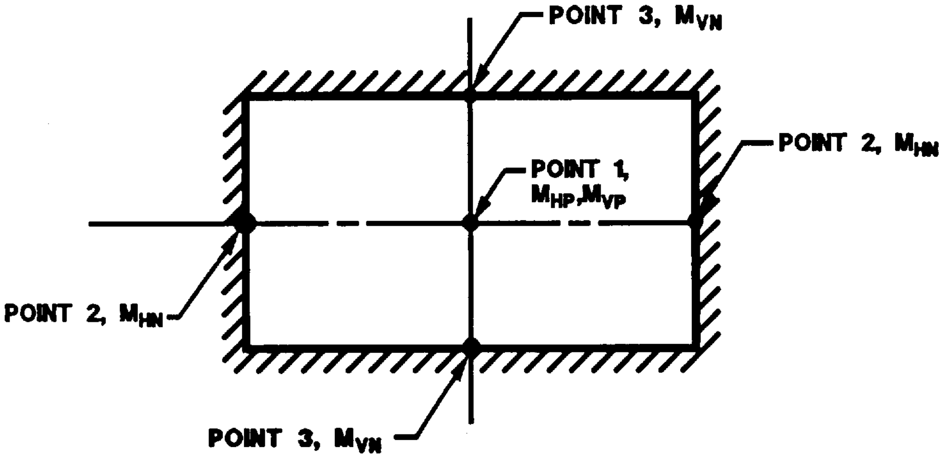


Figure 4A-3

$\beta_{1H} = 0.023$ $\qquad$ $\beta_2 = 0.056$

$\beta_{1V} = 0.031$ $\qquad$ $\beta_3 = 0.068$

$\gamma_1 = 0.0018$

a. $\quad M_{Hp} = 13.305$ in-lbs/in $\qquad$ $M_{HN} = 12,340$ in-lbs/in

$M_{Vp} = 16,689$ in-lbs/in $\qquad$ $M_{VN} = 15,531$ in-lbs/in

$$M = \beta r H^2 \quad \therefore r = \frac{M}{\beta H^2} \qquad \text{(eq. 3-25)}$$

$r_{1H} = 13,305/[0.023(144)^2] = 27.90$ psi

$r_{1V} = 16,689/[0.031\ (144)^2] = 25.96$ psi

$r_2 = 12,340/[0.056(144)^2] = 10.63$ psi

$r_3 = 15{,}531/[0.068(144)^2] = 11.01$ psi

First yield at point 2 (smallest r).

b. $r_e = 10.63$ psi

c. $M_{1H} = (0.023)(10.63)(144)^2 = 5{,}070$ in-lbs/in

$M_{1V} = (0.031)(10.63)(144)^2 = 6{,}833$ in-lbs/in

$M_3 = (0.068)(10.63)(144)^2 = 14{,}989$ in-lbs/in

d. $X_e = \gamma_1 r_e H^4/D$ (eq. 3-32)

$X_e = (0.0018)(10.63)(144)^4/305.34 \times 10^6 = 0.0269$ in

Step 14. Properties at second yield.

After first yield element assumes a simple-simple-fixed-fixed configuration, therefore, figure 3-34 for H/L = 0.80.

$\beta_{1H} = 0.020$

$\beta_{1V} = 0.039$ $\beta_3 = 0.076$

$\gamma_1 = 0.0022$

a. $M_{1H} = M_{HP} - M_{1H}$ (at r_e) $= 13{,}305 - 5{,}070 = 8{,}235$ in-lbs/in

$M_{1V} = M_{VP} - M_{1V}$ (at r_e) $= 16{,}689 - 6{,}833 = 9{,}856$ in-lbs/in

$M_3 = M_{VN} - M_3$ (at r_e) $= 15{,}531 - 14{,}989 = 542$ in-lbs/in

b. $$\Delta r = \frac{M}{\beta H^2}$$

$\Delta r_{1H} = 8{,}235/[0.020(144)^2] = 19.86$ psi

$\Delta r_{1V} = 9{,}856/[0.039(144)^2] = 12.19$ psi

$\Delta r_3 = 542/[0.076(144)^2] = 0.34$ psi

Second yield at point 3 (smaller Δr)

c. $\Delta r = 0.34$ psi

d. $r_{ep} = r_e + \Delta r = 10.63 + .34 = 10.97$ psi (eq. 3-26)

e. $M_{1H} = (.020)(0.34)(144)^2 + 5{,}070 = 5{,}211$ in-lbs/in

$M_{1V} = (0.039)(0.34)(144)^2 + 6{,}833 = 7{,}108$ in-lbs/in

f. $\Delta X = \gamma_1 \Delta r H^4/D$

$\Delta X = (0.0022)(0.34)(144)^4/305.34 \times 10^6 = 0.0011$ in

g. $X_{ep} = X_e + \Delta X = 0.0269 + 0.0011 = .028$ in

Step 15. Properties at final yield (ultimate unit resistance). After second yield element assumes a simple-simple-simple-simple configuration, therefore, from figure 3.36 for H/L = 0.80.

$\gamma_1 = 0.0054$

a. $r_u = 24.13$ psi (from step 8)

b. $\Delta r = r_u - r_{ep} = 24.13 - 10.97 = 13.16$ psi

c. $\Delta X = \gamma_1 \Delta r H^4/D$

$\Delta X = (0.0054)(13.16)(144)^4/305.34 \times 10^6 = 0.100$ in

d. $X_p = X_{ep} + \Delta X = 0.028 + 0.100 = 0.128$ in

Step 16. For actual resistance deflection curve, see figure 4A-4.

Step 17. Equivalent elastic deflection from equation 3-35.

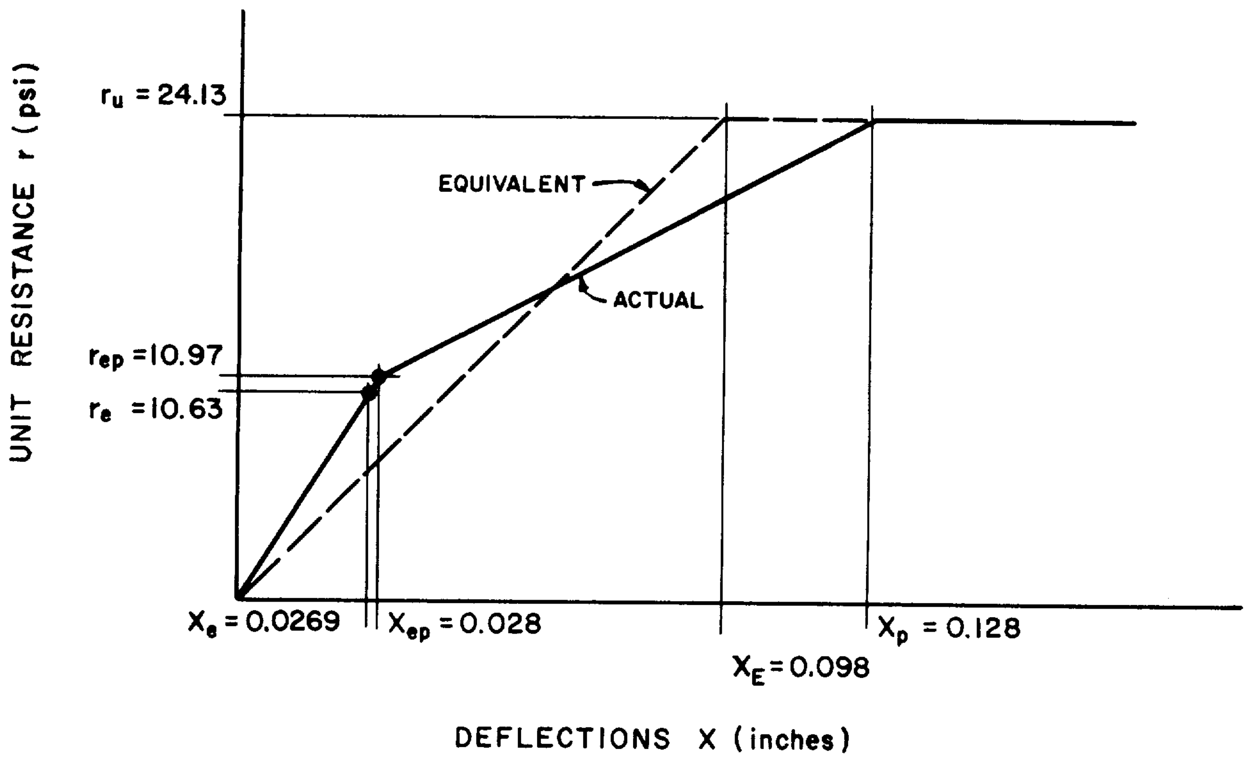

UNIT RESISTANCE r (psi)
r_u = 24.13
r_{ep} =10.97
r_e =10.63
EQUIVALENT
ACTUAL
X_e = 0.0269
X_{ep} = 0.028
X_p = 0.128
X_E = 0.098
DEFLECTIONS X (inches)

FIGURE 4A-4

$$X_E = X_e(r_{ep}/r_u) + X_{ep}[1 - (r_e/r_u)] + X_p[1 - (r_{ep}/r_u)]$$

$$X_E = 0.0269\ (10.97/24.13) + 0.028\ [1 - (10.63/24.13]$$

$$+ 0.128\ [1 - (10.97/24.13)] = 0.098 \text{ in}$$

The equivalent resistance-deflection curve is shown in figure 4A-4.

Step 18. Calculate equivalent elastic stiffness.

$$K_E = \frac{r_u}{X_E} = \frac{24.13}{0.098} = 246.2 \text{ psi/in} \qquad \text{(eq. 3-36)}$$

Step 19. Calculate effective mass of element.

a. Load mass factors (table 3-13 and fig. 3-44).

$L/H = 1.25$ $\qquad x/L = .405$

Elastic range - $K_{LM} = .61 + .16\ (1.25-1) = .65$

Elasto-plastic range:

two simple edges - $K_{LM} = .62 + .16(1.25-1) = .66$

four simple edges - $K_{LM} = .63 + .16(1.25-1) = .67$

Plastic range - $K_{LM} = .54$

$$K_{LM}(\text{avg. elastic and elasto-plastic}) = \frac{.65 + .66 + .67}{3} = .66$$

$$K_{LM}\ (\text{avg. elastic and plastic}) = \frac{.66 + .54}{2} = .60$$

b. Unit mass of element.

$$m = \frac{wT_c}{g} = \frac{150 \text{ X } (1) \text{ X } 10^6}{32.2\ (1728)} = 2{,}700 \text{ psi - ms}_2\text{/in}$$

c. Effective unit mass of element.

$$m_e = K_{LM}m = .60\ (2{,}700) = 1{,}620\ \text{psi-ms}^2/\text{in}$$

Step 20. Calculate natural period of vibration.

$$T_N = 2\pi \left[\frac{m_e}{K_E}\right]^{1/2} \qquad \text{(eq. 3-60)}$$

$$T_N = 2(3.14) \left[\frac{1{,}620}{246.2}\right]^{1/2} = 16.1\ \text{ms}$$

Step 21. Determine response chart parameters (fig. 3-64a).

Peak pressure	$P = 35$ psi (step 1)
Peak resistance	$r_u = 24.13$ psi (step 8)
Duration	$T = 10.5$ ms (step 1)
Period of vibration	$T_N = 16.1$ ms (step 20)
$P/r_u = 35/24.13 = 1.45$	$T/T_N = 10.5/16.1 = 0.65$

From figure 3-64a:

$X_m/X_E = 2.8 < 3$ (step 1)

∴ Use assumed section

$t_m/T_N = 0.60$ (fig. 3-64a)

$$t_m/t_o = t_m/T = \frac{t_m/T_N}{T/T_N} = \frac{0.60}{0.65} = 0.923$$

The correct procedure has been used since $t_m/t_o = 0.923$ is within the range, $0.1 < t_m/t_o < 3$.

Step 22. Check diagonal tension at d_e distance from support.

a. Ultimate shear stress (table 4-7).

$$V_{uH} = \frac{3r_u\ (1 - d_e/x)^2}{d_e\ /x\ (5 - 4d_e/x)} \quad \text{where } d_e = d_H \text{ (of negative moment)}$$

$$V_{uH} = \frac{3 \text{ X } 24.13\ (1 - 9.75/72.9)^2}{(9.75/72.9)\ [5 - 4\ \ (9.75/72.9)]} = 91.0 \text{ psi}$$

$$V_{uV} = \frac{3r_u\ (0.5 - d_e/H)(1 - x/L - 2\ d_e x/HL)}{d_e/H(3 - x/L - 8\ d_e x/HL)}$$

where $d_e = d_v$ (of negative moment)

$$V_{uV} = \frac{3 * 24.13(.5 - 10.25/144)(1 - .405 - 2*.405*10.25/144)}{(10.25/144)(3 - .405 - 8*.405*10.25/144)}$$

$= 99.1$ psi

b. Allowable shear stress (eq. 4-23).

$v_c = [1.9\ (f'_{dc})^{1/2} + 2{,}500\ p] \leq 3.50\ (f'_c)^{1/2} = 221.4$ psi

where p is the steel ratio at support

$$v_{cH} = 1.9\ (4{,}000)^{1/2} + \left[\frac{2{,}500\ (0.20)}{12\ (9.75)}\right] = 124.4 \text{ psi} > 91.0 \text{ psi}$$

$$v_{cV} = 1.9\ (4{,}000)^{1/2} + \left[\frac{2{,}500\ (0.24)}{12\ (10.25)}\right] = 125.0 \text{ psi} > 99.1 \text{ psi}$$

∴ No stirrups required

Step 23. Determine minimum area of the diagonal bars (cross section type I).

$A_d = v_c\ bd/f_{ds}\sin\alpha$ (eq. 4-31)

where d is equal to d_e at support.

Using $\alpha = 45°$,

$A_{dH} = 124.4\ (12 \text{ X } 9.75)/72{,}600\ (0.707) = .283\ \text{in}^2/\text{ft}$

$A_{dV} = 125.0\ (12 \text{ X } 10.25)/72{,}600\ (0.707) = .300\ \text{in}^2/\text{ft}$

use #5 diagonal bars @ 12"

Problem 4A-2. Preliminary Flat Slab Design for Large Deflection

Problem: Design a flat slab for large deflections.

Procedure:

Step 1. Establish design parameters:

a. Blast loads including pressure-time relationship (Chapter 2).
b. Deflection criteria.
c. Structural configuration including geometry and support conditions.
d. Type of section available to resist blast, type I, II or III depending upon the occurrence of spalling and/or crushing of the concrete cover.

Step 2. Select cross section of the slab and the column or column capital. Include concrete cover over reinforcement and maximum size of the reinforcing bars in the flat slab. Also determine allowable static stresses of concrete and reinforcing steel (Section 4-12).

Step 3. Determine dynamic increase factors for both concrete and reinforcement from table 4-1. Using the above DIF and the allowable static stresses of step 2, calculate the dynamic strength of materials.

Step 4. Determine the dynamic design stresses using table 4-2 and the results from step 3.

Step 5. Determine the ratio of the flexural stiffness of the wall to slab in both directions using equations 4-50, 4-51, 4-62 and 4-73.

Step 6. Proportion total span moments to unit column and midstrip moments in both directions using equations 4-52 through 4-60 and 4-63 through 4-71.

Note:
Use equivalent frame method for the direction(s) with only two spans.

Step 7. Adjust unbalanced negative unit moment at column and midstrip in both directions of the roof. Correct the corresponding positive moments to maintain the same total span moments.

Step 8. Calculate total external work done by r_u from equation 4-74 using yield line patterns similar to figure 4-24. Use uniform deflection (Δ) for all positive yield lines.

Step 9. Calculate total internal work done using equation 4-76 and the unit moments determined in steps 6 and 8.

Step 10. Equate the total external work to the internal work (equation 4-77). Solve the resulting equation for the ratio of r_u/M_{oH} . Use equations 4-61 and 4-72 to substitute M_{oL} with M_{oH}.

Step 11. Determine the minimum value of r_u/M_{oH} by trial and error procedure. Vary the assumed value of one of the yield location variables while assuming a constant value for the rest to find the minimum r_u/M_{oH}. Repeat until all yield line location variables are established (X, Y, W and Z). The last step will yield the final minimum value of r_u/M_{oH} to be used in the following steps.

Step 12. Calculate the load-mass factor for the flat slab using the procedure outlined in Chapter 3, for two-way elements. Use equation 3-59 for the slab sectors with no drop panel and equation 3-58 for the slab sectors with drop panel.

Step 13. Calculate effective unit mass of the slab using the larger d_e of the assumed slab section from step 2 and equation 3-54.

Step 14. Calculate the maximum deflection of the flat slab using the shortest sector length (L_s).

Step 15. Determine the required unit resistance (r_{avail} in equation 4-90) to resist the given impulse loading (Chapter 2) and the values from steps 13 and 14. Check that the correct procedure was used.

Step 16. Determine the uniform dead load of the flat slab and calculate the ultimate resistance of the slab (r_u) from equation 4-90.

Step 17. Determine the required total panel moments in each direction using the ultimate resistance from step 16, the minimum value of r_u/M_{oH} from step 11 and the ratio of M_{oL} to M_{oH} established in step 10.

Step 18. Calculate the minimum required unit moments in each direction from step 6 or 7 using the values of M_{oL} and M_{oH} from step 17.

Step 19. Calculate the minimum moment capacity of the slab section in each direction by choosing reinforcing bars. These capacities should be equal to or slightly larger than the corresponding moments from step 18. Also check for minimum reinforcing ratios from table 4-3.

Step 20. Determine provided resistance in each direction by using the ratios provided to required unit moments from steps 18 and 19. Find the average of these values to establish the unit resistance of the flat slab.

Step 21. Determine ultimate tension membrane capacity of the flat slab using equation 4-85. Find the average of continuous steel in the mid and column strip using: unit moment ratios from step 6 or 7 and step 19. Use f_{ds} for bending from step 4 in calculating unit tension forces in the continuous reinforcement, in each span direction.

Step 22. Calculate diagonal tension stresses at d_e distance from the edge of wall supports according to Section 4-31.2 in both directions. Determine concrete capacity in diagonal tension from equation 4-23 using the ratios of unit moments from steps 6 or 7 and the reinforcing ratios from step 19. If the diagonal tension stresses are larger than the concrete capacity, single leg stirrups should be used or a drop panel be added along the wall in lieu of a change in flat slab cross section. If drop panels are used, the diagonal shear stress at d_e distance from the edge of wall drop panel must also be checked.

Step 23. Check punching shear $d_e/2$ distance out and around the column or column capital. Use the load area between positive yield lines minus the area supported by column or its capital. If the shear stress is larger than $4(f'_c)^{1/2}$, use a column drop panel and check the punching shear with the new thickness of the slab over the column.

Step 24. Determine the size of column drop panel by checking punching shear $d_e/2$ distance out and around the drop panel.

Step 25. Check one-way diagonal shear stress between positive yield lines d_e distance out from the column drop panel in each direction. Use equation 4-23 to find concrete capacity. Increase column drop panel size if required or use single leg shear stirrups according to Section 4-18.3.

Step 26. Check one-way diagonal shear stress between positive yield lines for an average d_e distance out from the column capital similar to step 25. Average d_e is based on the width of the drop panel to the total width. Increase column drop panel width or thickness if required.

Step 27. Assume preliminary reinforcement for the flat slab using unit moment ratios from step 6 and 7, M_{oL} and M_{oH} from step 17 and equation 4-19 with the slab thicknesses established throughout this procedure. Calculate all actual unit moment capacities.

Note:

Check the actual flat slab resistance using unit moments from step 27 and the established sizes and thicknesses of drop panels. Repeat steps 8 to 27 for the actual values in each direction. Also provide diagonal bars at wall and column according to sections 4-19 and 4-31.2.

Example 4A-2. Preliminary Flat Slab Design for Large Deflection

Required: Design of a flat slab with three equal spans in each direction for large deflections.

Solution:

Step 1. Given:

a. P = 96 psi, T = 15 ms and triangular loading.

b. Maximum support rotation of 8 degrees.

c. L = H = 240 in., continuous walls all around 207 in. high and 21 in. thick.

d. Type III cross section.

Step 2. Assume:

a. T_c = 15 in. thickness of flat slab.

b. D = 45 in. diameter of column capital.

c. Concrete cover: outside 2 in.
inside 3/4 in.

d. d = 3/4 in. largest bar diameter.

e. f'_c = 4,000 psi compressive strength of concrete.

f. f_y = 66,000 psi yield stress of reinforcing bars.

g. f_u = 90,000 psi ultimate stress of reinforcing bars.

Step 3. Determine dynamic stresses.

a. Dynamic increase factors. DIF (table 4-1).

Concrete:

Diagonal tension	- 1.00

Reinforcement:

Bending, yield stress	- 1.17
Bending, ultimate stress	- 1.05
Direct shear yield stress	- 1.10
Direct shear ultimate stress	- 1.00

b. Dynamic strength of materials.

Concrete:

Diagonal tension (f'_c) - 1.00 (4,000) = 4,000 psi

Reinforcement:

Bending (f_{dv}) - 1.17 X 66,000 = 77,220 psi

Bending (f_{du}) - 1.05 X 90,000 = 94,500 psi

Direct shear (f_{dv}) - 1.10 X 66,000 = 72,600 psi

Direct shear (f_{du}) - 1.00 X 90,000 = 90,000 psi

Step 4. Dynamic design stresses from table 4-2.

Concrete (f'_c):

Diagonal tension - 4,000 psi

Reinforcement (f_{ds}):

$f_{ds} = (f_{dy} + f_{du})/2$

Bending - 85,860 psi

Direct shear - 81,300 psi

Note:
Since the structure is symmetrical in both directions, the calculations will be done only in one direction in steps 5 through 9, 12, 17 through 22, 25 and 26.

Step 5. Determine the ratio of the flexural stiffness of the wall to the roof slab.

$$\alpha_{ecH} = \frac{T_w^3 H}{T_s^3 H_w} = \frac{21^3 * 240}{15^3 * 207} = 3.18 \qquad \text{(eq. 4-50)}$$

$$\alpha'_{ecH} = \frac{1}{1+1/\alpha_{ecH}} = \frac{1}{1 + 1/3.18} = 0.76 \qquad \text{(eq. 4-62)}$$

Step 6. Calculate unit moments using equations 4-52 through 4-60. See figure 4A-5 for locations.

$m^-_1 = 0.65\ \alpha'_{ecH}\ M_{oH}/L = 0.65\ (0.76)\ M_{oH}/240 = (0.494)\ M_{oH}/240$

$m^{+}_{2} = 0.40\ (0.63 - 0.28\ \alpha'_{ecH})\ M_{oH}/(L - H/2)$

$= 0.40(0.63 - 0.28 \times 0.76)\ M_{oH}/(240 - 240/2) = (0.334)\ M_{oH}/240$

$m^{-}_{3} = 3.25\ (0.75 - 0.10\ \alpha'_{ecH})\ M_{oH}/(L - H/2)$

$= 0.25(0.75 - 0.10 \times 0.76)\ M_{oH}/(240 - 240/2) = (0.337)\ M_{oH}/240$

$m^{-}_{4} = 0.25\ (0.65) M_{oH}/(L - H/2)$

$= 0.25 \times 0.65\ M_{oH}/(240 - 240/2) = (0.325) M_{oH}/240$

$m^{+}_{5} = 0.40\ (0.35) M_{oH}/(L - H/2)$

$= 0.40 \times 0.35\ M_{oH}/(240 - 240/2) = (0.280) M_{oH}/240$

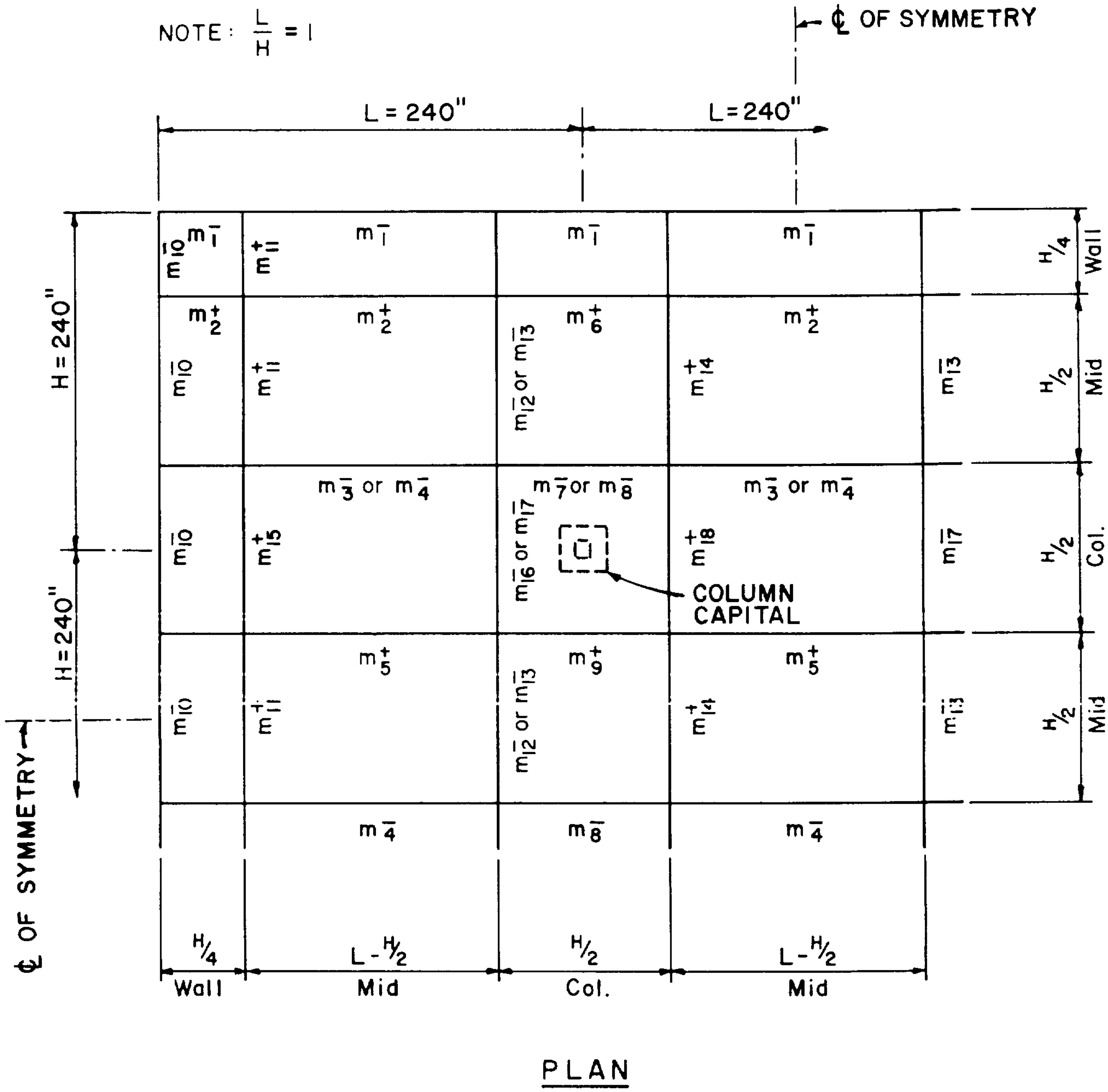
$m^{+}_{6} = 0.60\ (0.63 - 0.28\ \alpha'_{ecH})\ M_{oH}/(H/2)$

$= 0.60\ (0.63 - 0.28 \times 0.76)\ M_{oH}/(240/2) = (0.501) M_{oH}/240$

$m^{-}_{7} = 0.75\ (0.75 - 0.10\ \alpha'_{ecH})\ M_{oH}/(H/2)$

$= 0.75\ (0.75 - 0.10 \times 0.76)\ M_{oH}/(240/2) = (1.011) M_{oH}/240$

$m^{-}_{8} = 0.75(0.65) M_{oH}/(H/2) = 0.75(0.65) M_{oH}/(240/2) = (0.975) M_{oH}/240$



NOTE: L/H = 1
℄ OF SYMMETRY
L = 240"
L=240"
H = 240"
H=240"
℄ OF SYMMETRY
COLUMN CAPITAL
H/4
Wall
L - H/2
Mid
H/2
Col.
L - H/2
Mid
PLAN

FIGURE 4A-5

$m^-_9 = 0.60\ (0.35)M_{oH}/(H/2) = 0.60\ (0.35)M_{oH}/240/2 = (0.420)M_{oH}/240$

Step 7. a. Balance the negative unit moment over the column and midstrip.

$m^-_3 > m^-_4 \therefore m^-_3 = m^-_4 = (0.337)\ M_{oH}/240$

$m^-_7 > m^-_8 \therefore m^-_7 = m^-_8 = (1.011)\ M_{oH}/240$

b. Adjust the corresponding positive unit moment in order to keep total panel moments equal.

Adjusted $m^+_5 = m^+_5 - 2(m^-_3 - m^-_4)$

$\therefore\ m^+_5 = [0.280 - 2(0.337 - 0.325)]M_{oH}/240 = (0.256)\ M_{oH}/240$

Adjusted $m^+_9 = m^+_9\ \ 2(m^-_7 - m^-_8)$

$\therefore\ m^+_9 = [0.420 - 2\ (1.011 - 0.975)]M_{oH}/240 = (0.348)M_{oH}/240$

Step 8. Calculate total external work for an assumed deflection of Δ. Use one quarter of the roof slab due to symmetry in both directions. See figure 4A-6 for yield lines and sectors.

Equivalent square column capital, C X C.

$$C = \left[\frac{\pi D^2}{4}\right] = \left[\frac{\pi(45)^2}{4}\right]^{1/2} = 39.9 \text{ in. say } 40 \text{ X } 40 \text{ in.}$$

$$W_I = W_{IV} = r_u \left[\left(\frac{3L}{2} - X\right)(X)\frac{\Delta}{2} + \left(\frac{X^2}{2}\right)\frac{\Delta}{3} \right]$$

$$= r_u \left[\left(\frac{3 \text{ X } 240}{2} - X\right)(X)\frac{\Delta}{2} + \left(\frac{X^2}{2}\right)\frac{\Delta}{3} \right]$$

$$W_{II} = W_V = r_u\left[C\left[H - C/2 - X\right]\frac{\Delta}{2} + \frac{1}{2}\left[\frac{3L}{2} - X - C\right]\left[H - \frac{C}{2} - X\right]\left[\frac{2\Delta}{3}\right]\right]$$

$$= r_u\left[40(240 - \frac{40}{2} - X)\frac{\Delta}{2} + \frac{1}{2}(\frac{3*240}{2} - X - 40)(240 - \frac{40}{2} - X)(\frac{2\Delta}{3})\right]$$

$$W_{III} = W_{VI} = r_u\left[\frac{C}{2}(H - C)\frac{\Delta}{2} + \frac{1}{2}(\frac{3L}{2} - X - C)(\frac{H - C}{2})(\frac{2\Delta}{3})\right]$$

$$= r_u\left[\frac{40}{2}(240 - 40)\frac{\Delta}{2} + \frac{1}{2}\left[\frac{(3)(240)}{2} - X - 40\right](\frac{240 - 40}{2})(\frac{2\Delta}{3})\right]$$

From equation 4-74.

$$W = \Sigma\, r_u A\Delta$$

$$W = \sum_{i=I}^{VI} W_i = \frac{r_u\Delta}{3}(243200 - 320X)$$

Step 9. Calculate total internal work for the assumed deflection of Δ. Use one quarter of the roof slab due to symmetry in both directions. See figure 4A-6 for angles of rotation.

$$\theta_{1H} = \theta_{1V} = \frac{\Delta}{X}$$

$$\theta_{2H} = \theta_{2H} = \frac{\Delta}{H - C/2 - X} = \frac{\Delta}{240 - 40/2 - X} = \frac{\Delta}{220 - X}$$

$$\theta_{3H} = \theta_{3V} = \frac{\Delta}{(H - C)/2} = \frac{\Delta}{(240 - 40)/2} = \frac{\Delta}{100}$$

Assume $0 < X < 3H/4 = 180$ in.

$$E_I = E_{IV} = m^-{}_1\theta_{1H}\left(\frac{3L - X}{2}\right) + \frac{2}{3}m^-{}_1\theta_{1H}\left(\frac{X}{2}\right)$$

$$+ m^+{}_6\theta_{1H}\left(\frac{L}{2}\right) + m^+{}_2\theta_{1H}\left(L - \frac{X}{2}\right) + \frac{2}{3}m^+{}_2\theta_{1H}\left(\frac{X}{2}\right)$$

$$= \frac{M_{oH}\,\Delta}{(2)\,(240)\,X}\left[0.494\left[3\,(240) - X/3\right] + 0.501\,(240) + 0.334\left[2\,(240) - X/3)\right]\right]$$

$$E_{II} = E_V = m^+{}_6\theta_{2H}(\frac{L}{2}) + m^+{}_2\theta_{2H}(L - X) + m^-{}_7\theta_{2H}(\frac{L}{2}) + m^-{}_3\theta_{2H}(L - X)$$

$$= \frac{M_{oH}}{(2)\,(240)} \times \frac{\Delta}{220 - X}\left[(0.501 + 1.011)\,240 + 2\,(0.334 + 0.337)\,(240 - X)\right]$$

$$E_{III} = E_{VI} = m^+{}_9\theta_{3H}(\frac{L}{2}) + m^+{}_5\theta_{3H}(L-X) + m^-{}_7\theta_{3H}(\frac{L}{2}) + m^-{}_3\theta_{3H}(L-X)$$

$$= \frac{M_{oH}}{(2)(240)} \times \frac{\Delta}{100}\left[(0.348 + 1.011)\,240 + 2\,(0.256 + 0.337)\,(240 - X)\right]$$

From equation 4-75.

$$E = \Sigma\, m\, \theta l$$

$$E = \sum_{i=I}^{VI} E_i = \frac{M_{oH}\,\Delta\,[\,139972.8 + 1331.76X - 9.7832X^2 + 0.01186X^3\,]}{240X\,(220 - X)}$$

Step 10. Set the external work equal to the internal work and solve the equation for the ratios of r_u/M_{oH}.

$$W = E$$

$$\frac{r_u\Delta}{3}\,(243200-320X) = \frac{M_{oH}\Delta[139972.8 + 1331.76X - 9.7832X^2 + 0.01186X^3]}{240X\,(220 - X)}$$

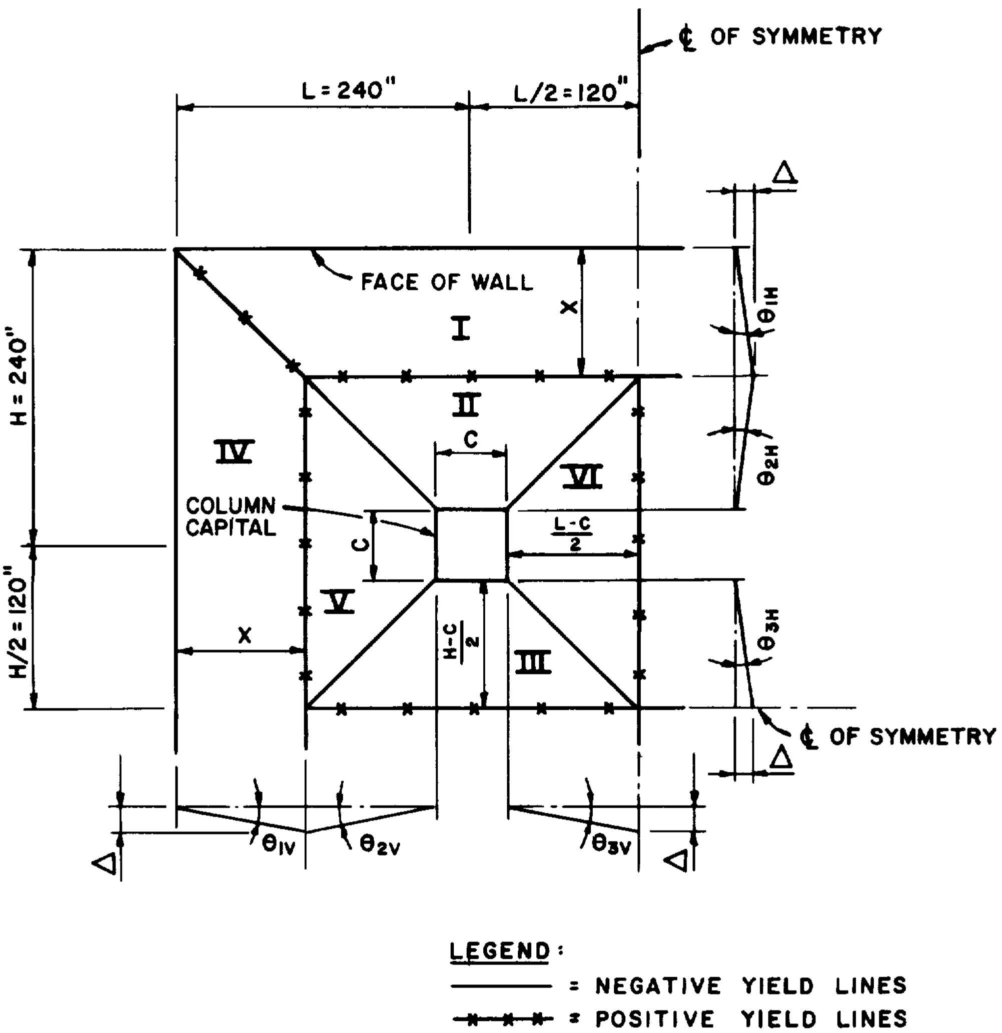


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FIGURE 4A-6

$$\therefore \frac{r_u}{M_{oH}} = \frac{139972.8 + 1331.76\,X - 9.7832X^2 + 0.01186X^3}{80X\,(220 - X)(243200 - 320X)}$$

Step 11. Minimize value of r_u/M_{oH} using the equation from step 10 by assuming various values for X to locate the yield line locations.

X (in)	r_u/M_{oH} (psi/in-lbs)	
110	913.212×10^{-9}	
115	911.686×10^{-9}	
120	912.445×10^{-9}	
116	911.654×10^{-9}	*minimum
117	911.713×10^{-9}	

Step 12. Calculate plastic load mass factor for the flat slab. See Section 4-30.5 and figure 4A-6.

$$\left(\frac{I}{cL_1}\right)_I = \left(\frac{I}{cL_2}\right)_{IV} = \left(\frac{2}{3}\right) X \left(\frac{3L}{2} - X + \frac{1}{2}\left(\frac{X^2}{2}\right)\right) = \frac{2}{3}(116)\left(\frac{(3)(240)}{2} - 116\right) + \frac{1}{2}\left(\frac{116^2}{28}\right)$$

$$\left(\frac{I}{cL_1}\right)_{II} = \left(\frac{I}{cL_1}\right)_{V} = \frac{2}{3} C\left(H - \frac{C}{2} - X\right) + \frac{3}{4}\left(H - \frac{C}{2} - X\right)\left[\frac{3L}{2} - C - X\right]/2$$

$$= \frac{2}{3}(40)\left(240 - \frac{40}{2} - 116\right)$$

$$+ \frac{3}{4}\left(240 - \frac{40}{2} - 116\right)\left[\frac{3 \times 240}{2} - 40 - 116\right]/2$$

$$\left(\frac{I}{cL_1}\right)_{III} = \left(\frac{I}{cL_1}\right)_{VI} = \frac{2}{3} C\,(H - C)/2 + \frac{3}{4}(H - C)\left[\frac{3L}{2} - C - X\right]/4$$

$$= \frac{2}{3} (40) \left[240 - 40\right]/2$$

$$+ \frac{3}{4} (240 - 40) \left[\frac{3 X 240}{2} - 40 - 116\right] /4$$

$$\sum_{i=I}^{VI} \left(\frac{I}{CL_1}\right)_i = 86558.67 \text{ in}^2$$

$$A_I = A_{IV} = \frac{X}{2} (3L - X) = \frac{116}{2} (3 X 240 - 116)$$

$$A_{II} = A_V = (H - \frac{C}{2} - X) \left[\frac{3L}{2} + C - X\right] /2$$

$$= (240 - \frac{40}{2} - 116) \left[\frac{3 X 240}{2} + 40 - 116\right] /4$$

$$A_{III} = A_{VI} = (H - C) \left[\frac{3L}{2} + C - X\right]/4$$

$$= (240 - 40)\left[\frac{3 X 240}{2} + 40 - 116\right]/4$$

$$\sum_{i=I}^{VI} A_i = 128,000.0 \text{ in}^2$$

$$K_{LM} = \frac{\Sigma \left(\frac{I}{CL_1}\right)_i}{\Sigma \; A_i} = \frac{86558.67}{128,000.0} = 0.676 \qquad \text{(eq. 3-59)}$$

Step 13. Calculate the unit mass of the slab.

$$d_c = 15 - 2 - .75 - (2 X .75)/2 = 11.5 \text{ in.}$$

$$m = \frac{W}{g} = \frac{11.5 X 150 X 1000^2}{12 X 32.2 X 1728} = 2583.5 \text{ psi-ms}^2 \text{ /in}$$

Determine effective unit mass from equation 3-54.

$$m_e = K_{LM} X m = .676 X 2583.5 = 1746.4 \text{ psi-ms}^2\text{/in}$$

Step 14. Find maximum deflection using shortest sector length.

$$L_s = \left(\frac{H - C}{2}\right) = \frac{240 - 40}{2} = 100. \text{ in}$$

$$X_m = L_s \tan \Theta_{max} = 100 \text{ X } \tan 8 = 14.05 \text{ in.}$$

Step 15. Determine impulse load and required resistance for blast.

$$i_b = PT/2 = (96.0 \text{ X } 15.0) / 2 = 720.0 \text{ psi-ms}$$

$$\frac{i_b^2}{2m_u} = r X_m \qquad \text{(eq. 3-93)}$$

$$\therefore r = \frac{720.0^2}{2 \text{ X } 1746.4 \text{ X } 14.05} = 10.56 \text{ psi} = r_{avail}$$

Check for correct procedure.

$$t_m = \frac{i_b}{r} = \frac{720}{10.56} = 68.2 \text{ ms}$$

$$t_m/T = 68.2 / 15 = 4.55 > 3 \quad \text{O.K. (section 3-20)}$$

Step 16. Calculate uniform dead load of slab and the required ultimate resistance. Assume 150 psf concrete.

$$r_{DL} = 15/12 \text{ X } 150/12^2 = 1.30 \text{ psi}$$

$$r_{avail} = r_u - \frac{f_{ds}}{f_{dy}} r_{DL} \qquad \text{(eq. 4-90)}$$

$$\text{Required } r_u = 10.56 + \left(\frac{85,860}{66,000}\right) 1.30 = 12.25 \text{ psi}$$

Step 17. Calculate required panel moment using the required r_u from step 16 and the r_u/M_{oH} from step 11.

$$\text{Required } M_{oH} = (r_u)/\left(\frac{r_u}{M_{oH}}\right) = \frac{12.25}{911.654 \text{ X } 10^{-9}} = 13{,}437{,}115 \text{ in-lbs}$$

Step 18. Calculate the minimum required unit moments from step 7.

$$\text{Minimum unit moment} = m^+_5 = \frac{.256\ M_{oH}}{240}$$

$$\therefore m^+_5 = \frac{.256 \text{ X } 13{,}437{,}115}{240} = 14333 \text{ in-lbs/in}$$

Step 19. Calculate actual moment capacity at m^+_5 , assume No. 4 reinforcing bars 12 in. o.c.

$$d_e \text{ in H direction} = d_H = 15 - 2 - .75 - 2 \text{ X } .75^* + 2 \text{ X } \frac{.5}{2} = 11.25 \text{ in}$$

$$d_e \text{ in L direction} = d_V = 15 - 2 - .75 - 2 \text{ X } .75^* - 2 \text{ X } \frac{.5}{2} = 10.25 \text{ in}$$

* Assumed No. 6 reinforcing in step 2.

$$M_u = \frac{A_s\ f_{ds}\ d_c}{b} \qquad \text{(eq. 4-19)}$$

$$M^+_{14} = \frac{.2 \text{ X } 85{,}860 \text{ X } 11.25}{12} = 16099 \text{ in-lb/in} > 14333 \text{ O.K.}$$

$$M^+_5 = \frac{.2 \text{ X } 85{,}860 \text{ X } 10.25}{12} = 14668 \text{ in-lbs/in} > 14333 \text{ O.K.}$$

$$p = \frac{A_s}{bd_e} \qquad \text{(eq. 4-13)}$$

$$p_{m^+14} = \frac{.20}{12 \text{ X } 11.25} = .0015 = .0015 \quad \text{O.K.}$$

$$P_m{+}_5 = \frac{.20}{12 \text{ X } 10.25} = .0016 > .0015 \qquad \text{O.K.}$$

Step 20. Calculate the provided unit resistance of the flat slab.

$$r_u \text{ (provided)} = r_e \text{ (required) X } \frac{\text{m (unit moment provided)}}{\text{m (unit moment required)}}$$

$$(r_u)_{m14} = 12.25 \text{ X } \frac{16099}{14333} = 13.76 \text{ psi}$$

$$(r_u)_{m5} = 12.25 \text{ X } \frac{14668}{14333} = 12.54 \text{ psi}$$

$$r_u = \frac{(r_u)_{m14} + (r_u)_{m5}}{2} = \frac{13.76 + 12.54}{2} = 13.15 \text{ psi} > 12.25 \text{ o.k.}$$

Step 21. Estimate minimum area of continuous steel in column strip using unit moment ratios from step 7.

$$(A_s)_{m9} = \frac{m^+_9}{m^+_5} \text{ X } (A_s)_{m5} = \frac{.348 \dfrac{M_{oH}}{240}}{.256 \dfrac{M_{oH}}{240}} \text{ X } (0.20) = .27 \text{ in}^2/\text{ft}$$

Calculate the average unit tension force in continuous steel.

$$T_L = \frac{(A_s)_{m5}\, H/2 + (A_s)_{m9}\, (L - H/2)}{b \text{ X } L} \text{ X } f_{ds} \text{ X } 2$$

$$= \frac{.20 \text{ X } 240/2 + .27 \text{ X } (240 - 240/2)}{12 \text{ X } 240} \text{ X } 85{,}860 \text{ X } 2 = 3.362 \text{ lbs/in}$$

Calculate tension membrane resistance from equation 4-85.

$$r_T = \frac{\pi^3\, 1.5 \text{ X } T_H/L_H{}^2}{4 \sum\limits_{n=1,3,5} \left\{ \dfrac{1}{n^3} (-1)^{(n-1)/2} \left[1 - \dfrac{1}{\cosh\left[\dfrac{n\pi L_L}{2L_H} \left[\dfrac{T_H}{T_L} \right]^{1/2} \right]} \right] \right\}}$$

$H_L = H - C = 240 - 40 = 200$ in

$$r_T = \frac{\pi^3\ 1.5 \times 14.05 \times 3{,}363 / 200^2}{4 \sum_{n=1,3,5} \left(\frac{1}{n^3} (-1)^{(n-1)/2} \left[1 - \frac{1}{\cosh\left[\frac{n\pi(200)}{2(200)} \left[\frac{3{,}363}{3{,}363}\right]^{1/2} \right]} \right] \right)}$$

$$r_T = \frac{\pi^3 \times 1.5 \times 14.05 \times 3{,}363}{4 \times (200)^2\ (.6015 - .0364 + .0080)} = 23.97 \text{ psi} > 13.15 \text{ psi O.K.}$$

Step 22. Calculate diagonal tension stresses at d_e distance from the edge of wall supports according to section 4-31.2 in both directions.

$$r_u \times \text{Area (Sector I)} = V_V\left(\frac{3L - X}{2}\right) + \frac{2}{3} V_V \frac{X}{2}$$

$$13.15 \left(\frac{3 \times 240 - 116}{2}\right) 116 = V_V \left(\frac{3 \times 240 - 116}{2}\right) + \frac{2}{3} V_V \frac{116}{2}$$

$$\therefore V_V = \frac{460670.8}{340.67} = 1352.3 \text{ lbs/in}$$

Total diagonal shear load in L direction.

$$V_{uV} = V_V \left(\frac{3L - X}{2}\right) + \frac{2}{3} V_V \left(\frac{X}{2} - d_V\right)$$

$$= 1352.3 \left[\frac{3 \times 240 - 116}{2} + \frac{2}{3} \times \left[\frac{116}{2} - 10.25\right]\right] = 451{,}443 \text{ lbs.}$$

Diagonal shear stress in L direction.

$$v_{uV} = \frac{V_{uV}}{\left(\frac{3L}{2} - d\right) d_V} = \frac{451,443}{\left(\frac{3 \times 240}{2} - 10.25\right) 10.25} = 125.9 \text{ psi}$$

Estimate reinforcing ratio at support using the ratio of unit moments.

$$r_{m1} = r_{m5} \left(\frac{m1}{m5}\right) = .0016 \left(\frac{.494 \frac{M_{oH}}{240}}{.256 \frac{M_{oH}}{240}}\right) = .0031$$

Calculate diagonal shear capacity of concrete.

$$v_c = 1.9 \, (f'_c)^{1/2} + 2500 \, p \qquad \text{(eq. 4-23)}$$

$$v_{cV} = 1.9 \, (4000)^{1/2} + 2500 \, (0.0031) = 127.9 \text{ psi} > 125.9 \text{ psi}$$

∴ No stirrups or wall drop panel required.

Note:
Diagonal shear at d_e distance from the H direction wall will be less than the one in L direction due to symmetry and larger d_e in H direction. Calculation is not required.

Step 23. Check punching shear around column capital.

Use average d_e.

$$d_{avg} = \frac{d + d}{2} = \frac{10.25 + 11.25}{2} = 10.75 \text{ in}$$

Diameter of punching. $D_p = D + d_{avg} = 45 + 10.75 = 55.75$ in.

Find area between positive yield lines minus column capacity.

$$\text{Area} = \left[\frac{3L}{2} - X\right]^2 - \frac{\pi D_p^2}{4} = \left[\frac{3 \times 240}{2} - 116\right]^2 - \frac{\pi \times 55.75^2}{4}$$

$$= 57095 \text{ in}^2$$

$$v\ (\text{punching}) = \frac{r_u \times \text{Area}}{\pi\ D_p\ d_{avg}} = \frac{13.15 \times 57095}{\pi \times 55.75 \times 10.75} = 398.8\ \text{psi}$$

$$v_c = 4\ (f'_c)^{1/2} = 4\ (4000)^{1/2} = 253.0\ \text{psi} < 398.8\ \text{psi}$$

∴ Need drop panel, assume 6 in.

$$d_{avg}\ (\text{revised}) = 10.75 + 6 = 16.75\ \text{in.}$$

$$D_p\ (\text{revised}) = 45 + 16.75 = 61.75\ \text{in.}$$

$$\text{Area (revised)} = \left[\frac{3 \times 240}{2} - 116\right]^2 - \frac{\pi \times 61.75^2}{4} = 5641\ \text{in}^2$$

$$V\ (\text{punching}) = \frac{13.15 \times 56541}{\pi \times 61.75 \times 16.75} = 228.8\ \text{psi} < 253.0 \quad \text{O.K.}$$

Step 24. Assume 63 X 63 in. drop panel. Check punching shear.

$$\text{Punching Length} = l_p = l + d_{avg} = 63 + 10.75 = 73.75\ \text{in}$$

$$\text{Area} = \left[\frac{3L}{2} - X\right]^2 - l_p^2 = \left[\frac{3 \times 240}{2} - 116\right]^2 - 73.752^2 = 54097\ \text{in}^2$$

$$v\ (\text{punching}) = \frac{r_u\ \text{Area}}{4\ l_p\ d_{avg}} = \frac{13.15 \times 54097}{4 \times 73.75 \times 10.75} = 224.3\ \text{psi} < 253.0\ \text{psi}$$

Step 25. Check one-way diagonal shear d_e distance away from column drop panel and between positive yield lines.

$$\text{Width in L direction} = \frac{3L}{2} - X = \frac{3 \times 240}{2} - 116 = 244\ \text{in.}$$

$$\text{Area in L direction} = \text{Width}\ (H - X - \frac{1}{2} - d_v)$$

$$= 244\,(240 - 116 - \frac{63}{2} - 10.75) = 19947 \text{ in}^2$$

$$v = \frac{r_u \text{ Area}}{d_v \text{ Width}} = \frac{13.15 \times 19947}{10.75 \times 244} = 100. \text{ psi} < v_c^{1/2} \qquad \text{O.K.}$$

Note:
Diagonal shear in H direction will be less than the one in L direction due to symmetry and larger d_e in H direction. Calculation is not required.

Step 26. Check one-way diagonal shear at an average d distance away from column capital and between positive yield lines.

$(\text{Width})_L = 244$ in (step 25)

$$d_{avg} = d_v + (\frac{1}{\text{width}}) \times \text{drop panel depth}$$

$$d_{avg} = 10.25 + \frac{63}{244} \times 6 = 11.80 \text{ in}$$

$$\text{Area in L direction} = \text{Width } (H - X - \frac{C}{2} - d_{avg})$$

$$= 244\,(240 - 116 - \frac{40}{2} - 11.80) = 22496.8 \text{ in}^2$$

$$v = \frac{r_u \text{ Area}}{d_{avg} \text{ Width}} = \frac{13.15 \times 22496.8}{11.80 \times 244} = 102.7 \text{ psi} < v_c \qquad \text{O.K.}$$

Note:
Diagonal shear in H direction will be less than the one in L direction due to symmetry and larger d_e in H direction. Calculation is not required.

Step 27. Calculate all remaining required moments similar to step 18. Assume reinforcing bars for each, and determine actual provided unit moment capacities similar to step 19.

Problem 4A-3, Elements Designed for Impulse-Large Deflections

Problem: Design an element subjected to an impulse load for a large deflection.

Procedure:

Step 1. Establish design parameters:

a. Impulse load and duration (Chapter 2).

b. Deflection criteria.

c. Geometry of element.

d. Support conditions.

e. Type of section available to resist blast, type II or III depending upon the occurrence of spalling.

f. Materials to be used and corresponding static design strengths.

g. Dynamic increase factors (table 4-1).

Step 2. Determine dynamic yield strength and dynamic ultimate strength of reinforcement.

Step 3. Determine dynamic design stress for the reinforcement according to the deflection range (support rotation) required by the desired protection level (table 4-2).

Step 4. Determine optimum distribution of the reinforcement according to the deflection range considered (sect. 4-33.4 and figs. 4-37 and 4-38). Step not necessary for one-way elements.

Step 5. Establish design equation for deflection range considered and type of section (type II or III) available.

Step 6. Determine impulse coefficient C_1 and/or C_u for optimum p_V/p_H ratio and L/H ratio.

Note:
If the desired deflection X_m is not equal to X_1 or X_u, determine yield line location (figs. 3-4 through 3-20) for optimum p_V/p_H ratio and L/H ratio and calculate X_m, X_1, and, if necessary, X_u (table 3-5 or 3-6).

Step 7. Substitute known parameters into equation of step 5 to obtain relationship between p_H and d_c.

Step 8. Assume value of d_c and calculate p_H and from optimum p_V/p_H calculate p_V. Select bar sizes and spacings necessary to furnish required reinforcement (see Sect. for limitations).

Step 9. For actual distribution of flexural reinforcement, establish yield line location (figs. 3-4 through 3-20).

Note:

$$\frac{L}{H}\left[\frac{M_{VN}+M_{VP}}{M_{HN}+M_{HP}}\right]^{1/2} = \frac{L}{H}\left[\frac{p_V}{p_H}\right]^{1/2} = \frac{L}{H}\left[\frac{A_{sV}}{A_{sH}}\right]^{1/2}$$

since:

$$M_N = M_P = p\, d_c^2\, f_{ds} = \frac{A_s}{b}\, d_c\, f_{ds}$$

Step 10. Determine the ultimate shear stress at distance d_c from the support in both the vertical (v_{uV} from eq. 4-119) and horizontal (v_{uH} from eq. 4-118) directions where the coefficients C_V and C_H are determined from figures 4-39 through 4-52 (see sect. 4-35.2 for an explanation of the figures and parameters involved).

Step 11. Determine the shear capacity v_c of the concrete in both the vertical and horizontal directions (use eq. 4-21).

Step 12. Select lacing method to be used (fig. 4-91). (Note: Lacing making an angle of 45° with longitudinal reinforcement is most efficient.)

Step 13. Determine the required lacing bar sizes for both the vertical and horizontal directions from equation 4-26 where the parameters b_l and s_l are determined from the lacing method used (fig. 4-91), and the angle of inclination of the lacing bars α is obtained from figure 4-15. The lacing bar size d_b must be assumed in order to compute d_l and R_l.

(Note: See sect. 4-18.3 for limitations imposed upon the design of the lacing).

Step 14. Determine required thickness T_c for assumed, d_c, selected flexural and lacing bar sizes, and required concrete cover. Adjust T_c to the nearest whole inch and calculate the actual d_c.

Step 15. Check flexural capacity based on either impulse or deflection. Generally, lacing bar sizes do not have to be checked since they are not usually affected by a small change in d_c.

a. Check of impulse. Compute actual impulse capacity of the element using the equation determined in step 5 and compare with anticipated blast load, Repeat design (from step 8 on) if capacity is less than required.

b. Check of deflection. Compute actual maximum deflection of the element using equation determined in step 5 and compare with deflection permitted by design criteria. Repeat design (from step 8 on) if actual deflection is greater than that permitted.

Step 16. Determine whether correct procedure has been used by first computing the response time of the element t_m (time to reach maximum deflection) from equation 3-95 or 3-96, depending on the deflection range considered in the design, and then compare response time t_m with duration of load t_o. For elements to be designed for impulse, $t_m > 3\ t_o$.

Step 17. Determine the ultimate support shear in both the vertical (V_{sV} from eq. 4-122) and horizontal (V_{sH} from eq. 4-121) directions. The coefficients C_{sV} and C_{sH} are determined from table 4-15 and figures 4-53 through 4-56 (see sect. 4-35.3 for an explanation of the figures and parameters involved).

Step 18. Determine the required diagonal bar sizes for the vertical and horizontal (intersecting elements may control) directions from equation 4-30. Diagonal bars should have the same spacing as the flexural reinforcement (fig. 4-95).

Note:
To obtain the most economical design repeat the above steps for several wall thicknesses and compare costs. Percentages of reinforcement may be used to reduce the amount of calculations. In determining the required quantities of reinforcement, lapping of the bars should be considered.

Example 4A-3, Elements Designed for Impulse-Large Deflections

Required: Design the back wall of the interior cell (fig. 4A-7) of a multicubicle structure for incipient failure.

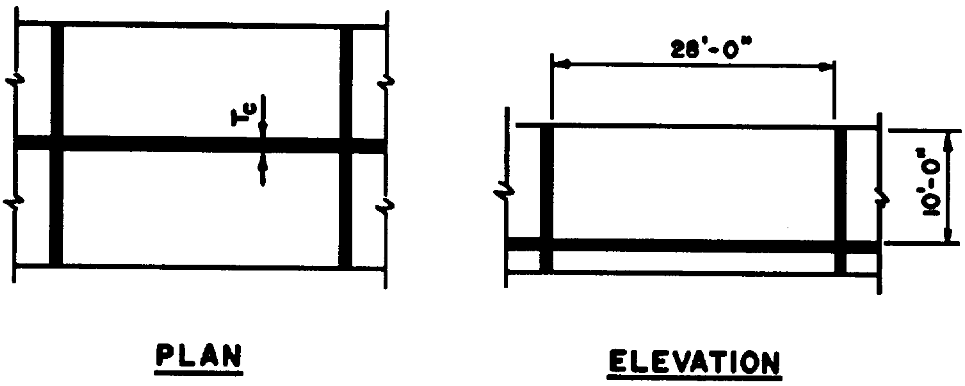


Figure 4A-7

Solution:

Step 1. Given:

a. i_b = 3,200 psi-ms and t_o = 5 ms

b. Incipient failure

c. L = 336 in, H = 120 in

d. Fixed on three edges and one edge free

e. Type III cross section

f. Reinforcement f_y = 66,000 psi and f_u = 90,000 psi

concrete f'_c = 4,000 psi

g. For reinforcement DIF = 1.23 for dynamic yield stress

DIF = 1.05 for ultimate dynamic stress

Step 2. Dynamic Strength of Materials

f_{dy} = DIF f_y = 1.23 x 66,000 = 81,180 psi

f_{du} = DIF f_u = 1.05 x 90,000 = 94,500 psi

Step 3. Dynamic Design Stress, from table 4-2

$$f_{ds} = \frac{(f_{dy} + f_{du})}{2} = \frac{(81,180 + 94,500)}{2} = 87,840 \text{ psi}$$

Step 4. From figure 4-38 for $\frac{L}{H} = \frac{336}{120} = 2.8$ and 3 edges fixed,

Optimum: $p_V/p_H = 1.41$

Step 5. Since $X_m = X_u$ (incipient failure):

$$\frac{i_b^2\ H}{p_H\ d_c^3\ f_{ds}} = C_u \qquad \text{(eq. 4-103)}$$

Step 6. L/H = 2.8 is not plotted on figure 4-34, therefore must interpolate for optimum p_V/p_H.

For $p_V/p_H = 1.41$

L/H	C_u
1.5	613.0
2.0	544.0
3.0	444.0
4.0	387.0

From figure 4A-8, $C_u = 461.0$.

Step 7.

$$p_H\ d_c^3 = \frac{i_b^2\ H}{C_u\ f_{ds}}$$

$$p_H\ d_c^3 = \frac{(3{,}200)^2\ (120)}{(461.0)\ (87{,}840)} = 30.3$$

Step 8. Assume $d_c = 21$ in:

$$p_H = \frac{30.3}{d_c^3} = \frac{30.3}{(21)^3} = 0.00327$$

$$p_V = \frac{p_V}{p_H}\ (p_H) = 1.41\ (0.00327) = 0.00461$$

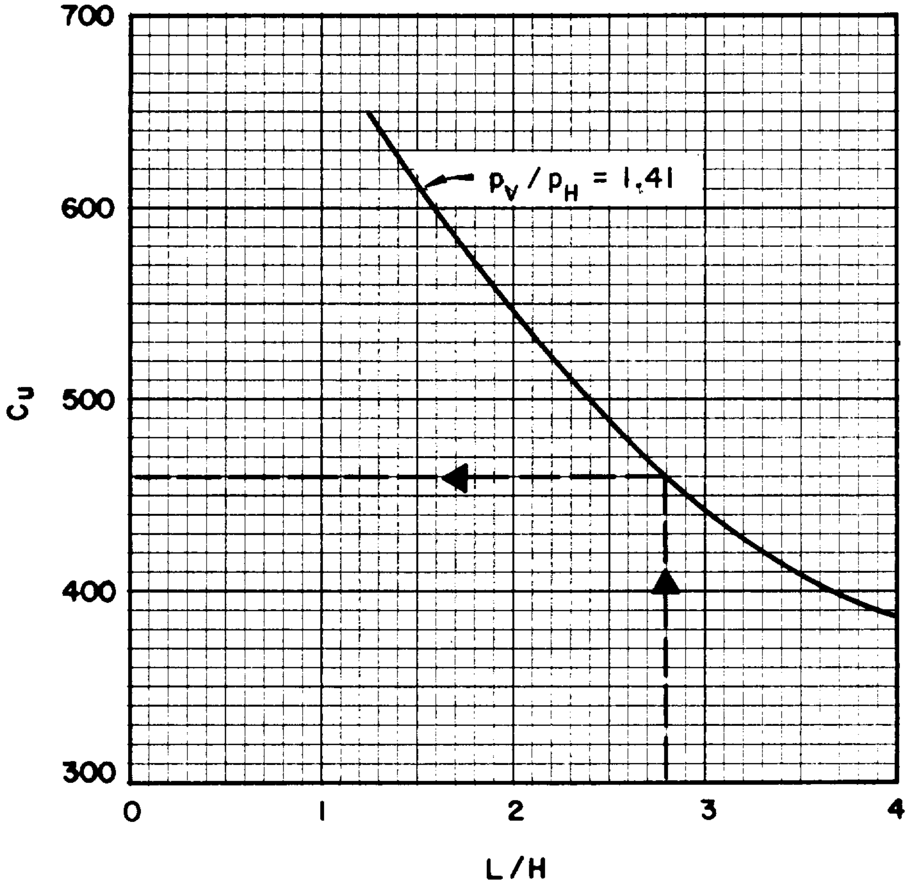


FIGURE 4A-8

$A_{sH} = 0.00327\ (12)\ (21) = 0.82\ in^2/ft$ - Use #8 @ 11 ($A_s = 0.86$)

$A_{sV} = 0.00461\ (12)\ (21) = 1.16\ in^2/ft$ - Use #9 @ 10 ($A_s = 1.20$)

Step 9. Yield line location.

Actual:

$$\frac{p_V}{2p_H} = \frac{A_{sV}}{2A_{sH}} = \frac{1.20}{2 \times 0.86} = 0.698$$

From figure 3-11 for $\frac{L}{H}\left[\frac{A_{sV}}{A_{sH}}\right]^{1/2} = 2.8\ (0.698)^{1/2} = 2.34$

and $M_{vp}/M_{vN2} = 1.00$

$x/L = 0.385, \quad x = 0.385\ (336) = 129$ in

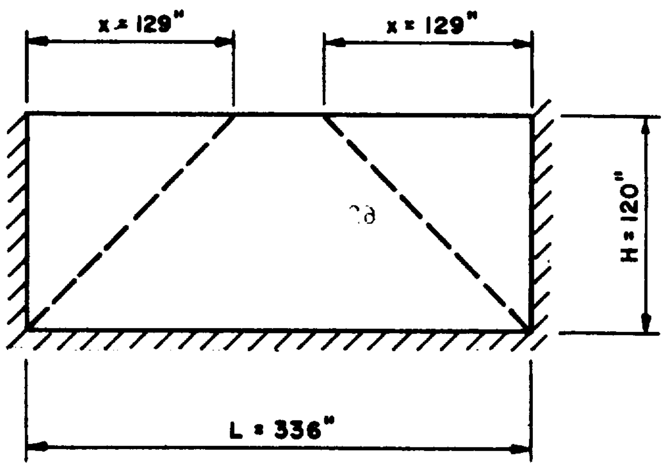


Figure 4A-9

Step 10. Ultimate shear stress at distance d_c from support.

a. Vertical Direction (along L):

For:

$$\frac{d_c}{H} = \frac{21}{120} = 0.175 \text{ and } x/L = 0.385$$

From figure 4-45:

$$\left(\frac{d_c}{H}\right)_M = 0.49 \quad \text{and} \quad C_M = 1.07$$

For:

$$\frac{d_c/H}{(d_c/H)_M} = \frac{0.175}{0.49} = 0.357$$

From figure 4-46:

$$\frac{C_V}{C_M} = 0.58$$

Therefore:

$$C_V = 1.07\ (0.58) = 0.62$$

$$p_V = \frac{A_{sV}}{b\ d_c} = \frac{1.20}{12(21)} = 0.00476$$

so that:

$$v_{uV} = C_V\ p_V\ f_{ds} \qquad \text{(eq. 4-119)}$$

$$= 0.62\ (0.00476)\ (87{,}840)$$

$$= 259 \text{ psi}$$

b. Horizontal Direction (along H)

For:

$$\frac{d_c}{x} = \frac{21}{129} = 0.163$$

From figure 4-40:

$$C_H = 0.81$$

$$p_H = \frac{A_{sH}}{b\ d_c} = \frac{0.86}{12(21)} = 0.00341$$

so that:

$$v_{uH} = C_H \, p_H \, f_{ds} \quad \text{(eq. 4-118)}$$

$$= 0.81 \, (0.00341) \, (87{,}840)$$

$$= 243 \text{ psi}$$

Step 11. Shear Capacity of Concrete (eq. 4-23)

a. Vertical Direction

$$v_c = (1.9 \, (f'_c)^{1/2} + 2{,}500 \, p_V)$$

$$= [1.9 \, (4{,}000)^{1/2} + 2{,}500 \, (0.00476)]$$

$$= 129 \text{ psi}$$

b. Horizontal Direction

$$v_c = (1.9 \, (f'_c)^{1/2} + 2{,}500 \, p_H)$$

$$= [1.9 \, (4{,}000)^{1/2} + 2{,}500 \, (0.00341)]$$

$$= 132 \text{ psi}$$

Step 12. Use lacing method No. 3 (see fig. 4-91).

Step 13. Lacing bar sizes:

a. Vertical Lacing Bars

$b_l = 10$ in $\quad s_l = 22$ in

Assume No. 6 Bars,

$$d_b = 0.75 \text{ in}$$

$$d_l = 21 + 1.13 + 2.00 + 0.75$$

$$= 24.88 \text{ in.}$$

Min $R_p = 4d_b$

For:

$$\frac{s_l}{d_l} = \frac{22}{24.88} = 0.884 \quad \text{(eq. 4-28)}$$

$$\frac{2\,R_1 + d_b}{d_1} = \frac{9\,d_b}{d_1} = \frac{9\,(0.75)}{24.88} = 0.271 \qquad \text{(eq. 4-29)}$$

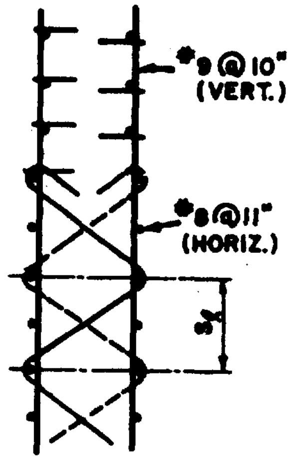


Figure 4A-10

From figure 4-15:

$\alpha = 53.0^{\circ}$

For shear:

$f_{dy} = 1.10 \times 66{,}000 = 72{,}600$ psi

$f_{du} = 1.00 \times 90{,}000 = 90{,}000$ psi

$f_{ds} = (72{,}600 + 90{,}000)/2 = 81{,}300$ psi

$$A_v = \frac{(v_{uV} - v_c)\,b_1\,s_1}{\phi\,f_{ds}\,(\sin\alpha + \cos\alpha)} \qquad \text{(eq. 4-26)}$$

$$= \frac{(259 - 132)\,(10)\,(22)}{0.85\,(81{,}300)\,(0.799 + 0.602)}$$

$= 0.289\ in^2$

Min $A_v = 0.0015\,b_1\,s_1$

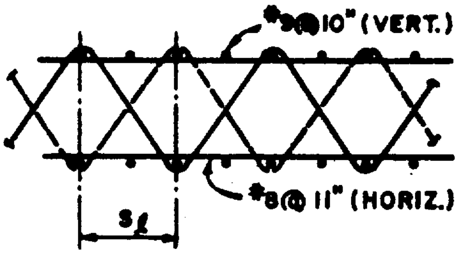
$= 0.0015\,(10)\,(22)$

$= 0.330\ in^2$

Use No. 6 bars:

$(A = 0.44 \text{ in}^2)$

b. Horizontal Lacing:



$b_1 = 11$ in $\quad s_1 = 20$ in

Figure 4A-11

Assume No. 6 Bars:

$$d_b = 0.75 \text{ in}$$

$$d_1 = 21.0 + 1.13 + 0.75 = 22.88 \text{ in}$$

$$\text{Min. } R_1 = 4\, d_b$$

For:

$$\frac{s_1}{d_1} = \frac{20}{22.88} = 0.874 \qquad \text{(eq. 4-28)}$$

$$\frac{2\, R_1 + d_b}{d_1} = \frac{9\, d_b}{d_1} = \frac{9\,(0.75)}{22.88} = 0.295 \qquad \text{(eq. 4-29)}$$

From figure 4-15:

$$\alpha = 53.5'$$

$$A_H = \frac{(v_{uH} - v_c)\, b_1\, s_1}{\phi\, f_{ds}\, (\sin \alpha + \cos \alpha)} \qquad \text{(eq. 4-26)}$$

$$= \frac{(243 - 129)\,(11)\,(20)}{0.85\,(81{,}300)\,(0.804 + 0.595)}$$

$$= 0.259 \text{ in}^2$$

$$\text{Min. } A_v = 0.0015\, b_1\, s_1$$

$$= 0.0015\ (11)\ (20)$$

$$= 0.330\ \text{in}^2$$

Use No. 6 bars:

$(A = 0.44\ \text{in}^2)$

Step 14. Actual d_c depends upon vertical lacing.

Cover	- 2 x 0.75	= 1.50
Lacing	- 2 x 0.75	= 1.50
Horizontal	- 2 x 1.00	= 2.00
Vertical	- 1.13	= 1.13
		6.13 in

$$T_c = d_c + 6.13 = 21 + 6.13$$

$$= 27.13 \text{ in} \quad \therefore \quad \text{Use } 27 \text{ in}$$

$$\text{Actual } d_c = 27 - 6.13 = 20.87 \text{ in}$$

Step 15. Check capacity.

a. Actual impulse capacity.

For:

$$\frac{p_V}{p_H} = 1.40 \approx 1.41, \quad C_u = 461.0 \qquad \text{(fig. 4-34)}$$

For:

$$d_c = 20.87 \text{ in}, \quad p_H = \frac{0.86}{12(20.87)} = 0.00343$$

$$i_b = \frac{p_H\, d_c^{\,3}\, f_{ds}\, C_u}{H}$$

$$= \frac{0.00343(20.87)^3\ (87{,}840)\ (461.0)}{(120)}$$

$$i_b = 3244 \text{ psi-ms} > i_b = 3{,}200 \text{ psi-ms}$$

b. Actual maximum deflection.

For:

$$\frac{p_V}{p_H} = 1.40, \quad C_1 = 452.0 \qquad \text{(fig. 4-31)}$$

Note: Interpolation for C_1 not shown.

$$C_u = 461.0$$

From table 3-6 for x > H:

$$X_1 = H \tan 12^\circ = 120\ (0.2125) = 25.5 \text{ in}$$

$$X_u = x \tan \theta_{max} + \left(\frac{L}{2} - x\right) \tan\left[\theta_{max} - \tan^{-1} \frac{(\tan \theta_{max})}{x/H}\right]$$

$$= (129) \tan 12 + (168-129) \tan\left[12 - \tan^{-1}\left[\frac{\tan 12}{129/120}\right]\right]$$

$$= 27.42 + 0.56 = 27.98 \text{ in.}$$

From Step 5:

$$\frac{i_b^2 H}{p_H\, d_c^3\, f_{ds}} = C_1 + (C_u - C_1)\left[\frac{X_m - X_1}{X_u - X_1}\right]$$

$$\frac{(3200)^2\ (120)}{0.00343(20.87)^3(87{,}840)} =$$

$$452.0 + (461.0 - 452.0)\left[\frac{X_m - 25.5}{27.98 - 25.5}\right]$$

From which: $X_m = 24.58$ in $< X_1$

Note:
Since the deflection X_m is less than X_1, the above solution ($X_m = 24.58$) is incorrect because the equation used is for

the deflection range $X_1 \leq X_m \leq X_u$. Therefore, an equation for the deflection range $0 \leq X_m \leq X_1$ must be used to obtain the correct solution.

From Section 4-33.5 for Type III cross sections and valid for deflection range:

$$0 \leq X_m \leq X_1$$

$$\frac{i_b^2 H}{p_H d_c^3 f_{ds}} = C_1 \left(\frac{X_m}{X_1} \right)$$

$$X_m = \frac{i_b^2 H X_1}{p_H d_c^3 f_{ds} C_1}$$

$$= \frac{(3200)^2 (120) (25.5)}{(0.00343) (20.87)^3 (87,840) 452.0}$$

$$X_m = 25.3 \text{ in}$$

Note:
The element is slightly over-designed. To obtain a more economical design, the amount of flexural reinforcement may be reduced.

Step 16. The response time of the element is obtained from:

$$t_m = \frac{i_b}{r_u} \qquad \text{(eq. 3-96)}$$

where:

$$r_u = \frac{5 (M_{HN} + M_{HP})}{x^2} \qquad \text{(table 3-2)}$$

but:

$$M_{HN} = M_{HP} = \frac{A_{sH} f_{ds} d_c}{b}$$

$$= \frac{0.86\ (87{,}840)\ (20.87)}{12}$$

$$= 131{,}380 \text{ in-lbs/in}$$

therefore:

$$r_u = \frac{5\ (2)\ (131{,}380)}{(129)^2} = 78.9 \text{ psi}$$

so that:

$$t_m = \frac{i_b}{r_u} = \frac{3{,}200}{78.9} = 40.6 \text{ ms}$$

$$\frac{t_m}{t_o} = \frac{40.6}{5} = 8.12$$

The correct procedure has been used since:

$$\frac{t_m}{t_o} > 3$$

Step 17. Ultimate support shear.

a. Vertical Direction (along L):

From figure 4-54, for:

$$x/L = 0.385$$

$$C_{sV} = 4.40$$

$$V_{sV} = \frac{C_{sV}\ p_V\ d_c^2\ f_{ds}}{H} \qquad \text{(eq. 4-122)}$$

$$= \frac{4.40\ (1.20)\ (20.87)^2\ (87{,}840)}{12\ (20.87)\ (120)}$$

$$= 6720 \text{ lbs/in}$$

b. Horizontal Direction (along H):

From table 4-15:

$$C_{sH} = \frac{6}{(x/L)} = \frac{6}{0.385} = 15.6$$

$$V_{sH} = \frac{C_{sH}\ p_H\ d_c^{\ 2}\ f_{ds}}{L} \qquad \text{(eq. 4-121)}$$

$$= \frac{15.6\ (0.00343)\ (20.87)^2\ (87{,}840)}{336}$$

$$= 6{,}090 \text{ lbs/in}$$

Step 18. Diagonal bar sizes.

Note:
Place bars on a 45° angle.

$\therefore \sin \alpha = 0.707$

a. Vertical Direction (at floor slab):

$$A_d = \frac{V_{sV}\ b}{f_{ds} \sin \alpha} = \frac{6{,}720\ (10)}{81{,}300\ (0.707)} = 1.17 \text{ in}^2 \qquad \text{(eq. 4-30)}$$

Required area of bar:

$$\frac{A_d}{2} = 0.58 \text{ in}^2$$

Use No. 8 @ 10.

b. Horizontal Direction (at wall intersections):

$$A_d = \frac{V_{sH}\ b}{f_{ds} \sin \alpha} = \frac{6{,}090\ (11)}{81{,}300\ (0.707)} = 1.16 \text{ in}^2 \qquad \text{(eq. 4-30)}$$

Required area of bar:

$$\frac{A_d}{2} = 0.58 \text{ in}^2$$

Use No. 8 @ 11.

Problem 4A-4, Elements Designed for Impulse-Limited Deflections

Problem: Design an element which responds to the impulse loading of a close-in detonation.

Procedure:

Step 1. Establish design parameters:

a. Blast loads including pressure-time relationship (Chapter 2).

b. Deflection criteria.

c. Structural configuration including geometry and support conditions.

d. Type of cross section available depending upon the occurrence of spalling and/or crushing of the concrete cover.

Step 2. Select cross section of element including thickness and concrete cover over the reinforcement. Also determine the static design stresses of concrete and reinforcing steel (Section 4-12).

Step 3. Determine dynamic increase factors for both concrete and reinforcement from table 4-1. Using the above DIF's and the static design stresses of step 2, calculate the dynamic strength of materials.

Step 4. Determine the dynamic design stresses using table 4-2 and the results from step 3.

Step 5. Assume vertical and horizontal reinforcement bars to yield the optimum steel ratio. The steel ratio is optimum when the resulting yield lines make an angle of 45 degrees with the supports.

Step 6. Calculate d_e (d or d_c depending upon the type of cross section available to resist the blast load) for both the positive and negative moments in both the vertical and horizontal directions. Determine the reinforcing ratios. Also check for the minimum steel ratios from table 4-3.

Step 7. Using the area of reinforcement and the value of de from step 6, and the dynamic design stress of step 4, calculate the moment capacity (Sect. 4-17) of both the positive and negative reinforcement. Also calculate the p_V/p_H ratio and compare to the optimum steel ratio from step 5.

Step 8. Establish values of K_E, X_E and r_u similar to the procedures of problem 4A-1, steps 8 to 18.

Step 9. Determine the load-mass factor K_{LM}, for elastic, elasto-plastic and plastic ranges from table 3-13 and figure 3-44. The average load mass factor is obtained by taking the average K_{LM} for the elastic and elasto-plastic ranges and averaging this value with the K_{LM} of the plastic range. In addition, calculate the unit mass of the element (account for reduced concrete thickness if spalling is anticipated) and multiply this unit mass by K_{LM} for the element to obtain the effective unit mass of the element.

Step 10. Using the effective mass of step 9 and the equivalent stiffness of step 8, calculate the natural period of vibration T_N from equation 3-60.

Step 11. Determine the response chart parameters:

a. Peak pressure P (step 1).

b. Peak resistance r_u (step 8).

c. Duration of load T (step 1).

d. Natural period of vibration T_N (step 10).

Also calculate the ratios of peak pressure P to peak resistance r_u and duration T to period of vibration T_N. Using these ratios and the response charts of Chapter 3, determine the value of X_m/X_E. Compute the value of X_m.

Step 12. Determine the support rotation corresponding to the value of X_m from step 11 using the equations of table 3-6. Compare this value to maximum permissible support rotation of step 1, and if found to be satisfactory, proceed to step 13. If comparison is unsatisfactory, repeat steps 2 to 12.

Step 13. Using the ultimate resistance of step 8, the values of d_e of step 6 and the equations of table 4-6 or 4-7 (table 3-10 or 3-11 if shear at support is required), calculate the ultimate diagonal tension shear stress at a distance d_e from each support (or at each support). Also, calculate the shear capacity of the concrete from equation 4-23. If the capacity of the concrete is greater than that produced by the load, minimum shear reinforcement must be used. However, if the shear produced by the load is greater than the capacity of the concrete, then shear reinforcement in excess of the minimum required must be provided. Also check for maximum spacing of shear reinforcement.

Step 14. Using the equations of table 3-9. 3-10 or 3-11 and the ultimate resistance of step 8, calculate the shear at the supports. Determine the required area of diagonal bars using equation 4-30. However, if section type I is used, then the minimum diagonal bars must be provided.

Example 4A-4, Elements Designed for Impulse-Limited Deflections

Required: Design the side wall of cubicle with no roof or front wall and subject to the effects of a detonation of an explosive within the cubicle.

Solution:

Step 1: Given:

a. Pressure-time loading (fig. 4A-12).

b. Maximum support rotation equal to 2 degrees.

c. L = 180 in., H = 144 in. and fixed on two sides (fig. 4A-12).

d. Type III cross section.

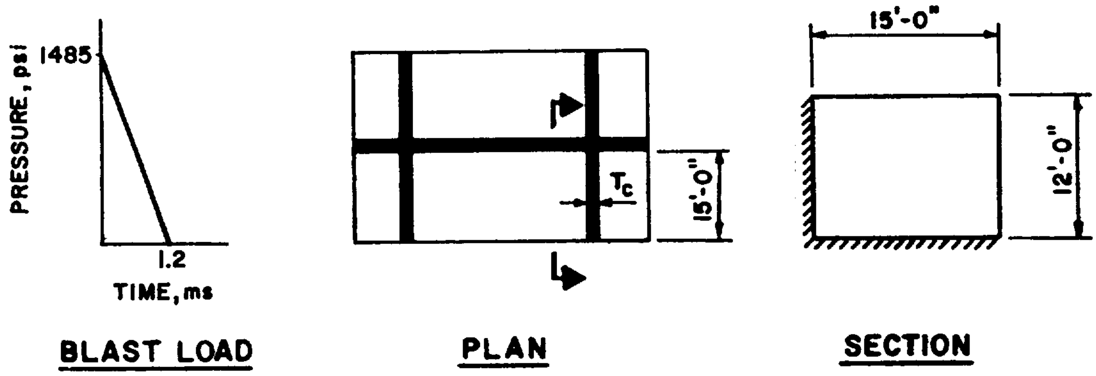


Figure 4A-12

Step 2. Select element thickness and static stress of reinforcement and concrete (fig 4A-13).

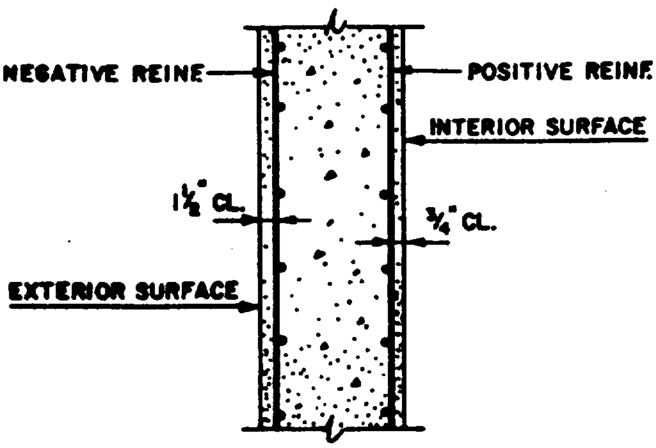


Figure 4A-13

$f'_c = 4{,}000$ psi

$f_y = 66{,}000$ psi

Assume $T_c = 22$ in. and concrete cover is as shown in figure 4A-13.

Step 3. Determine dynamic stresses.

a. Dynamic increase factors, DIF (from table 4-1).

Concrete:

Diagonal Tension - 1.00

Reinforcement:

Bending - 1.23

Diagonal Tension - 1.10

Direct Shear - 1.10

b. Dynamic strength of materials.

Concrete (f'_{dc}):

Diagonal Tension 1.00 (4,000) = 4,000 psi

Reinforcement (f_{dy}):

Bending 1.23 (66,000) = 81,180 psi

Diagonal Tension 1.10 (66,000) = 72,600 psi

Direct Shear 1.10 (66,000) = 72,600 psi

Step 4. Dynamic design stress from table 4-2.

Concrete (f'_{dc}):

Diagonal Tension - 4,000 psi

Reinforcement ($f_{ds} = f_{dy}$ for $0 < 2$)

Bending - 81,180 psi

Diagonal Tension - 72,600 psi

Direct Shear - 72,600 psi

Step 5. Determine the optimum steel ratio p_V/p_H. Set x = H to obtain 45 degree yield lines.

$$\frac{x}{L} = \frac{H}{L} = \frac{144}{180} = 0.80$$

From figure 3-4,

$$\frac{L}{H}\left(\frac{M_{vp}}{M_{HN} + M_{HP}}\right) = 1.08$$

Therefore,

$$\frac{M_{VP}}{M_{HN} + M_{HP}} = \left[\frac{1.08 \times 144}{180}\right]^2 = 0.75$$

or

$$M_{VP}/M_{HN} = 1.50$$

Try No. 7 bars at 8 in. o.c. in the vertical direction and No. 6 bars at 8 in. o.c. in the horizontal direction.

Step 6. Calculate d_c and the steel ratios for each direction.

Assume No. 3 stirrups.

$$d_{cV} = 22 - (2 \times 0.375) - 0.75 - 1.5 - (2 \times 0.875/2) = 18.125 \text{ in.}$$

$$d_{cH} = 18.125 - (2 \times 0.875/2) - (2 \times 0.75/2) = 16.50 \text{ in.}$$

$$p_V = \frac{A_{sV}}{bd_{cV}} = \frac{0.60}{8 \times 18.125} = 0.0041 > 0.0015 \text{ minimum}$$

$$p_H = \frac{A_{sH}}{bd_{cH}} = \frac{0.44}{8 \times 16.50} = 0.0033 \geq 0.0015 \text{ minimum}$$

Step 7. Calculate the moment capacity of both the positive and negative reinforcement in both directions (eq. 4-19).

$$M_u = \frac{A_s f_{ds} d_c}{b}$$

$$M_{VN} = M_{VP} = \frac{0.60\ (81{,}180)(18.125)}{8} = 110{,}354 \text{ in-lbs/in}$$

$$M_{HN} = M_{HP} = \frac{0.44\ (81{,}180)(16.50)}{8} = 73{,}671 \text{ in-lbs/in}$$

$$M_{VN}/M_{HN} = 110{,}354/73{,}671 \approx 1.5 = 1.5 \text{ from step 5 o.k.}$$

Step 8. Using the procedure in example 4A-1, steps 8 through 18 and the moment capacities from step 7. establish the values of K_E, X_E and r_u.

K_E = 36.7 psi/in.

X_E = 0.968 in.

r_u = 35.53 psi

Step 9. Calculate the effective mass of the element.

a. Load mass factors (table 3-13 and fig. 3-44)

x/L = 0.80

elastic range	K_{LM} = 0.65
elasto-plastic range:	
one simple edge	K_{LM} = 0.66
two simple edges	K_{LM} = 0.66
plastic range	K_{LM} = 0.54

K_{LM} (average elastic and elasto-plastic values)

$$= \frac{0.65 + 0.66 + 0.66}{3} = 0.66$$

K_{LM} (average elastic and plastic values)

$$= \frac{0.66 + 0.54}{2} = 0.60$$

b. Unit mass of element:

Using the larger d_c as the thickness of the element.

Due to spalling (Type III cross section) available thickness equals

$T_c = d_c = 18.125$ in.

$$m = \frac{w_c d_c}{g} = \frac{150\ (18.125)\ 10^6}{32.2 \times 12 \times 1728} = 4{,}072 \text{ psi-ms}^2\text{/in}$$

c. Effective unit mass of element:

$m_e = K_{LM}m = 0.60\ (4{,}072) = 2{,}443$ psi-ms^2/in

Step 10. Calculate the natural period of vibration.

$T_N = 2\ \pi\ (m_e/K_E)^{1/2}$ (eq. 3-60)

$T_N = 2\ (3.14)\ (2443/36.7)^{1/2} = 51.2$ ms

Step 11. Determine maximum response of element.

a. Response chart parameters:

Peak pressure, $P = 1485$ psi (step 1)

Peak resistance, $r_u = 35.53$ psi (step 8)

Duration of blast load, $T = 1.2$ ms (step 1)

Period of vibration $T_N = 51.2$ ms (step 6)

$P/r_u = 1485/35.53 = 41.8$

$T/T_N = 1.2/51.2 = 0.023$

b. From figure 3-64a:

$X_m/X_E = 5.0$

$X_m = 5.0 \times 0.968 = 4.84$ in

Step 12. Check support rotation (table 3-6).

Since $x = H = 144$ in. and $0 < X_m < X_1$

$X_m = x \tan \theta_H$

$\tan \theta_H = 4.84/144$

$\theta_H = 1.93° < 2°$ assumed section is O.K.

Step 13. Check diagonal tension at supports (internal loading).

a. Calculate ultimate shear stresses at support by dividing the values of the support shear from table 3-10 by their respective d_c.

$$V_{sH} = 3r_u x/5 = 3 \times 35.53 \times 144/5 = 3{,}070 \text{ lb/in}$$

$$V_{sV} = \frac{3r_u H \left(2 - \frac{X}{L}\right)}{\left(6 - \frac{x}{L}\right)} = \frac{3\ (35.53)\ 144\ (2 - 0.80)}{(6 - 0.80)} = 3{,}542 \text{ lb/in}$$

$$v_{uH} = V_{sH}/d_{cH} = 3{,}070/16.50 = 186.1 \text{ psi}$$

$$v_{uV} = V_{sV}/d_{cV} = 3{,}542/18.125 = 195.4 \text{ psi}$$

b. Allowable shear stresses (eq. 4-23)

$$v_c = 1.9\ (f'_c)^{1/2} + 2500\ p \le 3.5\ (f'_c)^{1/2} = 221.4 \text{ psi}$$

where p is the steel ratio at the support.

$$v_{cH} = 1.9\ (4000)^{1/2} + 2500\ (0.0033) = 128.4 \text{ psi} < 221.4 \text{ psi}$$

$$v_{cV} = 1.9\ (4000)^{1/2} + 2500\ (0.0041) = 130.4 \text{ psi} < 221.4 \text{ psi}$$

c. Required area of single leg stirrups.

$$A_V = \frac{(V_u - V_c)bs}{0.85\ (f'_{ds})} \qquad \text{(eq. 4-26)}$$

$$v_{uH} - v_{cH} = 186.1 - 128.4 = 57.7 < 0.85 v_{cH} = 108.8 \text{ psi}$$

Use 0.85 v_{cH} as minimum.

$v_{uV} - v_{cV} = 195.4 - 130.4 = 65.0 < 0.85v_{cV} = 110.8$ psi

Use 0.85 v_{cV} as minimum.

Tie every reinforcing bar intersection, therefore,

$b = s = 8$ in. $< d_c/2$ (maximum spacing) O.K.

$$A_{vH} = \frac{108.8 \times 8 \times 8}{0.85 \times 72{,}600} = 0.11 \text{ in}^2$$

minimum $A_v = 0.0015$ bs $= 0.10$ in^2 < 0.11 O.K.

$$A_{vV} = \frac{110.8 \times 8 \times 8}{0.85 \times 72{,}600} = 0.11 \text{ in}^2$$

minimum $A_v = 0.0015$ bs $= 0.10$ in^2 < 0.11 O.K.

The area of No. 3 bar is 0.11 in^2, so bar assumed in step 6 is O.K.

Step 14. Determine required area of diagonal bars using the values of the shear at the support from step 13.

$$A_d = \frac{V_s b}{f_{ds} \sin \alpha} \qquad \text{(eq. 4-30)}$$

Assume diagonal bars are inclined at 45 degrees.

$$A_{dH} = \frac{3.070 \times 8}{72{,}600 \times 0.707} = 0.48 \text{ in}^2 \text{ at 8 in. o.c.}$$

$$A_{dV} = \frac{3{,}542 \times 8}{72{,}600 \times 0.707} = 0.55 \text{ in}^2 \text{ at 8 in. o.c.}$$

Use No. 7 bars ($A_b = 0.60$ in^2) at 8 in. o.c. at both supports

Problem 4A-5. Elements Designed for Impulse-Composite Construction

Problem: Design a composite (concrete-sand-concrete) wall to resist a given blast output for incipient failure.

Procedure:

Step 1. Establish design parameters.

a. Structure configuration.

b. Charge weight.

c. Blast impulse load (Chapter 2).

d. Thicknesses of concrete and sand portions of wall.

e. Blast impulse resisted by concrete panels.

f. Density of concrete and sand.

Step 2. Determine scaled thicknesses of concrete and sand using:

$\bar{T}_c = T_c/W^{1/3}$ and $\bar{T}_s = T_s/W^{1/3}$

Step 3. Determine scaled blast impulse resisted by each concrete panel using:

$\bar{i}_{bd} = i_{bd}/W^{1/3}$ (donor panel)

$\bar{i}_{ba} = i_{ba}/W^{1/3}$ (acceptor panel)

Step 4. Correct scaled blast impulse resisted by concrete (Step. 3.) to account for the increased mass produced by the sand and the reduction of the concrete mass produced by spalling and scabbing of the concrete panels using:

$$\text{(Corr.)} \qquad \bar{i}_{bd} = \bar{i}_{bd} \left[\frac{\dfrac{T_c + d_c}{2} + \left(\dfrac{w_s}{w_c}\right)\left(\dfrac{T_s}{2}\right)}{d_c} \right]^{1/2}$$

$$\text{(Corr.)} \qquad \bar{i}_{ba} = \bar{i}_{ba} \left[\frac{\dfrac{T_c + d_c}{2} + \left(\dfrac{w_s}{w_c}\right)\left(\dfrac{T_s}{2}\right)}{d_c} \right]^{1/2}$$

Step 5. Determine scaled blast impulse attenuated by acceptor panel and the sand $\bar{i}_a$ from figure 4-57 or 4-58, for w_s equal to 85 and 100 pcf., respectively.

Step 6. Calculate total impulse resisted by the wall using:

$$\bar{i}_{bt} = \bar{i}_a + \bar{i}_{bd}$$

Step 7. Compare blast impulse which is resisted by wall to that of the applied blast loads.

Example 4A-5, Elements Designed for Impulse-Composite Construction

Required: Design the composite wall shown below for incipient failure conditions.

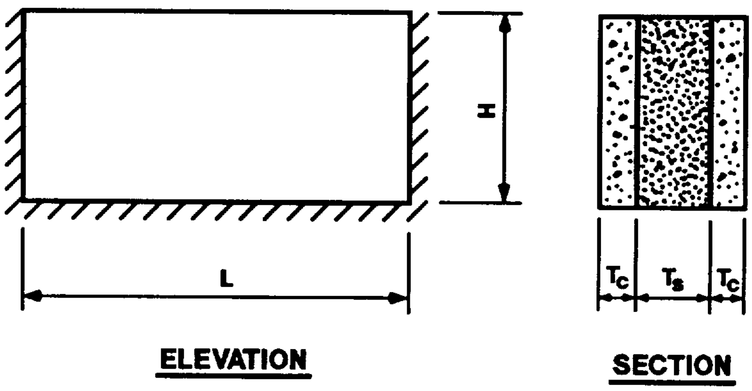


Figure 4A-14

Step 1. Given:

a. Structural configuration as shown in figure 4A-14.

b. $W = 1{,}000$ lbs.

c. $i_b = 4{,}800$ psi-ms (Chapter 2).

d. $T_c = 1$ ft, $T_s = 2$ ft, and $d_c = 0.833$ ft.

e. $i_{bd} = i_{ba} = 1{,}500$ psi-ms (sect. 4-33).

f. $w_c = 150$ pcf and $w_s = 100$ pcf.

Step 2. Scaled thicknesses of concrete and sand:

$$\bar{T}_c = T_c/W^{1/3} = 1/(1000)^{1/3} = 0.1 \text{ ft/lb}^{1/3} \qquad \text{(eq. 4-125)}$$

$$\bar{T}_s = T_s/W^{1/3} = 2/(1000)^{1/3} = 0.2 \text{ ft/lb}^{1/3} \qquad \text{(eq. 4-126)}$$

Step 3. Scaled blast impulse resisted by individual concrete panels.

$$\bar{i}_{bd} = \bar{i}_{ba} = \frac{1500}{(1000)^{1/3}} = 150 \text{ psi-ms/lb} \qquad \text{(eq. 4-127)}$$

Step 4. Correction of scaled impulse resisted by concrete panel used in composite walls.

$$(\text{Corr.}) \quad \bar{i}_{bd} = \bar{i}_{bd} \left[\frac{\dfrac{T_c + d_c}{2} + \left(\dfrac{w_s}{w_c}\right)\left(\dfrac{T_s}{2}\right)}{d_c} \right]^{1/2}$$

$$= 150 \left[\frac{\dfrac{1.0 + 0.833}{2} + \dfrac{100}{150}\left(\dfrac{2}{2}\right)}{0.833} \right]^{1/2}$$

$$= 207 \text{ psi-ms/lb}^{1/3} = i_{ba} \text{ (corr.)}$$

Step 5. Scaled blast impulse attenuated by acceptor panel and sand.

$$\bar{i}_a = 280 \text{ psi-ms/lb} \qquad \text{(fig. 4-58)}$$

Step 6. Total scaled blast impulse resisted by wall.

$$\bar{i}_{bt} = \bar{i}_a + \bar{i}_{bd} = 280 + 207 = 487 \text{ psi-ms/lb}^{1/3}$$

Step 7. Comparison of wall capacity and applied blast load.

$$\bar{i}_{bt} = 487 \approx \bar{i}_b = 480 \text{ psi-ms/lb}^{1/3} \qquad \text{O.K.}$$

Problem 4A-6, Design of a Beam in Flexure

Problem: Design an interior beam of a roof subjected to an overhead blast load.

Solution:

Step 1. Establish design parameters:

a. Structural configuration.

b. Pressure-time loading.

c. Maximum allowable support rotation.

d. Material properties

Step 2. From table 4-1, determine the dynamic increase factors, DIF. For the deflection criteria given in Step 1c, find the equation for the dynamic design stress from table 4-2. Using the DIF and the material properties from Step 1d, calculate the dynamic design stresses.

Step 3. Assuming reinforcing steel and concrete cover, calculate the distance from the extreme compression fiber to the centroid of the tension reinforcement, d.

Step 4. Calculate the reinforcement ratio of the steel assumed in Step 3. Check that this ratio is greater than the minimum reinforcement required by equation 4-137 but less than the maximum reinforcement permitted by equation 4-132.

Step 5. Using equations 4-129 and 4-130, the dynamic design stresses from Step 2, and the value of d from Step 3. calculate the ultimate moment capacity of the beam.

Step 6. Compute the ultimate unit resistance of the beam using the moment capacity of Step 4 and an equation from table 3-1.

Step 7. Calculate the modulus of elasticity of concrete E_c and steel E_s (equations 4-4 and 4-5, respectively) and the modular ratio n (equation 4-6). Determine the average moment of inertia I_a of the beam according to Section 4-15.

Step 8. From table 3-8, find the correct equation for the equivalent elastic stiffness K_E. Evaluate this equation using the values of E_c and I_a from Step 7.

Step 9. With the ultimate resistance from Step 6 and the stiffness K_E from Step 8, use equation 3-36 to calculate the equivalent elastic deflection X_E.

Step 10. Find the values for the load-mass factor K_{LM} in the elastic, elasto-plastic and plastic ranges from table 3-12. Average these values according to Section 3-17.4 to determine the value of K_{LM} to be used in design.

Step 11. Determine the natural period of vibration T_N using equation 3-60, K_{LM} from Step 10, K_E from Step 8 and the mass of the beam. The mass includes 20 percent of the adjacent slabs.

Step 12. Calculate the non-dimensional parameters T/T_N and r_u/P. Using the appropriate response chart by Chapter 3 determine the ductility ratio, μ.

Step 13. Compute the maximum deflection X_m using the ductility ratio from Step 12 and X_E from Step 9. Calculate the support rotation corresponding to X_m using an equation from table 3-5. Compare this rotation with the maximum allowable rotation of Step 1c.

Step 14. Verify that the ultimate support shear V_s given in table 3-9 does not exceed the maximum shear permitted by equation 4-142. If it does, the size of the beam must be increased and Steps 2 through 13 repeated.

Step 15. Calculate the diagonal tension stress v_u from equation 4-139 and check that it does not exceed $10(f_{dc}')^{1/2}$.

Step 16. Using the dynamic concrete strength f_{dc}' from Step 2 and equation 4-140, calculate the shear capacity of the unreinforced web, v_c.

Step 17. Design the shear reinforcement using equation 4-140, and the excess shear stress $(v_u - v_c)$ or the shear capacity of concrete v_c' whichever is greater.

Step 18. Check that the shear reinforcement meets the minimum area and maximum spacing requirements of Section 4-39.4.

Step 19. With T/T_N and X_m/X_E from Step 12, enter figure 3-268 and read the required resistance of the beam in rebound.

Step 20. Repeat Steps 3 through 6 to satisfy the required rebound resistance.

Example 4A-6, Design of a Beam in Flexure

Required: Design of an interior of a roof beam subjected to an overhead blast load.

Solution:

Step 1. Given:

a. Structural configuration is shown in figure 4A-15a.

b. Pressure-time loading is shown figure 4A-15c.

c. Maximum support rotation of one degree.

d. Yield stress of reinforcing steel, $f_y = 66,000$ psi

Concrete compressive strength, $f'_c = 4,000$ psi

Weight of concrete, $w = 150$ lbs/ft^3

Step 2. a. Dynamic increase factors from table 4-1 for intermediate and low pressure range.

Reinforcing steel	- bending, DIF	= 1.17
	- direct shear, DIF	= 1.10
Concrete	- compression, DIF	= 1.19
	- direct shear, DIF	= 1.10
	- diagonal tension, DIF	= 1.00

b. From table 4-2, for $\theta_m \leq 2°$:

$$f_{ds} = f_{dy}$$

c. Dynamic design stresses from equation 4-3.

Reinforcing steel	- bending	f_{dy}	= 1.17 x 66,000 = 77,220 psi
	- diagonal tension	f_{dy}	= 1.00 x 66,000 = 66,000 psi
Concrete	- compression	f'_{dc}	= 1.19 x 4,000 = 4,760 psi
	- direct shear	f'_{dc}	= 1.10 x 4,000 = 4,400 psi
	- diagonal tension	f_{dy}	= 1.00 x 4,000 = 4,000 psi

Step 3. Assume 5 No. 6 bars for bending:

$A_s = 5 \times .44 = 2.20$ in^2

For concrete cover and beam sections see figure 4A-15b.

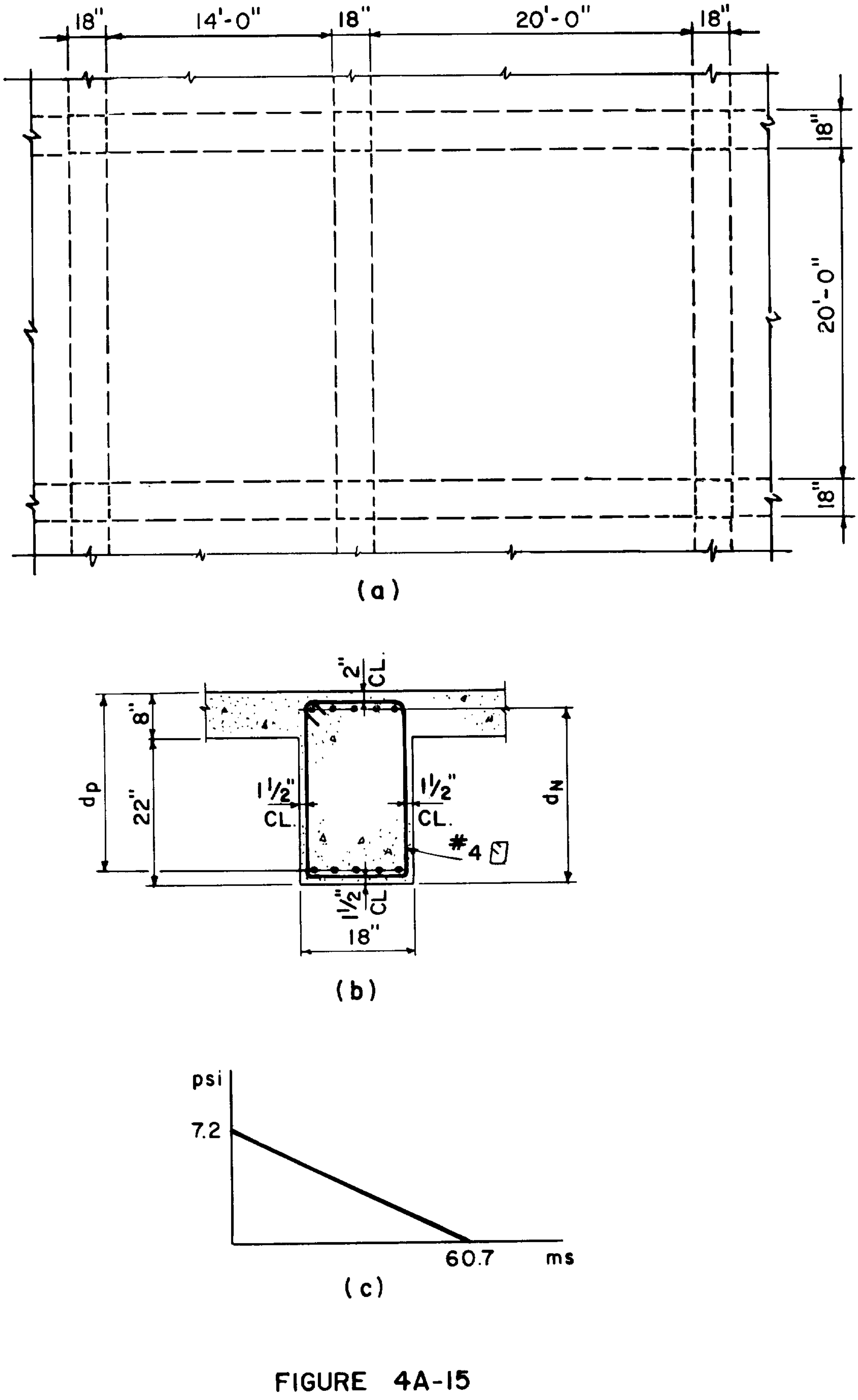

18"
14'-0"
18"
20'-0"
18"
18"
20'-0"
18"
(a)
2" CL.
8"
22"
dp
1 1/2" CL.
1 1/2" CL.
#4
dN
1 1/2" CL
18"
(b)
psi
7.2
60.7
ms
(c)

FIGURE 4A-15

Step 4. Check reinforcement requirements:

a. Calculate d negative (support) and positive (mid-span) for checking bending reinforcement ratios.

$$d = h - d' \text{ (cover)} - \phi ' \text{ (tie)} \frac{\phi}{2} \text{ (Bending Bar)}$$

$$d_N = 30 - 2 - 0.5 - 0.75/2 = 27.125 \text{ in}$$

$$d_p = 30 - 1.5 - 0.5 - 0.75/2 = 27.625 \text{ in}$$

b. Calculate reinforcement ratio:

From equation 4-131,

$$p = A_s/bd$$

$$p_N = 2.2/(18 \times 27.125) = 0.0045$$

$$p_p = 2.2/(18 \times 27.615) = 0.0044$$

c. Maximum reinforcement:

Maximum reinforcing ratio $p_{max} = 0.75 \times p_b$

From equation 4-132;

$$p_b = \frac{0.85K_1 \ f'_{dc}}{f_{dy}} + \frac{87{,}000}{87{,}000 + f_{dy}}$$

where:

$$K_1 = 0.85 - \frac{0.05 \ (f'_{dc} - 4{,}000)}{1{,}000} = 0.812$$

$$p_b = \frac{0.85 \times 0.812 \times 4760}{77{,}220} \left(\frac{87{,}000}{87{,}000 + 77{,}220}\right) = 0.0225$$

$p_{max} = 0.75 \times 0.0225 = 0.0169 > p_N = 0.0045$ and

$p_p = 0.0044$ O.K.

d. Check for minimum reinforcing ratio using equation 4-138

$$p_{min} = 200 / f_y$$

$$p_{min} = \frac{200}{60,000} = 0.0033 < p_N = 0.0045 \text{ O.K.}$$

$$< p_p = 0.0044 \text{ O.K.}$$

Step 5. Moment capacity of the beam using equations 4-129 and 4-130 is:

$$M_u = A_s \ f_{dy} \ (d-a/2)$$

where:

$$a = \frac{A_s \ f_{dy}}{0.85b \ f'_{dc}}$$

$$a = \frac{2.20 \times 77,220}{0.85 \times 18 \times 4,760} = 2.333 \text{ in}$$

at support:

$$M_N = 2.20 \times 77,220 \times (27.125 - 2.333/2)$$

$$= 4,409,934 \text{ in-lbs}$$

at mid-span:

$$M_p = 2.20 \times 77,220 \times (27.625 - 2.333/2)$$

$$= 4,494,876 \text{ in-lbs}$$

Step 6. From table 3-1, ultimate resistance of a uniformly loaded beam with fixed ends is:

$$r_u = \frac{8 \ (M_N + M_p)}{L^2}$$

$$r_u = \frac{8 \ (4,409,934 + 4,494,934)}{240^2} = 1,236.79 \text{ lbs/in}$$

Step 7. From Section 4-15, calculate average moment of inertia of the beam section.

a. Concrete modulus of elasticity (eq. 4-4):

$$E_c = w^{1.5} \times 33 \times (f'_c)^{1/2}$$

$$E_c = 150^{1.5} \times 33 \times (4{,}000)^{1/2} = 3.8 \times 10^6 \text{ psi}$$

b. Steel modulus of elasticity (eq. 4-5):

$$E_s = 29 \times 10^6 \text{ psi}$$

c. Modular ratio (eq. 4-6):

$$n = \frac{E_s}{E_c}$$

$$n = \frac{29 \times 10^6}{3.8 \times 10^6} = 7.6$$

d. From figure 4-11 and having n, p_N and p_p, the coefficients for moment of inertia of cracked sections are:

$$F_N = 0.0235 \text{ at support}$$

$$F_p = 0.0230 \text{ at mid-span}$$

Cracked moment of inertia from equation 4-8b is:

$$I_c = Fbd^3$$

$$I_{cN} = 0.0235 \times 18 \times 27.125^3 = 8{,}442 \text{ in}^4$$

$$I_{cp} = 0.0230 \times 18 \times 27.625^3 = 8{,}728 \text{ in}^4$$

Average:

$$I_c = (I_{cN} + I_{cp}) / 2$$

$$I_c = \frac{8{,}442 + 8{,}728}{} = 8{,}585 \text{ in}^4$$

e. Gross moment of inertia (eq. 4-8):

$$I_g = \frac{bh^3}{12}$$

$$I_g = \frac{18 \times 30^3}{12} = 40{,}500 \text{ in}^4$$

f. Average moment of inertia of the beams from equation 4-7:

$$I_a = \frac{I_g + I_c}{2}$$

$$I_a = \frac{40{,}500 + 8{,}585}{2} = 24{,}542.5 \text{ in}^4$$

Step 8. From table 3-8, K_E of a uniformly loaded beam with fixed ends is:

$$K_E = \frac{307\ E_c\ I_a}{L^4}$$

$$K_E = \frac{307 \times 3.8 \times 10^6 \times 24{,}542.5}{240^4}$$

$$= 8{,}629.70 \text{ lbs/in/in}$$

Step 9. Equivalent elastic deflection from equation 3-36 is:

$$X_E = \frac{r_u}{K_E} = \frac{1{,}236.79}{8{,}629.70} = 0.1433 \text{ in}$$

Step 10. Load-mass factor from table 3-12 for a plastic range of a uniformly loaded beam with fixed ends is:

K_{LM} - elastic = 0.77
- elasto-plastic = 0.78
- plastic = 0.66

K_{LM} for plastic mode deflections; from Section 3-17.4 from Chapter 3:

$$K_{LM} = \left[\left[\frac{0.77 + 0.78}{2} \right] + 0.66 \right] / 2 = 0.72$$

Step 11. Natural period of the beam from equation 3-60 is:

$$T_N = 2\pi \, (K_{LM} \, m/K_E)^{1/2}$$

Where m is the mass of the beam plus 20% of the slabs span perpendicular to the beam:

$$m = w/g$$

$$m = (30 \times 18 + 2 \times 8 \times 102 \times 0.20)$$

$$\times \frac{150}{12^3} \times \frac{1{,}000^2}{32.2 \times 12}$$

$$= 194{,}638.50 \text{ lbs-ms}^2\text{/in/in}$$

$$T_n = 2\pi \left[\frac{0.72 \times 194{,}638.50}{8{,}629.70}\right]^{1/2} = 25.3 \text{ ms}$$

Step 12. Find μ, ductility ratio from figure 3-54.

From Step 1:

$$T/T_N = 60.7/25.3 = 2.40$$

$$P = (18 + 84 + 120) \times 7.2$$

$$= 1{,}598.40 \text{ lbs/in}$$

$$r_u/P = \frac{1{,}236.79}{1{,}598.40} = 0.77$$

$$\mu = 9.0$$

Step 13. From table 3-5 support rotation is:

$$X_m = \frac{L \tan \Theta}{2}$$

$$X_m = \mu \times X_E$$

$$X_m = 9.0 \times 0.1433 = 1.29 \text{ in}$$

$$\tan \Theta = \frac{2 \times 1.29}{240} = 0.01075$$

$$\Theta = 0.620 \leq 1° \quad \text{O.K.}$$

Step 14. Direct Shear from table 3-9 is:

$$V_s = \frac{r_u L}{2}$$

$$V_s = \frac{1236.79 \times 240}{2} = 148{,}415 \text{ lbs}$$

Section capacity in direct shear from equation 4-142:

$$V_d = 0.18 f'_{dc} bd$$

$$V_d = 0.18 \times 4{,}400 \times 18 \times 27.125$$

$$= 386{,}694 \text{ lbs} > V_s = 148{,}415 \quad \text{O.K.}$$

Step 15. Diagonal tension stress from equation 4-139:

$$v_u = \frac{V_u}{bd} \leq 10 (f'_c)^{1/2}$$

Total shear d distance from the face of support:

$$V_u = (L/2 - d) r_u$$

$$= (\frac{240}{2} - 27.125) 1236.79 = 114{,}867 \text{ lb}$$

$$V_u = \frac{114867}{18 \times 27.125} = 235.2 \text{ psi}$$

$$10 (f'_{dc})^{1/2} = 10 \times (4{,}000)^{1/2}$$

$$= 632.5 \text{ psi} > 235.2 \text{ psi O.K.}$$

Step 16. Unreinforced web shear capacity using equation 4-140 is:

$$v_c = [1.9 (f'_{dc})^{1/2} + 2{,}500 p]$$

$$< 3.5 (f'_{dc})^{1/2}$$

$$v_c = [1.9\ (4{,}000)^{1/2} + 2{,}500 \times 0.0045]$$

$$= 131.4 \text{ psi}$$

$$3.5\ (f'_{dc})^{1/2} = 3.5 \times (4{,}000)^{1/2}$$

$$= 221.4 \text{ psi} > 131.4 \text{ psi} \quad \text{O.K.}$$

Step 17. Area of web reinforcing from equation 4-141:

$$A_v = [(v_u - v_c) \times b \times s_s] / \phi \times f_{dy}; \qquad v_u - v_c \geq v_c$$

$$v_u - v_c = 235.2 - 131.4 = 104 < v_c \text{ use } v_c$$

Assume:

$$s_s = 9 \text{ in}$$

$$A_v = 131.4 \times 18 \times 9/(0.85 \times 66{,}000)$$

$$= 0.38 \text{ in } /9 \text{ in}^2$$

Use No. 4 tie:

$$A_v = 0.40 \text{ in}^2$$

Step 18. Minimum tie reinforcing area:

$$A_v \text{ (min)} = 0.0015\ bs_s$$

$$A_v \text{ (min)} = 0.0015 \times 18 \times 9$$

$$= 0.24 \text{ in}^2 < 0.40 \text{ in}^2 \quad \text{O.K.}$$

Maximum tie spacing:

$$(f'_{dc})^{1/2} = 4 \times (4{,}000)^{1/2}$$

$$= 253 \text{ psi} > v_c = 131 \text{ psi}$$

$$> v_u - v_c = 104 \text{ psi}$$

$$s_{max} = d/2$$

$$s_{max} = 27.125 / 2 = 13.56 \text{ in} > 9 \text{ in} \quad \text{O.K.}$$

Step 19. Determine required resistance for rebound r^- from figure 3-268:

$r^-/r_u = 0.50$ for $T/T_N = 2.40$ and $X_m/X_E = 9.0$

Required:

$r^- = 0.50 \times 1236.79 = 618.4$ lbs/in

Step 20. Repeat Steps 3 to 6:

Assume:

$A^-_s = 1.64$ in^2 , 2 No. 7 + 1 No. 6

p^-_N at support = 0.0033 = $200/f_y$

p^-_p at mid-span = 0.0034 > $200/f_y$

N^-_N at support = 3,388,275 in-lbs

M^-_p at mid-span = 3,324,954 in-lbs

r^- = 932.4 lbs/in

> 618.4 lbs/in O.K.

Problem 4A-7, Design of a Beam Subject to Torsion

Problem: Design a beam for a uniformly distributed torsional load.

Procedure:

Step 1. Design the beam and adjacent slabs in flexure for the applied blast load.

Step 2. Calculate the unbalanced slab support shears, V_T using the ultimate resistance of the slabs from Step 1.

$$V_T = \frac{r_{u1} L_1}{2} + \frac{r_{u2} L_2}{2}$$

Using the unbalanced slab support shears, compute the torsional load at d distance from the face of the support from:

$$T_u = \left(\frac{L}{2} - d\right) \frac{b}{2} (V_T)$$

Step 3. Using the torsional load from Step 2, compute the nominal torsional stress in the vertical direction from equation 4-143 and in the horizontal direction from equation 4-144. Verify that the torsional stresses do not exceed the maximum stress permitted in Sec. 4-41.5.

Note:
If the height of the beam is greater than width, the horizontal torsional stresses will not be critical and may be ignored.

Step 4. Determine the shear and torsional capacity of an unreinforced web, v_c and v_{tc}, from equations 4-145 and 4-146 or 4-147, the torsional stress from Step 3 and the shear stress from step 1.

Step 5. Find the excess shear stress (v_u - v_c) where the nominal shear stress v_u is from Step 1, and the shear capacity of the unreinforced web v_c is from Step 4. Using the excess shear stress and equation 4-141, determine the area of web reinforcing for shear.

Step 6. With torsional capacity of the concrete from Step 4, the torsional stresses from Step 3, and equations 4-148 and 4-149, calculate the area of web reinforcement for torsion in the vertical and, if required, in the horizontal directions.

Step 7. Add the area of shear reinforcement from Step 5 and the area of torsion reinforcement in the vertical direction, and compare with the area of torsion reinforcement required in the horizontal direction. The larger of the two values will control for the design of the closed ties. (If height of beam is greater than width, see note at Step 3.)

Step 8. Check minimum area and maximum spacing requirements of ties according to section 4-41.5.

Step 9. Calculate the required area of longitudinal torsion reinforcement from equations 4-151a and 4-151b, the torsional stress from Step 3 and the torsion capacity of concrete from Step 4.

Step 10. Determine the distribution of flexural and longitudinal steel at the supports and at the midsection.

Example 4A-7, Design of Beam in Torsion

Required: Design of beam in example 4A-6, for torsional load due to unequal spans of adjacent slab.

Solution:

Step 1. Given:

a. Beam designed for flexure in example 4A-6 where:

$$L = 240 \text{ in}$$

$$d = 27.125 \text{ in}; \qquad b = 18 \text{ in}$$

$$v_u = 235.2 \text{ psi}$$

b. Slabs designed for flexure where:

$$r_{u1} = 15.0 \text{ psi} \quad L_1 = 14 \text{ ft} = 168 \text{ in}$$

$$r_{u2} = 7.85 \text{ psi} \quad L_2 = 20 \text{ ft} = 240 \text{ in}$$

Step 2. Calculate torsional load.

a. Unbalanced slab support shears:

$$V_T = \frac{r_{u1} \; L_1}{2} - \frac{r_{u2} \; L_2}{2}$$

$$= \frac{15(168)}{2} - \frac{7.85(240)}{2} = 320 \text{ lb/in}$$

b. Torsional load at d from the support:

$$T_u = (\frac{L}{2} - d) \frac{b}{2} (V_T)$$

$$= (\frac{240}{2} - 27.125) \times \frac{18}{2} \times 320 = 267{,}480 \text{ in-lb}$$

Step 3. Maximum torsional stress:

Since h > b, the torsional stress in the vertical direction is not critical and will be ignored.

a. Torsional stress:

$$v_{(tu)V} = \frac{3T_u}{b^2 h} \qquad \text{(eq. 4-143)}$$

$$= \frac{3 \times 267{,}480}{18^2 \times 30} = 82.5 \text{ psi}$$

b. Shear stress: $v_u = 235.2$ psi

c. Check maximum allowable torsional stress.

$$\max v_{tu} = \frac{12(f'_{dc})^{1/2}}{\left[1 + \left[\frac{1.2\ v_u}{v_{tu}}\right]^2\right]^{1/2}}$$

$$= \frac{12 \times (4000)^{1/2}}{\left[1 + \left[\frac{1.2 \times 235.2}{82.5}\right]^2\right]^{1/2}} = 212.9 > 82.5 \text{ O.K.}$$

Step 4. Find shear and torsional capacity of unreinforced web.

a. Shear capacity:

$$v_c = \frac{2\ (f'_{dc})^{1/2}}{\left[1 + \left[\frac{v_{tu}}{1.2 \times v_u}\right]^2\right]^{1/2}} \quad \text{(eq. 4-145)}$$

$$v_c = \frac{2 \times (4000)^{1/2}}{\left[1 + \left[\frac{82.5}{1.2 \times 235.2}\right]^2\right]^{1/2}} = 121.4 \text{ psi}$$

b. Torsional capacity:

$$v_{tc} = \frac{2.4\ (f'_{dc})^{1/2}}{\left[1 + \left[\frac{1.2v_u}{v_{tu}}\right]^2\right]^{1/2}} \quad \text{(eq. 4-146)}$$

howdy

$$v_{tc} = \frac{2.4 \times (4,000)^{1/2}}{\left[1 \left[\frac{1.2 \times 235.2}{82.5}\right]^2\right]^{\frac{1}{2}}}$$

$$= 42.6 \text{ psi (Vertical Face)}$$

Step 5. Area of web reinforcing for shear using equation 4-141:

$$A_v = (v_u - v_c) \times b \times s/(\phi f_{dy})$$

Assume s = 12 in.

$$A_v = (235.2 - 121.4) \times 18 \times 12/(0.85 \times 66,000)$$

$$= 0.438 \text{ in}^2/\text{ft}$$

Step 6. Web reinforcing for torsional stress using equation 4-148:

$$A_t = \frac{(v_{tu} - v_{tc})^2 b\ hs}{3\ \phi\ \alpha_t\ b_t\ h_t\ f_{dy}} \quad \text{Vertical}$$

where:

$$\alpha_t = 0.66 + 0.33 \frac{h_t}{b_t} \leq 1.50$$

$$h_t = 30.0 - 2.0 - 1.5 - (2 \times 0.5/2) = 26 \text{ in}$$

See figure 4A-15.

$$b_t = 18.0 - 1.5 - 1.5 - (2 \times 0.5/2) = 14.5 \text{ in}$$

See figure 4A-15.

$$\alpha_t = 0.66 + 0.33 \times \frac{26}{14.5} = 1.25 < 1.50 \quad \text{O.K.}$$

$$A_t = \frac{(82.5 - 42.6)\ 18^2 \times 30 \times 12}{3 \times 0.85 \times 1.25 \times 14.5 \times 26 \times 66{,}000}$$

$$= 0.059 \text{ in}^2/\text{ft}$$

Step 7. Total web reinforcement:

$$A_t + A_v/2 = 0.059 + 0.438/2 = 0.278 \text{ in}^2/\text{ft/Leg}$$

Use No. 4 ties @ 8 in = 0.300 in^2/ft/Leg.

Step 8. Minimum torsion reinforcement (sect. 4-41.5):

a. Minimum tie reinforcing area:

A_v (min) = A_v shear alone from example 4A-6

Use No. 4 ties @ 9 in.

$$A_v\ (\text{min}) = \frac{0.38}{2} \times \frac{12}{9}$$

$$= 0.253 \text{ in}^2/\text{ft/Leg} < 0.300 \text{ in}^2/\text{ft/Leg} \quad \text{O.K.}$$

b. Maximum spacing:

$$s_{max} = \frac{h_t + b_t}{4}$$

$$s_{max} = \frac{26 + 14.5}{4}$$

$$= 10.125 \text{ in} > 7 \text{ in} \quad \text{O.K.}$$

Step 9. Required area of longitudinal steel is the greater of the two values from equations 4-151a or 4-151b.

$$A_l = 2A_t \times \frac{b_t + h_t}{s}$$

$$A_l = 2 \times 0.06 \times \frac{14.5 + 26.0}{12} = 0.40 \text{ in}^2$$

or:

$$A_l = \left[\frac{400 \times b \times s}{f_{dy}} \; \frac{v_{tu}}{v_{tu} + v_u} - 2A_t \right] \times \frac{b_t + h_t}{s}$$

where:

$$2A_t = \frac{50bs}{f_{dy}}$$

$$\frac{50bs}{f_{dy}} = \frac{50 \times 18 \times 12}{66{,}000} = 0.16 \text{ in}^2/\text{ft } 2A_t$$

$$A_l = \left[\frac{400 \times 18 \times 12}{66{,}000} \; \frac{(82.5)}{82.5 + 235.2} - 0.12 \right]$$

$$\times \frac{14.5 + 26.0}{12} = 0.74 \text{ in}^2$$

Step 10. Distribute A_l, A_s, and A_s^- as follows (see fig. 4A-16):

Distribute A_l equally between four corners of the beam and one on each face of depth, a total of six locations to satisfy maximum spacing of 12 inches.

$$A_l/6 = 0.74/6 = 0.12 \text{ in}^2$$

Vertical Face:

One (1) No. 4 bar

$= 0.20 \text{ in}^2 > 0.12$ O.K.

Horizontal Face at Top:

Support = 2.20 (bending) + 2 x 0.12 (torsion)

= 2.44 in^2

Two (2) No. 7 at corners + three (3) No. 6

= 2.52 in^2 O.K.

Midspan = 1.64 (rebound)

Two (2) No. 7 at corners + one (1) No. 6

= 1.64 in^2 O.K.

Horizontal Face at Bottom:

Support = Greater of rebound (1.64 in^2)
or torsion (2 x 0.12)

Two (2) No. 7 at corners + one (1) No. 6

= 1.64 in^2 O.K.

Midspan = 2.20 (bending)

Two (2) No. 7 at corners + one (1) No. 6 + two (2) No. 5

= 2.26 in^2 > 2.20 O.K.

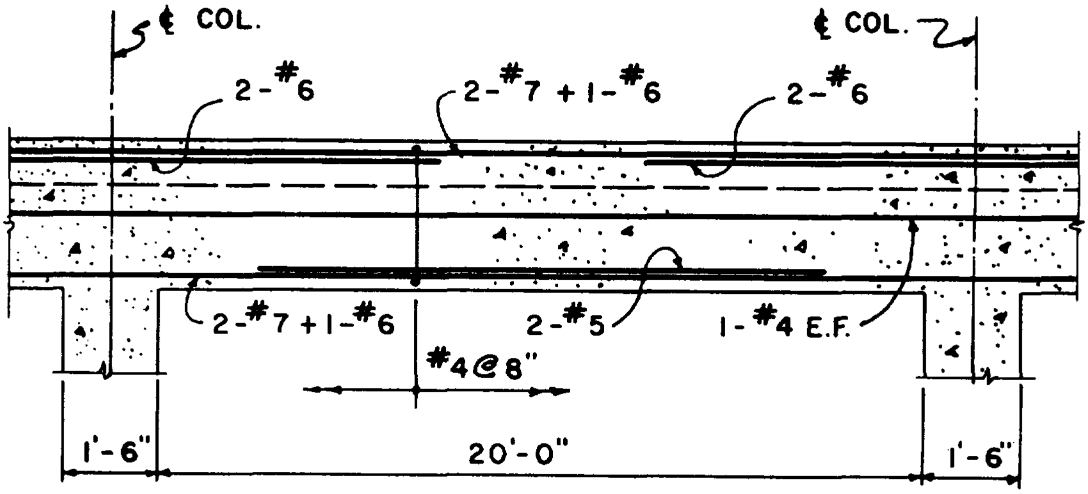


Figure 4A-16

Problem 4A-8, Column Design

Problem: Design an interior column of a one-story structure with shear walls.

Procedure:

Step 1. Establish design parameters:

a. End conditions of column.

b. Clear height of column.

c. Dynamic loads from roof.

d. Static material properties.

Step 2. Find equivalent static loads on the column by increasing the dynamic loads 20 percent (sect. 4-47).

Step 3. From table 4-1, determine the dynamic increase factors, DIF. Using the DIF, the material properties from Step 1d and equation 4-3, calculate the dynamic design strength of the concrete and the reinforcement.

Step 4. Assume a column section and reinforcing steel.

Step 5. Calculate the slenderness ratio of the column section assumed in Step 4, using either equation 4-167 or 4-168. If the slenderness ratio is less than 22, slenderness effects may be neglected. If it is greater than 22 and less than 50, the moment magnifier must be calculated from equation 4-170 and the moments increased according to equation 4-169. The column section must be increased if the slenderness ratio is greater than 50.

Step 6. Divide the moment by the axial load to obtain the design eccentricity in both directions. Verify that the design eccentricities are greater than the minimum eccentricity of 0.1h for a tied column and 0.0707D for a spiral column.

Step 7. Compute the balanced eccentricity e_b of the column using equation 4-156 or 4-158 (for a rectangular and circular column, respectively), the dynamic material properties from Step 3, and the section properties from Step 4. Compare the balanced eccentricity with the design eccentricity from Step 6. Determine if the column failure is controlled by compressive strength of the concrete ($e_b > e$) or tensile strength of reinforcement ($e_b < e$).

Step 8. Calculate the ultimate axial load capacity, at the actual eccentricity, in both directions. If compression controls use equation 4-160 or 4-161. If tension controls use equation 4-162 or 4-166.

Step 9. Using equation 4-154, compute the pure axial load capacity of the section.

Step 10. Compute the ultimate capacity of the column section, using the load capacities at the actual eccentricities from Step 8, the pure axial load capacity from Step 9, and equation 4-176. Verify that the ultimate load capacity is greater than the equivalent static load from Step 2.

Step 11. Provide ties according to section 4-48.4 for a tied column, or section 4-49.4 for a spiral reinforced column.

Example 4A-8, Column Design

Required: Design of a rectangular, tied interior column.

Solution:

Step 1. Given:

a. Both ends of column fixed

b. Clear height of column, $l = 120$ in

c. Axial load 491,000 lbs

Moment about x-axis, 2,946,000 in-lbs
No calculated moment about y-axis

d. Reinforcing steel, $f_y = 66{,}000$ psi

Concrete, $f'_c = 4{,}000$ psi

Step 2. Equivalent static loads.

a. Axial load

$P = 491{,}000 \times 1.2 = 589{,}200$ lbs

b. Moment about x-axis

$M_x = 2{,}946{,}000 \times 1.2 = 3{,}535{,}200$ in-lbs

c. Moment about y-axis

$M_y = 0$ in-lbs

Step 3. Dynamic material strengths:

a. Reinforcing Steel.

$f_{dy} = f_y \times DIF$

f_{dy} = 66,000 x 1.10 = 72,600 psi

b. Concrete.

$f'_{dc} = f'_c \times DIF$

f'_{dc} = 4,000 x 1.12 = 4,480 psi

Step 4. Use an 18" x 18" column section with 12 No. 7 reinforcing bars (see fig. 4A-17).

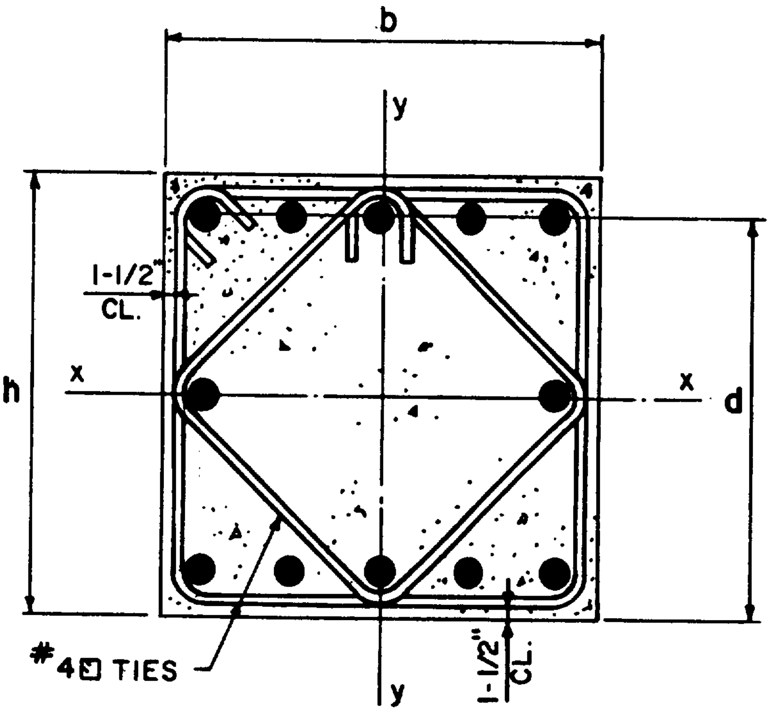


Figure 4A-17

Step 5. Radius of gyration for rectangular section is equal to 0.3 of depth.

$r_x = r_y = 0.3 \times 18 = 5.4$ in

From section 4-46:

k = 0.9

$$\frac{kl}{r} = \frac{0.9 \times 120}{5.4} = 20 < 22 \qquad \text{(eq. 4-168)}$$

Therefore slenderness effects may be neglected.

Step 6. Minimum eccentricity in both directions,

e_{min} = 0.1 x 18 = 1.8 in

e_x = M_x/P = 3,535,200/589,200 = 6 in > 1.8 in O.K.

e_y = M_y/P = 0/589,200 = 0 < 1.8, use 1.8 in

Step 7. Balanced eccentricity:

a. From figure 4A-17,

$d_x = d_y$ = 18 - 1.5 - 0.5 - 0.875/2 = 15.56 in

A_{sx} = 4 x 0.6 = 2.40 in^2

A_{sy} = 2 x 0.6 = 1.20 in^2

b. Find value of m,

$m = f_{dy}/(0.85\ f'_{dc})$ = 72,600/(0.85 x 4,480) = 19.06

c. Using equation 4-156,

e_b = 0.20 h + 1.54_s A m/b

e_{bx} = 0.20 x 18 + (1.54 x 2.40 x 19.06)/18

= 7.51 in > 6 in, compression controls

e_{by} = 0.20 x 18 + (1.54 x 1.2 x 19.06)/18

= 5.56 in > 1.8 in, compression controls

Step 8. Axial load from equation 4-160:

$$P_u = \frac{A_s\ f_{dy}}{e/(2d - h) + 0.5} + \frac{bhf'_{dc}}{3he/d^2 + 1.18}$$

a. When only eccentricity e_x is present:

$$P_x = \frac{2.4 \times 72{,}600}{6/(2 \times 15.56 - 18) + 0.5} + \frac{18 \times 18\ 4{,}480}{3 \times 18\ 6/(15.56)^2 + 1.18}$$

= 758,417 lbs

b. When only eccentricity e_y is present:

$$P_y = \frac{1.2 \times 72{,}600}{1.8/(2 \times 15.56 - 18) + 0.5}$$

$$+ \frac{18 \times 18 \times 4{,}480}{3 \times 18 \times 1.8/(15.56)^2 + 1.18} = 1{,}054{,}557 \text{ lbs}$$

Step 9. Compute pure axial load capacity from equation 4-154.

$$P_o = 0.85\, f'_{dc}\, (A_g - A_{st}) + A_{st}\, f_{dy}$$

$$A_g = 18 \times 18 = 324 \text{ in}^2$$

$$A_{st} = 12 \times 0.6 = 7.2 \text{ in}^2$$

$$P_o = 0.85 \times 4480\, (324 - 7.2) + 7.2 \times 72{,}600$$

$$= 1{,}729{,}094 \text{ lbs}$$

Step 10. Ultimate capacity of the column from equation 4-176:

$$\frac{1}{P_u} = \frac{1}{P_x} + \frac{1}{P_y} - \frac{1}{P_o}$$

$$\frac{1}{P_u} = \frac{1}{758{,}417} + \frac{1}{1{,}054{,}557} - \frac{1}{1{,}729{,}094} = \frac{1}{592{,}254} \text{ 1/lbs}$$

$$P_u = 592{,}254 \text{ lbs} > 589{,}200 \quad \text{O.K.}$$

Step 11. Provide ties, according to section 4-48.4.

For #7 longitudinal bars, use #3 ties.

$$s \leq 16\phi \text{ (longitudinal bars)} = 16 \times 0.875 = 14 \text{ in}$$

and:

$$s \leq 48\phi \text{ (ties)} = 48 \times 0.375 = 18 \text{ in}$$

and:

$$s \leq h/2 = 9 \text{ in}$$

Use two (2) #3 ties at 9 inches arranged as shown in figure 4A-17.

Problem 4A-9, Brittle Mode, Post-Failure Fragments

Problem: Design an element, which responds to blast impulse, for controlled post-failure fragments.

Procedure:

Step 1. Establish design parameters:

a. Impulse load and duration (Chapter 2).

b. Maximum average velocity of post-failure fragments v_f as required by receiver sensitivity.

c. Geometry of element.

d. Support conditions.

e. Materials to be used and corresponding static design strengths.

f. Dynamic increase factors (table 4-1).

Step 2. Determine dynamic yield strength and ultimate strength of reinforcement from equation 4-3.

Step 3. Determine the dynamic design stress for the flexural reinforcement according to the deflection range (support rotation) from table 4-2.

Step 4. Substitute known quantities of i_b (step 1a), f_{ds} (step 3), H (step 1c), and v_f (step 1b) into equation 4-194.

Step 5. Obtain optimum ratio of vertical to horizontal reinforcement p_V/p_H for a given wall thickness T_c:

a. Assume a value of d_c and substitute it into the equation obtained in step 4.

b. Read optimum: p_V/p_H ratio from figure 4-38.

Note:
For one-way elements, the ratio of the main to secondary reinforcement is always 4 to 1 unless minimum conditions govern (table 43). Obtain C_u from table 4-11 for the given support condition. C_f is always equal to 22,500 (Sect. 4-58)

Step 6. For the optimum p_V/p_H ratio, determine C_u (from figure 4-33, 4-34 or 4-35) and C_f (from figure 4-73, 4-74 or 4-75). Calculate p_v and p_H. Select bar sizes and spacings necessary to furnish the required reinforcement ratios.

Step 7. Determine the required lacing and diagonal bars. (Procedure is exactly the same as that for elements designed for the ductile mode for incipient failure or less. See problem 4A-3.)

Step 8. Determine the required T_c for the assumed d_c selected flexural and lacing bar sizes and required concrete cover. Adjust T_c to the nearest whole inch and calculate the actual d_c.

Step 9. Check flexural capacity of the element based on either blast impulse of post-failure fragment velocity. Generally, lacing bar sizes do not have to be checked since they are not usually affected by a small change in d_c.

a. Compute the actual impulse capacity of the element using equation 4-194 and compare with the anticipated blast load. Repeat design (from step 5 on) if the capacity is less than that required.

b. Compute the actual post-failure fragment velocity using equation 4-194 and compare with the value permitted by the acceptor sensitivity. Repeat design (from step 5 on) if the actual velocity is greater than that permitted.

Step 10. Determine whether the correct design procedure has been utilized by first computing the response time of the element t_u (time to reach ultimate deflection) from equation 4-192 or 4-193. Then compare the response time t_u with the duration of the blast load t_o. For elements that respond to impulse loading $t_u/t_o > 3$.

Note:
To obtain the most economical design, repeat steps 5 through 10 for several wall thicknesses and compare their costs. Percentages of reinforcement can be used to reduce the amount of calculations. In determining the required quantities of reinforcement, the length of the lap splice should be considered.

Example 4A-9, Brittle Mode, Post-Failure Fragments

Required: Design the back wall of an interior cell (fig. 4A-18) of a multicubicle structure for controlled post-failure fragments.

Solution:

Step 1. Given:

a. i_b = 4800 psi-ms, and t_o = 1.0 ms.

b. v_f = 100 fps = 1.2 in/ms.

c. L = 360 in., H = 120 in.

d. Fixed on three edges and one edge free.

e. Reinforcing bars; f_y = 66,000 psi and f_u = 90,000 psi

Concrete, f'_c = 4,000 psi

f. For reinforcement, DIF = 1.23 for yield stress

DIF = 1.05 for ultimate stress

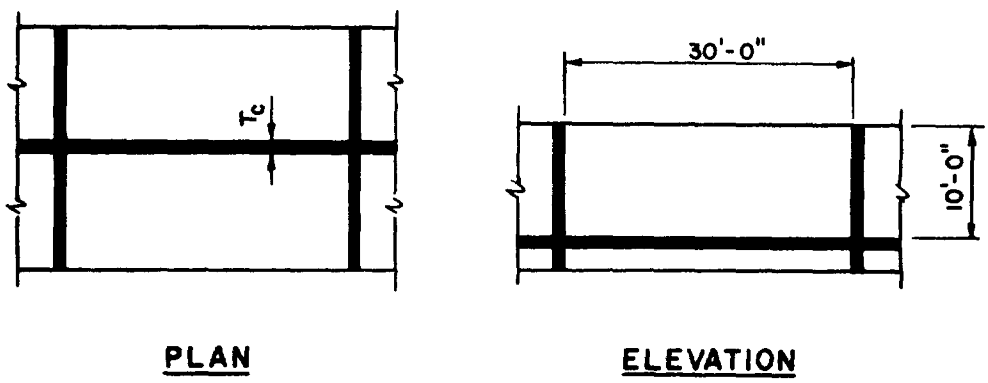


Figure 4A-18

Step 2. Dynamic strength of materials.

f_{dy} = DIF f_y = 1.23 X 66,000 = 81.180 psi

f_{du} = DIF f_u = 1.05 X 90,000 = 94,500 psi

Step 3. Dynamic design stress from table 4-2.

$$f_{ds} = \frac{f_{dy} + f_{du}}{2} = \frac{81,180 + 94,500}{2} = 87,840 \text{ psi}$$

Step 4. Substitute known quantities into equation 4-194.

$$i_b^2 = C_u \left[\frac{p_H \, d_c^3 \, f_{ds}}{H} \right] + C_f \, d_c^2 v_f^2$$

$$4800^2 = \frac{C_u \, p_H \, d_c^3 \, (87{,}840)}{120} + C_f \, d_c^2 (1.2)^2$$

$$23.04 \text{ X } 10^6 = 732 \; C_u \, p_H \, d_c^3 + 1.44 \, C_f \, d_c^2$$

Step 5. Optimum reinforcement ratio.

a. Assume $d_c = 21$ in. and substitute into equation 4-194.

$$23.04 \text{ X } 10^6 = 732 \, C_u \, p_H \, (21)^3 + 1.44 \, C_f \, (21)^2$$

Therefore:

$$p_H = \frac{23.04 \text{ X } 10^6 - 635 \, C_f}{6.78 \text{ X } 10^6 \, C_u}$$

b. Read optimum p_V/p_H value from figure 4-38 for $L/H = 3$.

$$p_V/p_H = 1.58$$

Step 6. For optimum $p_V/p_H = 1.58$

From figure 4-34, $C_u = 477$

From figure 4-74, $C_f = 1.583 \text{ X } 10^4$

$$p_H = \frac{[23.04 \text{ X } 10^6 - 635 \, (1.583 \text{ X } 10^4)]}{(6.78 \text{ X } 10^6) \, (477)} = 0.00402$$

$$p_V = 1.58 \, p_H = 1.58 \, (0.00402) = 0.00635$$

$$A_{sH} = 0.00402 \, (12) \, (21) = 1.013 \text{ in}^2/\text{ft.},$$

use #8 @ 9 in. (A_s = 1.05 in^2)

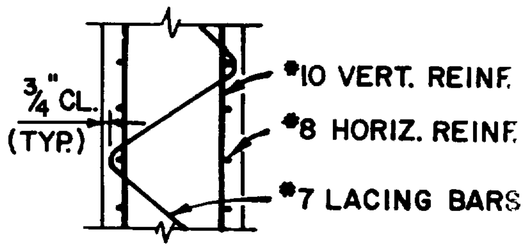
A_{sV} = 0.00635 (12) (21) = 1.60 in^2/ft.,

use #10 @ 9 in. (A_s = 1.69 in^2)

$$\text{Actual } \frac{p_V}{p_H} = \frac{A_{sV}}{A_{sH}} = \frac{1.69}{1.05} = 1.610$$

Step 7. Using lacing method No. 3, No. 7 vertical lacing bars are required. (Calculations are not shown since they are similar to those presented in example 4A-3 for incipient failure design. Also, the remainder of the design for shear will not be shown.)

Step 8. The actual d_c depends upon the details of the base of the wall (region of vertical lacing).



cover	= 2 X 0.75	= 1.50
lacing	= 2 X 0.875	= 1.75
horizontal bars	= 2 X 1.00	= 2.00
vertical bars	= 1.27	= 1.27
		6.52 in.

Figure 4A-19

$T_c = d_c + 6.52 = 27.52$ in., use 28 in.

actual $d_c = 28 - 6.52 = 21.48$ in.

Step 9. Actual capacity of element.

a. Actual impulse capacity.

For $p_V/p_H = 1.610$, $C_u = 481$

and $C_f = 1.588 \times 10^4$

$d_c = 21.48$ in. $p_H = \dfrac{1.05}{12\ (21.48)} = 0.00407$

$$i_c^2 = C_u \left[\frac{p_H\ d_c^3\ f_{ds}}{H}\right] + C_f\ d_c^2\ v_f^2$$

$$i_c^2 = 481 \left[\frac{(0.00407)\ (21.48)^3\ (87{,}840)}{120}\right]$$

$$+ (1.588 \times 10^4)\ (21.48)^2\ (1.2)^2$$

$$i_c^2 = 24.75 \times 10^6$$

$i_c = 4{,}975$ psi-ms $> i_b = 4{,}800$ psi-ms O.K.

b. Actual post-failure fragment velocity.

$$i_b^2 = C_u \left[\frac{p_H\ d_c^3\ f_{ds}}{H}\right] + C_f\ d_c^2 v_f^2$$

$$4{,}800^2 = 481\ \frac{0.00407\ (21.48)^3\ (87{,}840)}{120}$$

$$+ (1.588 \times 10^4)\ (21.48)^2\ v_f^2$$

$v_f = 1.10$ in/ms $= 92$ fps < 100 fps O.K.

Step 10. Response time of element t_u:

$M_{VN2} = M_{VP}$ and $M_{HN1} = M_{HN1} = M_{HN3} = M_{HP}$

$$M_{HP} = A_{sH}\, f_{ds}(d_c\ /b) = \frac{(1.05)\ (87,840)\ (21.48)}{12}$$

$$= 165,095 \text{ in-lbs/in}$$

$$M_{VP} = A_{sV}\, f_{ds}\, d_c\ /b = \frac{(1.69)\ (87,840)\ (21.48)}{12}$$

$$= 265,725 \text{ in-lbs/in}$$

$x_2/x_1 = 1.0$ (symmetrical yield lines)

$$\frac{L}{H}\left[\frac{M_{VP}}{M_{HN1} + M_{HP}}\right]^{1/2} = \frac{360}{120}\left[\frac{26,725}{2 \times 165,095}\right]^{1/2} = 2.69$$

$$\frac{M_{VP}}{M_{VN2}} = \frac{265,725}{265,725} = 1.0$$

From figure 3-11,

$x_1/L = 0.358$

$x_1 = x_2 = 0.358\ (360) = 128.88$

$$r_u = \frac{5(M_{HN1} + M_{HP})}{{x_1}^2} \quad \text{for } x_1 < L/2 \qquad \text{(table 3-2)}$$

$$r_u = \frac{5\ (2 \times 165,095)}{120^2} = 99.4 \text{ psi}$$

$m = 225d_c = 225\ (21.48) = 4833\ psi\text{-}ms^2/in$

$(K_{LM})_u = 0.557$ (fig. 3-44)

$m_u = 0.557\ (4833) = 2692\ psi\text{-}ms^2/in$

$x_1 = 128.88 > H = 120$

∴ Vertical supports fail first and the post-ultimate range resistance is:

$$r_{up} = \frac{8\ (M_{HN1} + M_{HP})}{L^2} = \frac{8\ (2 \times 165{,}095)}{360^2} = 20.4\ psi$$

$m_{up} = 0.66 \times 4833 = 3{,}190\ psi\text{-}ms^2/in$

$X_1 = H \tan\Theta_{max} = 12 \times \tan 12° = 25.51\ in.$ (table 3-6)

Use equation 4-192 for t_u.

$$t_u = \frac{i_b}{m_u} + \left[\frac{m_{up}}{m_{up}\ r_{up}} - \frac{1}{r_u}\right]\left[i_b^2 - 2m_u r_u X_1\right]^{1/2} - \left[\frac{m_u}{r_{up}}\right] V_f$$

$$t_u = \frac{4{,}800}{2{,}692} + \left[\frac{3190}{2692 \times 20.4} - \frac{1}{99.4}\right]\left[4800^2 - 2 \times 2692 \times 99.4 \times 25.51\right]^{1/2}$$

$$- \left(\frac{2692}{20.4}\right) 1.1 = 3.78\ ms$$

$\frac{t_u}{t_o} = 3.78 / 1.0 = 3.78 > 3$ ∴ correct procedure has been used

Problem 4A-10, Maximum Fragment Penetration

Problem: Determine the maximum penetration of a primary metal fragment into a concrete wall and determine if perforation occurs.

Procedure:

Step 1. Establish design parameters:

a. Type of fragment

b. Weight W_f and diameter d of fragment

c. Fragment striking velocity v_s

d. Thickness T_c and the ultimate compressive strength f'_c of concrete wall

Step 2. Determine the maximum penetration X_f from figure 4-78 (or equation 4-200 or 4-201) for the values of W_f and v_s in step 1. X_f is the maximum penetration of an armor-piercing steel fragment into 4,000 psi concrete.

Step 3. To determine the depth of penetration into concrete with ultimate strength other than 4,000 psi, use X_f from step 2, the concrete strength from step 1d and equation 4-202:

$$X'_f = X_f\ (4000/f'_c)^{1/2}$$

Step 4. To obtain the maximum penetration into concrete by metal fragments other than those of armor-piercing fragments use the penetration from <u>step 3</u>, the penetrability coefficient k from table 4-16 and equation 4-203.

$$X'_f = k\ X_f$$

Step 5. Calculate the limiting thickness of concrete at which perforation will occur from equation 4-204.

$$T_{pf} = 1.13\ X\ \ d^{0.1} + 1.311\ d$$

Where applicable, replace X_f with X'_f from step 3 or 4. If T_{pf} is less than T_c embedment will occur; if T_{pf} is greater than T_c the fragment will perforate the wall.

Example 4A-10, Maximum Fragment Penetration

Required: Maximum penetration of a primary fragment into a concrete wall; determine if perforation occurs.

Solution:

Step 1. Given:

a. Type of fragment: mild steel

b. Primary fragment weight: W_f = 30 ounces

Primary fragment diameter: d = 2.15 inches

c. Striking velocity: v_s = 3,500 fps

d. Thickness of wall: T_c = 18 inches

Ultimate concrete compressive strength: f'_c = 4,000 psi

Step 2. Maximum penetration:

For:

W_f = 30 oz and v_s = 3,500 fps

X_f = 14.5 inches from figure 4-78

or from equation 4-199

$$X_f = 2.04 \times 10^{-6}\, d^{1.2}\, v_s^{1.8} + d \geq 2\, d$$

$$= 2.04 \times 10^{-6}\, (2.15)^{1.2}\, (3500)^{1.8} + 2.15$$

$$= 14.4 \text{ inches}$$

$$X_f \geq 2\, d = 2 \times 2.15 = 4.30 \quad \text{O.K.}$$

Step 3. Concrete strength adjustment (eq. 4-202)

$$X'_f = X_f\,(4000/f'_c)^{1/2}$$

$$= 14.5 \times 1 = 14.5 \text{ in.}$$

Step 4. Fragment material adjustment:

a. $k = 0.70$ (table 4-16)

b. $X'_f = kX_f$ (eq. 4-203)

$$= 0.7 \times 14.5 = 10.15 \text{ in}$$

Step 5. Calculate minimum thickness to prevent perforation.

$$T_{pf} = 1.13\, X'_f\, d^{0.1} + 1.311\, d \qquad \text{(eq. 4-204)}$$

$$= 1.13\,(10.15)\,(2.15)^{0.1} + 1.311\,(2.15)$$

$$T_{pf} = 15.20 \text{ inches} \leq 18 \text{ in}$$

Since T_{pf} is less than T_c the fragment does not perforate the slab.

Problem 4A-11, Determination of the Occurrence and Effects of Perforation

Problem: Determine the residual velocity of a primary fragment if it perforates a concrete wall.

Procedure:

Step 1. Determine the type, weight W_f, and striking velocity v_s of the primary fragment. Also, the thickness of the concrete wall T_c and the ultimate compressive stress f'_c of the concrete must be known.

Step 2. Proceed through steps 2, 3, 4, and 5 of Problem 4A-10.If T_{pf} is greater than T_c, perforation will occur.

Step 3. If perforation is not indicated by the calculations in step 2 then discontinue the analysis. If perforation does result, then compute the value of T_c/T_{pf}.

Step 4. Utilizing the value of T_c/T_{pf}, obtain v_r/v_s from figure 4-79 or 4-80.

Step 5. With the values of v_s and v_r/v_s of Steps 1 and 4, calculate the residual velocity v_r.

Example 4A-11, Determination of the Occurrence and Effects of Perforation

Required: Residual velocity of a primary fragment if it perforates a concrete wall.

Solution:

Step 1. Given:

a. Type of metal: mild steel

b. Primary fragment weight: $W_f = 20$ ounces

c. Primary fragment diameter: $d = 1.89$ inches

d. Striking velocity: $v_s = 4{,}700$ fps

e. Thickness of wall: $T_c = 12$ inches

f. Ultimate concrete compressive stress: $f'_c = 4{,}500$ psi

Step 2. For given conditions:

$$X_f = 19.75 \text{ inches} \qquad \text{(fig. 4-78)}$$

Actual maximum penetration:

$$X'_f = 19.75\ (4000/4500)^{1/2} \qquad \text{(eq. 4-202)}$$

$$= 18.6 \text{ inches}$$

$$k = 0.70 \qquad \text{(table 4-16)}$$

$$X'_f = 0.7\ (18.6) = 13.0 \text{ inches} \qquad \text{(eq. 4-203)}$$

Since T_{pf} is greater than X'_f, and X'_f (13.0 inches) already exceeds the wall thickness (12 inches), perforation will occur. It is necessary to determine T_{pf} (eq. 4-204) in order to calculate the residual fragment velocity. Hence:

$$T_{pf} = 1.13\ (13.0)\ (1.89)^{0.1} + 1.311\ (1.89)$$

$$= 15.6 + 2.5 = 18.1 \text{ inches}$$

Step 3. $T_c/T_{pf} = 12.0/18.1 = 0.663$

Step 4. Since the given conditions correspond to a case

where:

$$X_f > 2d,$$

determine:

$$v_r/v_s$$

for: $T_c/T_{pf} = 0.663$ from figure 4-80.

Obtain:

$$v_r/v_s = 0.55$$

Step 5. $v_r = 0.55\ v_s$

$$= 0.55\ (4{,}700) = 2{,}585 \text{ fps}$$

Problem 4A-12, Determination of the Occurrence of Spalling

Problem: To determine if spalling of a concrete wall occurs if there is no perforation by the primary fragment.

Procedure:

Step 1. Determine the type, weight W_f and striking velocity v_s of the primary fragment. Also, the thickness of the concrete wall and the ultimate compressive stress of the concrete f'_c must be known.

Step 2. Proceed through Steps 2, 3, 4, and 5 of Problem 4A-10.

Step 3. If embedment of the fragment occurs, compute the limiting concrete thickness at which spalling will occur according to:

$$T_{sp} = 1.215\ X_f\ d^{0.1} + 2.12\ d \qquad \text{(eq. 4-207)}$$

If T_{sp} is greater than T_c, spalling will occur.

Example 4A-12, Determination of the Occurrence of Spalling

Required: Determine if spalling of a concrete wall occurs due to penetration by a primary fragment.

Solution:

Step 1. Given:

a. Type of metal: armor-piercing steel

b. Primary fragment weight: $W_f = 40$ ounces

c. Primary fragment diameter: 2.38 inches

d. Striking velocity: $v_s = 3{,}000$ fps

e. Thickness of wall: $T_c = 19$ inches

f. Ultimate concrete compressive stress: $f'_c = 5{,}000$ psi

Step 2. For given conditions:

$X_f = 12.8$ inches (fig. 4-78)

$X'_f = 12.8\ (4000/5000)^2$ (eq. 4-202)

$= 11.5$ in.

$k = 1.00$ (table 4-16)

$X'_f = X_f$

Then:

$T_{pf} = 1.13\ (11.5)\ (2.38)^{0.1}$ (eq. 4-204)

$+ 1.311\ (2.38)$

$= 14.2 + 3.1 = 17.3$ inches

Since T_c is greater than 17.3 inches, embedment occurs.

Step 3. Determine minimum concrete thickness to prevent spalling from equation 4-207:

$$T_{sp} = 1.215\ (11.5)\ (2.38)^{0.1} + 2.12\ (2.38)$$

$$= 20.3 \text{ inches}$$

Since T_{sp} is greater than 19 inches, spalling will occur.

Problem 4A-13. Determination of the Effects of a Primary Fragment on a Composite Wall

Problem: Determine the maximum penetration by a primary fragment into a composite wall and the resulting effects on the donor panel, sand layer and acceptor panel.

Procedure:

Step 1. Determine the type, weight W_f, and striking velocity v_s of the primary fragment. Also, the thicknesses of the concrete donor panel T_c (donor), the sand layer T_s, and the concrete receiver panel T_c (acceptor) and the ultimate compressive stress f'_c of the concrete must be known.

Step 2. Proceed through Steps 2, 3, 4, and 5 of Problem 4A-10.

If embedment of the fragment in the donor panel occurs, perform Step 3 of Problem 4A-12 to determine if spalling takes place.

If perforation of the donor panel occurs, compute:

$$T_c \text{ (donor) } / T_{pf}$$

and perform Steps 4 and 5 of Problem 4A-11 to find the residual velocity v_r.

Step 3. Utilizing v_r (donor) as the v_s of the sand layer and W_f, obtain X_s from figure 4-81.

If X_s is less than T_s, the fragment is embedded in the sand layer and the analysis is discontinued.

If X_s is greater than T_s, perforation of the sand occurs.

Step 4. Compute T_s/X_s if perforation results. Use this value and figure

4-80 to obtain v_r/v_s of the sand layer.

Calculate v_r(sand).

Step 5. Utilizing v_r(sand) as the v_s of the acceptor wall and W_f, obtain X_f (acceptor) from figure 4-78.

Proceed through Steps 3 and 4 of Problem 4A-10, if necessary.

If embedment of the fragment in the acceptor panel occurs, perform Step 3 of Problem 4A-12 to determine if spalling takes place.

If perforation of the acceptor wall occurs, compute T_c (acceptor) and perform Steps 4 and 5 of Problem 4A-11 to find v_r.

Example 4A-13. Determination of the Effects of a Primary Fragment on a Composite Wall

Required: Maximum penetration by a primary fragment into a composite wall and the resulting effects on the donor panel, sand layer and the acceptor panel.

Solution:

Step 1. Given:

a. Type of metal: armor-piercing steel

b. Primary fragment weight: W_f = 20 ounces

c. Primary fragment diameter: d = 1.89 inches

d. Striking velocity: v_s = 4,200 fps

e. Thicknesses:

T_c (donor) = 12 inches

T_s = 24 inches

T_c (acceptor) = 12 inches

f. Ultimate concrete compressive stress:

f'_c = 5,000 psi

Step 2. For given conditions:

$X_f = 16.5$ inches (fig. 4-78)

$X'_f = 16.5\ (4000/5000)^2$ (eq. 4-202)

$= 14.7$ inches

$k = 1.00$ (table 4-16)

$X'_f = X_f$

Since X'_f is greater than 12 inches, perforation of the donor panel will certainly occur. T_{pf} must be calculated in order to determine the residual velocity of the fragment.

$$T_{pf} = 1.13\ (14.7)\ (1.89)^{0.1} + 1.311\ (1.89)$$

$$= 17.7 + 2.5 = 20.2 \text{ inches}$$

$$T_c/T_{pf} = 12/20.2 = 0.594$$

Since X_{pf} is greater than 2d, $v_r/v_s = 0.61$ from figure 4-80.

v_r (donor) $= 0.61\ (4,200) = 2,562$ fps

Step 3. For:

v_s (sand) $= 2,562$ fps, (fig. 4-81)

$X_s = 64.0$ inches

Since T_s is less than 64.0 inches, perforation of the sand layer will occur.

Step 4. $T_s/X_s = 24/64.0 = 0.375$

from figure 4-80

$v_r/v_s = 0.77$

Step 5. v_r (sand) $= 0.77\ v_s$

$v_r = 0.77\ (2,562) = 1,970$ fps

Step 6. For:

v_s (acceptor) = 1,970 fps

X_f (acceptor) = 5.6 inches (fig. 4-78)

$X'_f = 5.6\ (4000/5000)^{1/2}$ (eq. 4-202)

$= 5.0$ inches

$k = 1.00$ (table 4-16)

$X'_f = X_f$

$T_{pf} = 1.13\ (5.0)\ (1.89)^{0.1}$ (eq. 4-204)

$+ 1.311\ (1.89)$

$= 6.0 + 2.5 = 8.5$ inches

Since T_{pf} is less than T_c (acceptor), the fragment is embedded in the receiver panel. Calculate T_{sp} to determine if spalling occurs:

$T_{sp} = 1.215\ (5.0)\ (1.89)^{0.1}$ (eq. 4-207)

$+ 2.12\ (1.89)$

$= 6.5 + 4.0 = 10.5$ inches

Since T_{sp} is less than T_c (acceptor), spalling will not occur.

APPENDIX 4B

LIST OF SYMBOLS

a (1) acceleration (in./ms^2)
(2) depth of equivalent rectangular stress block (in.)
(3) long span of a panel (in.)

a_o velocity of sold in air (ft./sec.)

a_x acceleration in x direction (in./ms^2)

a_y acceleration in y direction (in./ms^2)

A (1) area (in.^{2})
(2) explosive composition factor (oz.$^{1/2}$in.$^{-3/2}$)

A_a area of diagonal bars at the support within a width b (in.^{2})

A_b area of reinforcing bar (in.^{2})

A_d (1) door area (in.^{2})
(2) area of diagonal bars at the support within a width b (in.^{2})

A_D drag area (in.^{2})

A_f net area of wall excluding openings (ft.^{2})

A_g area of gross section (in.^{2})

A_H maximum horizontal acceleration of the ground surface (g's)

A_l area of longitudinal torsion reinforcement (in.^{2})

A_L lift area (in.^{2})

A_n (1) net area of section (in.^{2})
(2) area of individual wall subdivision (ft.^{2})

A_o area of openings (ft.^{2})

A_{ps} area of prestressed reinforcement (in.^{2})

A_s area of tension reinforcement within a width b (in.^{2})

A'_s area of compression reinforcement within a width b (in.^{2})

A_s^- area of rebound reinforcement (in.^{2})

A_{sH} area of flexural reinforcement within a width b in the horizontal direction on each face (in.^{2})*

A_{sp} area of spiral reinforcement (in.^{2})

* See note at end of symbols

A_{st} total area of reinforcing steel ($in.^2$)

A_{sV} area of flexural reinforcement within a width b in the vertical direction on each face ($in.^2$)*

A_t area of one leg of a closed tie resisting torsion within a distance s ($in.^2$)

A_v total area of stirrups or lacing reinforcement in tension within a distance, s_s or s_l and a width b_s or b_l ($in.^2$)

A_V maximum vertical acceleration of the ground surface (g's)

A_w area of wall ($ft.^2$)

A_I, A_{II} area of sector I and II, respectively ($in.^2$)

b
- (1) width of compression face of flexural member (in.)
- (2) width of concrete strip in which the direct shear stresses at the supports are resisted by diagonal bars (in.)
- (3) short span of a panel (in.)

b_f width of fragment (in.)

b_s width of concrete strip in which the diagonal tension stresses are resisted by stirrups of area A_v (in.)

b_l width of concrete strip in which the diagonal tension stresses are resisted by lacing of area A_v (in.)

b_o failure perimeter for punching shear (in.)

b_t center-to-center dimension of a closed rectangular tie along b (in.)

B explosive constant defined in table 2-7 ($oz.^{1/2}in.^{7/6}$)

c
- (1) distance from the resultant applied load to the axis of rotation (in.)
- (2) damping coefficient
- (3) width of column capital (in.)

c_I, c_{II} distance from the resultant applied load to the axis of rotation for sectors I and II, respectively (in.)

c_s dilatational velocity of concrete (ft./sec.)

C
- (1) shear coefficient
- (2) deflection coefficient for flat slabs

C_C deflection coefficient for the center of interior panel of flat slab

* See note at end of symbols

C_{cr} critical damping

C_d shear coefficient for ultimate shear stress of one-way elements

C_D drag coefficient

C_Dq drag pressure (psi)

C_Dq_o peak drag pressure (psi)

C_E equivalent load factor

C_f post-failure fragment coefficient ($lb.^2\ ms^4/in.^8$)

C_H shear coefficient for ultimate shear stress in horizontal direction for two-way elements*

C_L (1) leakage pressure coefficient from figure 2-235
(2) deflection coefficient for midpoint of long side of interior flat slab panel
(3) lift coefficient

C_M maximum shear coefficient

C_m equivalent moment correction factor

C_p compression wave seismic velocity in the soil from Table 2-10 (in./sec.)

C_r sound velocity in reflected region from figure 2-192 (ft./ms)

C_R force coefficient for shear at the corners of a window frame

$C_{r\alpha}$ Peak reflected pressure coefficient at angle of incidence α

C_s shear coefficient for ultimate support shear for one-way elements

C_{sH} shear coefficient for ultimate support shear in horizontal direction for two-way elements*

C_{sV} shear coefficient for ultimate support shear in vertical direction for two-way elements*

C_S deflection coefficient for midpoint of short side of interior flat slab panel

C_u impulse coefficient at deflection X_u ($psi\text{-}ms^2/in.^2$)

C'_u impulse coefficient at deflection X_m ($psi\text{-}ms^2/in.^2$)

C_v shear coefficient for ultimate shear stress in vertical direction for two-way elements*

* See note at end of symbols

C_x shear coefficient for the ultimate shear along the long side of window frame

C_y shear coefficient for the ultimate shear along the short side of window frame

C_L confidence level

C_1 (1) impulse coefficient at deflection X_1 (psi-ms^2/in.^{2})
(2) ratio of gas load to shock load

C'_1 impulse coefficient at deflection X_m (psi-ms^2/in.^{2})

C_2 ratio of gas load duration to shock load duration

d (1) distance from extreme compression fiber to centroid of tension reinforcement (in.)
(2) diameter (in.)
(3) fragment diameter (in.)

d' distance from extreme compression fiber to centroid of compression reinforcement (in.)

d_b diameter of reinforcing bar (in.)

d_c distance between the centroids of the compression and tension reinforcement (in.)

d_{cH} distance between the centroids of the horizontal compression and tension reinforcement (in.)

d_{co} diameter of steel core (in.)

d_{cV} distance between the centroids of the vertical compression and tension reinforcement (in.)

d_e distance from support and equal to distance d or d_c (in.)

d_i average inside diameter of explosive casing (in.)

d'_i adjusted inside diameter of casing (in.)

d_l distance between center lines of adjacent lacing bends measured normal to flexural reinforcement (in.)

d_p distance from extreme compression fiber to centroid of prestressed reinforcement (in.)

d_{sp} depth of spalled concrete (in.)

d_1 diameter of cylindrical portion of primary fragment (in.)

D (1) unit flexural rigidity (lb-in.)
(2) location of shock front for maximum stress (ft.)
(3) minimum magazine separation distance (ft.)
(4) caliber density ($lb/in.^3$)
(5) overall diameter of circular section (in.)
(6) damping force (lb.)
(7) displacement of mass from shock load (in.)

D_E equivalent loaded width of structure for non-planar wave front (ft.)

D_H maximum horizontal displacement of the ground surface (in.)

DIF dynamic increase factor

D_s diameter of the circle through centers of reinforcement arranged in a circular pattern (in.)

D_{sp} diameter of the spiral measured through the centerline of the spiral bar (in.)

DLF dynamic load factor

D_V maximum vertical displacement of the ground surface (in.)

e (1) base of natural logarithms and equal to 2.71828...
(2) distance from centroid of section to centroid of pre-stressed reinforcement (in.)
(3) actual eccentricity of load (in.)

e_b balanced eccentricity (in.)

$(2E')^{1/2}$ Gurney Energy Constant (ft./sec.)

E (1) modulus of elasticity
(2) internal work (in.-lbs.)

E_c modulus of elasticity of concrete (psi)

E_m modulus of elasticity of masonry units (psi)

E_s modulus of elasticity of reinforcement (psi)

f (1) unit external force (psi)
(2) frequency of vibration (cps)

f'_c static ultimate compressive strength of concrete at 28 days (psi)

f'_{dc} dynamic ultimate compressive strength of concrete (psi)

f'_{dm} dynamic ultimate compressive strength of masonry units (psi)

f_{ds} dynamic design stress for reinforcement (a function of f_y, f_u, and Θ) (psi)

f_{du} dynamic ultimate stress of reinforcement (psi)

f_{dy} dynamic yield stress of reinforcement (psi)

f'_m static ultimate compressive strength of masonry units (psi)

f_n natural frequency of vibration (cps)

f_{ps} average stress in the prestressed reinforcement at ultimate load (psi)

f_{pu} specified tensile strength of prestressing tendon (psi)

f_{py} yield stress of prestressing tendon corresponding to a 1 percent elongation (psi)

f_r reflection factor

f_s static design stress for reinforcement (psi)

f_{se} effective stress in prestressed reinforcement after allowances for all prestress losses (psi)

f_u static ultimate stress of reinforcement (psi)

f_y static yield stress of reinforcement (psi)

F
(1) total external force (lbs.)
(2) coefficient for moment of inertia of cracked section
(3) function of C_2 and C_1 for bilinear triangular load

F_o force in the reinforcing bars (lbs.)

F_E equivalent external force (lbs.)

F_D Drag force (lbs.)

F_F frictional force (lbs.)

F_L lift force (lbs.)

F_N vertical load supported by foundation (lbs.)

g acceleration due to gravity (32.2 ft./sec.^{2})

G shear modulus (psi)

h
(1) charge location parameter (ft.)
(2) height of masonry wall

h_n average clearing distance for individual areas of openings from Section 2-15.4.2

h_t center-to-center dimension of a closed rectangular tie along h (in.)

h' clear height between floor slab and roof slab

H
(1) span height (in.)*
(2) distance between reflecting surface(s) and/or free edge(s) in vertical direction (ft.)
(3) minimum transverse dimension of mean presented area of object (ft.)

H_c height of charge above ground (ft.)

H_s height of structure (ft.)

H_T height of triple point (ft.)

H_w height of wall (ft.)

H^c heat of combustion (ft.-lb./lb.)

H^d heat of detonation (ft.-lb./lb.)

i unit positive impulse (psi-ms)

i_a sum of blast impulse capacity of the receiver panel and the least impulse absorbed by the sand (psi-ms)

i_{ba} blast impulse capacity of receiver panel (psi-ms)

i^- unit negative impulse (psi-ms)

$\bar{i}_a$ sum of scaled unit blast impulse capacity of receiver panel and scaled unit blast impulse attenuated through concrete and sand in a composite element (psi-ms/lb.$^{1/3}$)

i_b unit blast impulse (psi-ms)

$\bar{i}_b$ scaled unit blast impulse (psi-ms/lb.$^{1/3}$)

$\bar{i}_{ba}$ scaled unit blast impulse capacity of receiver panel of composite element (psi-ms/lb.$^{1/3}$)

$\bar{i}_{bd}$ scaled unit blast impulse capacity of donor panel of composite element (psi-ms/lb.$^{1/3}$)

$\bar{i}_{bt}$ total scaled unit blast impulse capacity of composite element (psi-ms/lb.$^{1/3}$)

i_c impulse capacity of an element (psi-ms)

i_d total drag and diffraction impulse (psi-ms)

i_e unit excess blast impulse (psi-ms)

i_{fs} required impulse capacity of fragment shield (psi-ms)

* See note at end of symbols

i_g gas impulse (psi-ms)

i_r unit positive normal reflected impulse (psi-ms)

i_r^- unit negative normal reflected impulse (psi-ms)

$i_{r\alpha}$ peak reflected impulse at angle of incidence α (psi-ms)

i_s unit positive incident impulse (psi-ms)

i_s^- unit negative incident impulse (psi-ms)

i_{st} impulse consumed by fragment support connection (psi-ms)

I (1) moment of inertia (in.^{4}/in. for slabs) (in.^{4} for beams)
(2) total impulse applied to fragment

I_a average of gross and cracked moments of inertia (in.^{4}/in. for slabs) (in.^{4} for beams)

I_c moment of inertia of cracked concrete section (in.^{4}/in. for slabs) (in.^{4} for beams)

I_{ch} moment of inertia of cracked concrete section in horizontal direction (in.^{4}/in.)*

I_{cV} moment of inertia of cracked concrete section in vertical direction (in.^{4}/in.)*

I_g moment of inertia of gross concrete section (in.^{4}/in. for slabs) (in.^{4} for beams)

I_m mass moment of inertia (lb.-ms^2-in.)

I_n moment of inertia of net section of masonry unit (in.^{4})

I_s gross moment of inertia of slab (in.^{4}/in.)

I_{st} impulse consumed by the fragment support connection (psi-ms)

I_w gross moment of inertia of wall (in.^{4}/in.)

j ratio of distance between centroids of compression and tension forces to the depth d

k (1) constant depending on the casing metal
(2) effective length factor

k_v velocity decay coefficient

K (1) unit stiffness (psi/in. for slabs) (lb./in./in. for beams) (lb./in. for springs)
(2) constant defined in paragraph 2-18.2

* See note at end of symbols

K_e elastic unit stiffness (psi/in. for slabs) (lb./in./in. for beams)

K_{ep} elasto-plastic unit stiffness (psi/in. for slabs) (lb./in./in. for beams)

K_E (1) equivalent elastic unit stiffness (psi/in. for slabs) (lb./in./in. for beams)
(2) equivalent spring constant (lb./in.)

K_L load factor

K_{LM} load-mass factor

$(K_{LM})_u$ load-mass factor in the ultimate range

$(K_{LM})_{up}$ load-mass factor in the post-ultimate range

K_M mass factor

K_R resistance factor

KE kinetic energy

l charge location parameter (ft.)
(1) length of the yield line (in.)
(2) width of ½ of the column strip (in.)

l_d basic development length of reinforcing bar (in.)

l_{dh} development length of hooked bar (in.)

l_c length of cylindrical explosive (in.)

l_p spacing of same type of lacing bar (in.)

l_s span of flat slab panel (in.)

L (1) span length (in.)*
(2) distance between reflecting surface(s) and/or free edge(s) in horizontal direction (ft.)

L_{cyl} length of cylinder (in.)

L_f length of fragment (in.)

L_H clear span in short direction (in.)

L_1 length of lacing bar required in distance s_1 (in.)

L_L clear span in long direction (in.)

L_o embedment length of reinforcing bars (in.)

* See note at end of symbols

L_s length of shaft (in.)

L_u unsupported length of column (in.)

L_w wave length of positive pressure phase (ft.)

L_w^- wave length of negative pressure phase (ft.)

L_x clear span in long direction (in.)

L_y clear span in short direction (in.)

L_{wb}, L_{wd} wave length of positive pressure phase at points b and d, respectively (ft.)

L_1 total length of sector of element normal to axis of rotation (in.)

m (1) unit mass (psi-ms^2/in. for slabs) [beams, (lb./in-ms^2)/in.]
(2) ultimate unit moment (in.-lbs./in.)
(3) mass of fragment (lbs.-ms^2/in.)

m_a average of the effective elastic and plastic unit masses (psi-ms^2/in. for slabs) [beams, (lb./in-ms^2)/in]

m_e effective unit mass (psi-ms^2/in. for slabs) [beams, (lb/in-ms^2)/in]

m_{sp} mass of spalled fragments (psi-ms^2/in.)

m_u effective unit mass in the ultimate range (psi-ms^2/in. for slabs) [beams, (lb/in-ms^2]/in.]

m_{up} effective unit mass in the post-ultimate range (psi-ms^2/in.)

M (1) unit bending moment (in.-lbs./in. for slabs) (in.-lbs. for beams)
(2) total mass (lb.-ms^2/in.)
(3) design moment (in.-lbs.)

M_e effective total mass (lb.-ms^2/in.)

M_u ultimate unit resisting moment (in.-lbs./in.^{2} for slabs) (in.-lbs. for beams)

M_u^- ultimate unit rebound moment (in.-lbs./in. for slabs) (in.-lbs. for beams)

M_c moment of concentrated loads about line of rotation of sector (in.-lbs.)

M_A fragment distribution factor

M_E equivalent total mass (lb.-ms^2/in.)

M_{HN} ultimate unit negative moment capacity in horizontal direction (in.-lbs./in.)*

M_{HP} ultimate unit positive moment capacity in horizontal direction (in.-lbs./in.)*

M_{OH}, M_{OL} total panel moment for direction H and L respectively (in.-lbs.)

M_N ultimate unit negative moment capacity at supports (in.-lbs./in. for slabs) (in.-lbs. for beams)

M_P ultimate unit positive moment capacity at midspan (in.-lbs./in. for slabs) (in.-lbs. for beams)

M_{VN} ultimate unit negative moment capacity in vertical direction (in.-lbs./in.)*

M_{VP} ultimate unit positive moment capacity in vertical direction (in.-lbs./in.)*

M_1 value of smaller end moment on column

M_2 value of larger end moment on column

n (1) modular ratio
(2) number of time intervals
(3) number of glass pane tests
(4) caliber radius of the tangent ogive of fragment nose

N (1) number of adjacent reflecting surfaces
(2) nose shape factor

N_f number of primary fragments larger than W_f

N_u axial load normal to the cross section

N_T total number of fragments

p reinforcement ratio equal to A_s/bd or A_s/bd_c

p' reinforceaent ratio equal to A'_s/bd or A'_s/bd_c

p_b reinforcement ratio producing balanced conditions at ultimate strength

p_o ambient atmospheric pressure (psi)

p_p prestressed reinforcement ratio equal to A_{ps}/bd_p

p_m mean pressure in a partially vented chamber (psi)

p_{mo} peak mean pressure in a partially vented chamber (psi)

* See note at end of symbols

p_r average peak reflected pressure (psi)

p_H reinforcement ratio in horizontal direction on each face*

p_T total reinforcement ratio equal to $p_H + p_V$

p_V reinforcement ratio in vertical direction on each face*

p(x) distributed load per unit length

P (1) pressure (psi)
(2) concentrated load (lbs.)

P^- negative pressure (psi)

P_c critical axial load causing buckling (lbs.)

P_g maximum gas pressure (psi)

P_i interior pressure within structure (psi)

ΔP_i interior pressure increment (psi)

P_f fictitious peak pressure (psi)

P_{max} maximum average pressure acting on interior face of wall (psi)

P_o (1) peak pressure (psi)
(2) maximum axial load (lbs.)
(3) atmospheric pressure (psi)

P_r peak positive normal reflected pressure (psi)

P_r^- peak negative normal reflected pressure (psi)

$P_{r\alpha}$ peak reflected pressure at angle of incidence α (psi)

P_{RIB} maximum average pressure on backwall (psi)

P_s positive incident pressure (psi)

P_{sb}, P_{se} positive incident pressure at points b and e, respectively (psi)

P_{so} peak positive incident pressure (psi)

P_{so}^- peak negative incident pressure (psi)

P_{sob},P_{sod},P_{soe} peak positive incident pressure at points b, d, and e, respectively (psi)

P_u ultimate axial load at actual eccentricity e (lbs.)

P_x ultimate load when eccentricity e_x

* See note at end of symbols

P_y ultimate load when eccentricity e_y is present (lbs.)

q dynamic pressure (psi)

q_b, q_e dynamic pressure at points b and e, respectively (psi)

q_o peak dynamic pressure (psi)

q_{ob}, q_{oe} peak dynamic pressure at points b and e, respectively (psi)

r
(1) unit resistance (psi)
(2) radius of spherical TNT [density equals 95 lb./ft.^{3}] charge (ft.)
(3) radius of gyration of cross section of column (in.)

r^- unit rebound resistance (psi, for slabs) (lb./in. for beams)

r_{avail} dynamic resistance available (psi)

Δr change in unit resistance (psi, for slabs) (lb./in. for beams)

r_d radius from center of impulse load to center of door rotation (in.)

r_{DL} uniform dead load (psi)

r_e elastic unit resistance (psi, for slabs) (lb./in. for beams)

r_{ep} elasto-plastic unit resistance (psi, for slabs) (lb./in. for beams)

r_{fs} ultimate unit resistance of fragment shield (psi)

r_s radius of shaft (in.)

r_T tension membrane resistance (psi)

r_u ultimate unit resistance (psi, for slabs) (lb./in. for beams)

r_{up} post-ultimate unit resistance (psi)

r_1 radius of hemispherical portion of primary fragment (in.)

R
(1) total internal resistance (lbs.)
(2) slant distance (ft.)
(3) ratio of S/G
(4) standoff distance (ft.)

R_{eff} effective radius (ft.)

R_f
(1) distance traveled by primary fragment (ft.)
(2) distance from center of detonation (ft.)

R_g uplift force at corners of window frame (lbs.)

R_1 radius of lacing bend (in.)

R_t target radius (ft.)

R_A normal distance (ft.)

R_E equivalent total internal resistance (lbs.)

R_G ground distance (ft.)

R_u total ultimate resistance (lb.)

R_I, R_{II} total internal resistance of sectors I and II, respectively (lbs.)

s (1) sample standard deviation
(2) spacing of torsion reinforcement in a direction parallel to the longitudinal reinforcement (in.)
(3) pitch of spiral (in.)

s_s spacing of stirrups in the direction parallel to the longitudinal reinforcement (in.)

s_1 spacing of lacing in the direction parallel to the longitudinal reinforcement (in.)

S height of front wall or one-half its width, whichever is smaller (ft.)

S' weighted average clearing distance with openings (ft.)

SE strain energy

t time (ms)

Δt time increment (ms)

t_a any time (ms)

t_b, t_e, t_f time of arrival of blast wave at points b, e, and f, respectively (ms)

t_c (1) clearing time for reflected pressures (ms)
(2) average casing thickness of explosive charges (in.)

t'_c (1) adjusted casing thickness (in.)
(2) Clearing time for reflected pressures adjusted for wall openings (ms)

t_d rise time (ms)

t_E time to reach maximum elastic deflection (ms)

t_g fictitious gas duration (ms)

t_m time at which maximum deflection occurs (ms)

t_o duration of positive phase of blast pressure (ms)

t_o^- duration of negative phase of blast pressure (ms)

t_{of} fictitious positive phase pressure duration (ms)

t_{of}^- fictitious negative phase pressure duration (ms)

t_r fictitious reflected pressure duration (ms)

t_u time at which ultimate deflection occurs (ms)

t_y time to reach yield (ms)

t_A time of arrival of blast wave (ms)

t_{AG} time of arrival of ground shock (ms)

t_1 time at which partial failure occurs (ms)

T (1) duration of equivalent triangular loading function (ms)
(2) thickness of masonry wall (in.)
(3) toughness of material (psi-in./in.)

T_c thickness of concrete section (in.)

$\bar{T}_c$ scaled thickness of concrete section (ft./lb.$^{1/3}$)

T_g thickness of glass (in.)

T_H force in the continuous reinforcement in the short span direction (lbs.)

T_i angular impulse load (lb.-ms-in.)

T_L force in the continuous reinforcement in the long span direction (lbs.)

T_N effective natural period of vibration (ms)

T_{pf} minimum thickness of concrete to prevent perforation by a given fragment (in.)

T_r rise time (ms)

T_s (1) thickness of sand fill (in.)
(2) thickness of slab (in.)

T_{sp} minimum concrete thickness to prevent spalling (in.)

$\bar{T}_s$ scaled thickness of sand fill (ft./lb.$^{1/3}$)

T_u total torsional moment at critical section (in.-lbs.)

T_w thickness of wall (in.)

T_y force of the continuous reinforcement in the short direction (lbs.)

u particle velocity (ft./ms)

u_u ultimate flexural or anchorage bond stress (psi)

U shock front velocity (ft./ms)

U_s strain energy

v velocity (in./ms)

v_a instantaneous velocity at any time (in./ms)

v_b boundary velocity for primary fragments (ft./sec.)

v_c ultimate shear stress permitted on an unreinforced web (psi)

v_f maximum post-failure fragment velocity (in./ms)

v_f(avg.) average post-failure fragaent velocity (in./ms)

v_i velocity at incipient failure deflection (in./ms)

v_o initial velocity of primary fragaent (ft./sec.)

v_r residual velocity of primary fragment after perforation (ft./sec.)

v_s striking velocity of primary fragment (ft./sec.)

v_{tc} maximum torsion capacity of an unreinforced web (psi)

v_{tu} nominal torsion stress in the direction of v_u (psi)

v_u ultimate shear stress (psi)

v_{uH} ultimate shear stress at distance d_e from the horizontal support (psi)*

v_{uV} ultimate shear stress at distance d_e from the vertical support (psi)*

v_x velocity in x direction (in./ms.)

v_y velocity in y direction (in./ms.)

V (1) volume of partially vented chamber (ft.^{3})
(2) velocity of compression wave through concrete (in./sec.)
(3) velocity of mass under shock load (in./sec.)

* See note at end of symbols

V_d ultimate direct shear capacity of the concrete of width b (lbs.)

V_{dH} shear at distance d_e from the vertical support on a unit width (lbs./in.)*

V_{dV} shear at distance d_e from the horizontal support on a unit width (lbs./in.)*

V_f free volume (ft.^{3})

V_H maximum horizontal velocity of the ground surface (in./sec.)

V_o volume of structure (ft.^{3})

V_s shear at the support (lb./in., for panels) (lbs. for beam)

V_{sH} shear at the vertical support on a unit width (lbs./in.)*

V_{sV} shear at the horizontal support on a unit width (lbs./in.)*

V_u total shear on a width b (lbs.)

V_V maximum vertical velocity of the ground surface (in./sec.)

V_x unit shear along the long side of window frame (lb./in.)

V_y unit shear along the short side of window frame, (lbs./in.)

w applied uniform load (lbs.-in.^{2})

w_c (1) unit weight (psi, for panels) (lb./in. for beam)
(2) weight density of concrete (lbs./ft.^{3})

w_s weight density of sand (lbs./ft.^{3})

W (1) design charge weight (lbs.)
(2) external work (in.-lbs.)
(3) width of wall (ft.)

W_A weight of fluid (lbs.)

W_{ACT} actual quantity of explosives (lbs.)

W_c total weight of explosive containers (lbs.)

W_E effective charge weight (lbs.)

W_{E_g} effective charge weight for gas pressure (lb.)

W_{EXP} weight of explosive in question (lbs.)

W_f weight of primary fragment (oz.)

* See note at end of symbols

$\overline{W}_f$ average fragment weight (oz.)

W_F weight of frangible element (lb./ft.^{2})

W_{CI} weight of inner casing (lbs.)

W_{co} total weight of steel core (lbs.)

W_{CO} weight of outer casing (lbs.)

W_{c1}, W_{c2} total weight of plates 1 and 2, respectively (lbs.)

W_s width of structure (ft.)

WD work done

x yield line location in horizontal direction (in.)*

X (1) deflection (in.)
(2) distance from front of object to location of largest cross section to plane of shock front (ft.)

X_a any deflection (in.)

X_c lateral deflection to which a masonry wall develops no resistance (in.)

X_{DL} deflection due to dead load (in.)

X_e elastic deflection (in.)

X_E equivalent elastic deflection (in.)

X_{ep} elasto-plastic deflection (in.)

X_f maximum penetration into concrete of armor-piercing fragments (in.)

X_f' maximum penetration into concrete of fragments other than armor-piercing (in.)

X_m maximum transient deflection (in.)

X_p plastic deflection (in.)

X_s (1) maximum penetration into sand of armor-piercing fragments (in.)
(2) static deflection (in.)

X_u ultimate deflection (in.)

* See note at end of symbols

X_1 (1) partial failure deflection (in.)
(2) deflection at maximum ultimate resistance of masonry wall (in.)

y yield line location in vertical direction (in.)*

y_t distance from the top of section to centroid (in.)

Z scaled slant distance (ft./lb.$^{1/3}$)

Z_A scaled normal distance (ft./lb.$^{1/3}$)

Z_G scaled ground distance (ft./lb.$^{1/3}$)

α (1) angle formed by the plane of stirrups, lacing, or diagonal reinforcement and the plane of the longitudinal reinforcement (deg)
(2) angle of incidence of the pressure front (deg)
(3) acceptance coefficient
(4) trajectory angle (deg.)

α_{ec} ratio of flexural stiffness of exterior wall to flat slab

α_{ecH}, α_{ecL} ratio of flexural stiffness of exterior wall to slab in direction H and L respectively

β (1) coefficient for determining elastic and elasto-plastic resistances
(2) particular support rotation angle (deg)
(3) rejection coefficient
(4) target shape factor from figure 2-212

β_1 factor equal to 0.85 for concrete strengths up to 4,000 psi and is reduced by 0.05 for each 1,000 psi in excess of 4,000 psi

γ coefficient for determining elastic and elasto-plastic deflections

γ_p factor for type of prestressing tendon

δ moment magnifier

δ_n clearing factor

Δ deflection at sector's displacement (in.)

ϵ'_c average strain rate for concrete (in./in./ms)

ϵ_m unit strain in mortar (in./in.)

ϵ'_s average strain rate for reinforcement (in./in./ms)

ϵ_u rupture strain (in./in./ms)

* See note at end of symbols

θ (1) support rotation angle (deg)
(2) angular acceleration (rad/ms^2)

θ_{max} maximum support rotation angle (deg)

θ_H horizontal rotation angle (deg)*

θ_V vertical rotation angle (deg)*

λ increase in support rotation angle after partial failure (deg)

μ (1) ductility factor
(2) coefficient of friction

ν Poisson's ratio

ρ (1) mass density ($lbs.-ms.^2/in.^4$)
(2) density of air behind shock front ($lbs/ft.^3$)

ρ_a density of air ($oz./in.^3$)

ρ_c density of casing ($oz./in.^3$)

ρ_f mass density of fragment ($oz./in.^3$)

ρ_o mass density of medium ($lb.-ms.^2/in.^4$)

σ_u fracture strength of concrete (psi)

Σ_o effective perimeter of reinforcing bars (in.)

ΣM summation of moments (in.-lbs.)

ΣM_N sum of the ultimate unit resisting moments acting along the negative yield lines (in.-lbs.)

ΣM_p sum of the ultimate unit resisting moments acting along the positive yield lines (in.-lbs.)

τ_s maximum shear stress in the shaft (psi)

ϕ (1) capacity reduction factor
(2) bar diameter (in.)
(3) TNT conversion factor

ϕ_r assumed shape function for concentrated loads

$\phi(x)$ assumed shape function for distributed loads free edge

ω angular velocity (rad./ms)

* See note at end of symbols

═══════	simple support
///////	fixed support
XXXXXXX	either fixed, restrained, or simple support

* Note. This symbol was developed for two-way elements which are used as walls. When roof slabs or other horizontal elements are under consideration, this symbol will also be applicable if the element is treated as being rotated into a vertical position.

APPENDIX 4C

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